Heat Transfer

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Lecture - 06
Temperature Distribution in Radial Systems

In the last class we discussed about what would be the resistance for heat flow through a planar wall or a combination of several plane walls. And, there we saw that the temperature difference between two ends of the wall can be viewed as the potential difference which causes the heat to flow from high temperature to low temperature. And the resistance if the flow heat flow can be expressed in the form which is similar to that of ohms law where the potential difference is to be replaced by the temperature difference and the current is replaced by the heat flow. And whatever we have in the denominator of q equals delta T by r, this r is equivalent to thermal resistance.

And the thermal resistance we have evaluated for a plane wall. And also if we have several such walls sandwiched one after the other then the overall temperature difference between the edge between the outer edge of wall 1 and the outer edge of wall n this temperature difference is the overall temperature difference overall potential gradient. And the denominator is going to be a sum of all the individual resistances of each of these walls.

So, in essence the heat flow through a composite wall where cross sectional areas remain the same at steady state can be expressed as heat flow equal to the overall temperature difference by the sum of all resistances. And therefore, all these resistances it has been shown that they are in series.

So, based on that we will now and we clearly understood that what is the resistance of the plane wall it is going to depend on the geometry; that means, what is a cross sectional area, what is the thickness and it also going to depend on the thermo physical property which is the thermal conductivity of the specific wall that we are considering. So, the resistance is simply going to be L, where L is the thickness divided by k times a k the thermal conductivity of the material and a is the floor the area which is perpendicular to the direction of heat flow.

We have also seen that for a composite walls of different type in which you have wall number 1 whose length whose area is a then another wall of different thermal conductivity whose area is let us say a by 2 and another wall, wall number 3 whose area is a by 2, but it is of another material. So when such walls, composite walls of unequal cross sectional area are together then there are 2 ways by which you can represent the heat flow; one is you can assume that there is going to be a series there is going to be the top 3 are going to be in series whereas the top and the bottom series are going to in parallel. So, we discussed about that as well.

One of the characteristic features of heat flow through plane wall is that the heat flow is constant not only heat flow at steady state at steady state the heat flow is constant and not only the heat flow the heat flux is also constant since the cross sectional area for which is perpendicular to the direction of heat flow does not change. So, as in planar systems the cross sectional area does not change with x with the direction of heat flow therefore, both the heat rate and the heat flux are going to be constant.

However, I have mentioned in the previous class that for radial systems and for spherical systems in which as you move in the direction of r the cross sectional area keeps on changing. And therefore, even though the conservation equation demands that heat rate has to be a constant the flux may not be a constant. And therefore, we are going to have a difference different expression for the resistance to heat flow for the case of radial systems and for spherical systems. So, that is what we are going to we are going to analyze in today’s class.

So, it is going to be temperature distribution in radial temperature distribution at steady state in radial flow of heat and the mode of heat transfer is conduction, but you can always have convection at the outer or the inner edges of a radial of a wall. And therefore, it in that case it is going to become a convection conduction problem, but for the time being let us start first with conduction only case in a radial system.

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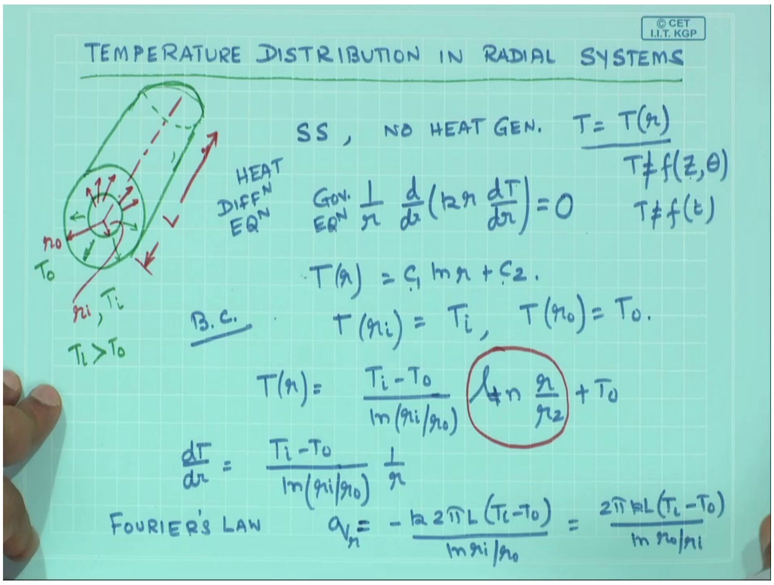

So, the figure that you see over here is that of a that of a hollow cylinder whose inner radius is equal to ri and the outer radius is r naught and the temperature over here in at the inside is maintained at T i whereas, the temperature at the outside is maintained at T o. So, this is a situation the length of the cylinder hollow cylinder is L and we are trying to find out what is the flow of heat through this area to outside. So, if we assume that T i is greater than T o if T i is greater than T o then this is the radial direction in which heat will flow through the solid material of the cylinder and we are trying to find out what is going to be the resistance for heat flow.

For this case for steady state condition the for steady state with no heat generation and the T is equal to as you can see is going to be a function of r only it is definitely not a function of theta and it is not a function of z. So, T is a function of r only therefore, the heat diffusion equation we for radial systems reduces to only this much, because we do not have to consider the variation of temperature with r or theta. So, T is not a function of either z or theta. So therefore, all the terms containing gradient of T with respect to z or with respect to theta can be cancelled it is a steady state one. So, this not T is not a function. So, T is not a function of time and it is no heat generation. So, therefore, if you look at the heat diffusion equation this is the form it is going to take ok.

And one can integrate this equation and what you get is T r equals C 1 ln r plus C 2 and the 2 boundary conditions this is the governing equation. So, this is your governing equation and in order to solve the governing equation you need 2 boundary conditions which are the 2 known temperatures in this case. So, T at ri is equal to T i and the second is T at r naught is equal to T naught. So, when you put these 2 boundary conditions to evaluate C 1 and C 2 the final expression, but you are going to get is T i minus T naught by ln ri by r 0 plus times ln r by r 2 plus T 0. So here, what you I think it we should appreciate is that the temperature distribution unlike the case of planar system is not a linear function of position, but it is going to be a logarithmic function of the radial location of the (Refer Time: 09:14).

So, what is so, that is very important in to find to note that it is no longer going to be linear distribution it is going to be logarithmic distribution, and as you move in the direction towards the outer radius the area of heat flow the area available for heat flow continuously increases. So, since it increases the temperate the temperature is not going to be linear and temperature here you can see it is going to be linear function of position.

Now, what is remaining is since we are trying to correlate cause and effect, the cause being the temperature difference and the effect being the heat flow and we are trying to find a relation or express our results in the form of something similar to ohms law between cause and effect. So, if you do that to this equation in the automatically the resistance to conductive heat transfer in radial systems will come out. So, that would be our next exercise to see what form the resistance would take in a radial system.

So, if you look at this equation once again then I can write this from this equation I can write d T d r is equal to T i minus T naught by ln ri by r naught times 1 by r. So, with this using Fourier's law the q is simply going to be q in the radial direction is simply going to be minus k 2 pi L L being the length of the cylinder times T i minus T naught by ln ri by r naught or it is 2 pi k L T i minus T naught by ln r 0 by ri.

So, if I take this to the next logical step and here you see this is the effect and this is the cause. So, if I bring twice by k L in the denominator what you have is something similar to ohms law, where the cause is the potential difference is a thermal potential difference which is expressed as temperature difference, and the effect is the radial flow of heat. And what I have in the denominator ln r naught by ri by twice by k L that simply is the resistance to heat transfer for radial systems.

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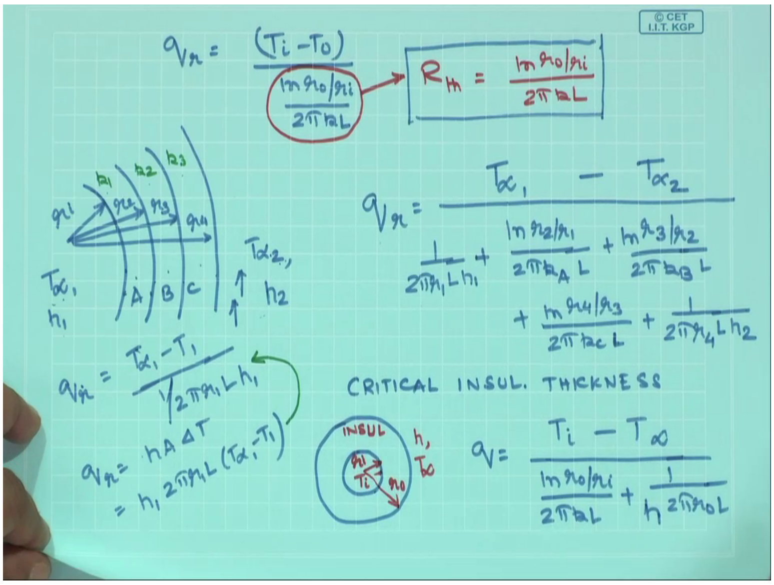

So, I am going to write in the next page as temperature difference. So, what you have here in the denominator is your resistance the thermal resistance which; obviously, is this. So, this is an important result which gives us what would be the resistance to heat transfer in such case. In this session that you have a composite situation in which you have a number of walls like this, so this is r 2. Similarly have r 3 and r 4 with each one having thermal conductivity of k 1, k 2 and k 3 and you also have convection inside as well as convection outside, with the inside condition given as the temperature of the of the fluid in here is T infinity 1 which is creating a convection coefficient of h 1 at the inner surface of the composite wall.

On the other side, you have T infinity 2 as the temperature of the fluid which is made to flow on the outside which is also producing heat thermal convective heat transfer coefficient of h 2. This is quite common in many of the practical situations for example, in heat exchangers which we would see where the hot fluid let us assume that it is flowing through a tube and somehow we it is going to come in contact in thermal contact with a cold fluid with which it will exchange heat. And therefore, the cold heat is going to be heated going to absorb going to gain energy out of this hot fluid.

And let us see hot fluid, fluid is flowing from point a to point b from where it is produced to where let us it is going to a reactor. So, in the transit you want to maintain the heat loss to a minimum you want to keep the heat loss to a minimum. So, how do you do that, you put insulation on top of the on top of the pipe top of that tube. So, this insulation the pipe and the insulation they are may be 2 or 3 different types of insulation one is and insulation which is thermally which is going to protect heat loss which is going to minimize heat lost and in order to protect the insulation you may have another outer cover.

So, as the heat travels from the inside to the outside it is going to experience different materials as it travels, and it is going to also see materials of different thermal conductivity. And as it travels it will see that the heat transfer area will keep on changing. So, under circumstances what you are going to get is the picture that I have drawn where you have a composite wall where at the in the inside you have some convection coefficient. And at the outside you have a different convection coefficient which is probably provided by the ambient air which flows over the over the lagged pipe.

So, what would happen to such a case is in is depicted over here and I am simply going to write what is the form of the final equation going to be. So, the radial flow of heat is going to be due to the temperature difference, the temperature difference inside minus temperature difference outside. So, what you have is T infinity 1 minus T infinity 2 that is the call that is the reason why the heat is flowing divided by the sum of all resistances.

So, what is the resistance for the first one as I have as we have discussed before over here. So, it simply going to be ln r 2 by r 1 divided by twice pi k of material let say this is A B and C, k A times L. And then the second one would take over. So, this is going to be a ln r 3 by r 2 by twice pi k B times l plus ln r 4 by r 3 by twice by k C into L.

So, this are the 3 conduction resistance through the material A B and C, I also has a convection which is taking place over here. So, what is the conviction the convective heat transfer q r between this point and this point in simply going to be T infinity 1 minus T, this is T 1 divided by 1 by twice pi r 1 1 by twice pi r 1 times length times h. This comes from Newton’s law of cooling with simply says that q r is h A times delta T and in this case h is h 1 times area is twice pi r 1 L, because this is the area which is exposed to convection environment times T infinity 1 minus T 1.

So, from here I have written it in the form of temperature difference by some sort of a resistance. So, this is the expression for convective heat transfer resistance in the case of radial systems. So, this is must be added to this overall resistance twice pi r 1 L h 1 plus and over here it is going to be 1 by twice pi the last r 4 L times h 2. So, I think it is clear to you now is that what happens in the in this case where you have multiple walls radial walls one after the other of different thermal conductivities the inside is exposed to a convection environment the outside is also exposed to another convection environment with 2 different temperatures and 2 different heat transfer coefficient.

So, the heat flow which has to remain constant in order to maintain equation of conservation of energy is the overall temperature difference which is T infinity 1 minus T infinity 2 by the sum of all resistances and the by the what you mean by the sum of all resistance is the convection resistance on the inside all the possible conduction resistance is that the heat flow is going to face. So, is there are 3 walls there is going to be 3 different conduction resistances and when it reaches the outer edge of the outer wall it experiences a convection once again. And from Newton's law of cooling we have seen how we can express the heat flow as temperature difference divided by a resistance and the convective heat transfer resistance is expressed as 1 by h A. And the area simply is going to be twice pi r times length that is the cross section that is the that is the inner area of the inner cylinder.

So, you this way you can quite easily write what would be the form of the equation for radial flow in systems and as I mentioned they are very common in many industrial situations. Now what I would do next is something very interesting is normally what we feel is that, if I am feeling cold I just wear a sweater which is nothing but an installation and this reduces the law loss of heat from my body to the ambient and I do not feel cold anymore.

So, in the way the purpose of the sweater or the jacket is to ensure that the heat flow from my body to the ambient get slow that is what is, that is what insulations do they will use the flow of heat from one from the hot object to the cold object, but is it ever possible that by adding and insulation your making more or higher flow of heat from the hot object to the cold object.

So, this is counter intuitive, but it may happen and I am going to show how under what conditions by adding insulations you simply increase the rate of flow of heat through that insulation. So, that is a very interesting concepts it is known as the critical thickness of insulation. So, if your thickness of insulation is below a certain limit then by adding insulation you increases heat flow, if it is above the critical insulation thickness by adding insulation what you get is what we commonly expect that the heat flow rate reduces, but the interesting part is that region in which your sizes are is your size is below that of critical thickness of insulation so, by adding insulation you are increasing the loss of heat.

So, you would first mathematically see what needs to happen for you to decide; what is this critical thickness of insulation. And then we will talk about some of the practical uses practical situations in which you are may expect to encounter critical thickness of insulation and it has certain advantages as well and what is the scientific reason for such thing to happen. So, let us look at the derivation once again. So, next we I go to critical thickness of insulation let say I have a cylinder and a solid cylinder and a insulation. So, this is ri and let us assume that the temperature here is maintained at T i temperature at the junction between the solid rod and the insulation. So, this is your installation and the insulation radius is r naught and on the outside it is exposed to a convection environment with temperature and heat transfer coefficient is h and T infinity.

So, it is a solid rod of radius ri the junction temperature is at T I, the insulation radius is r naught and it is exposed to h and T infinity. So, as before from here I can write that the heat flow is simply going to be T i minus T infinity divided by the resistance of heat flow through this which is ln r naught by ri exactly following what we have done before divided by twice pi k L where L is the length of the of the cylinder plus I have the convective resistance which is 1 by h twice pi r naught L.

So, there is nothing new I have simply use this for a system in for this case. So, if this is q then I am going to see is it possible to mathematical get at which point my heat flow is going to be maximum. So, if I can do that then I will probably get an idea of what is what is there some the concept of critical thickness of insulation.

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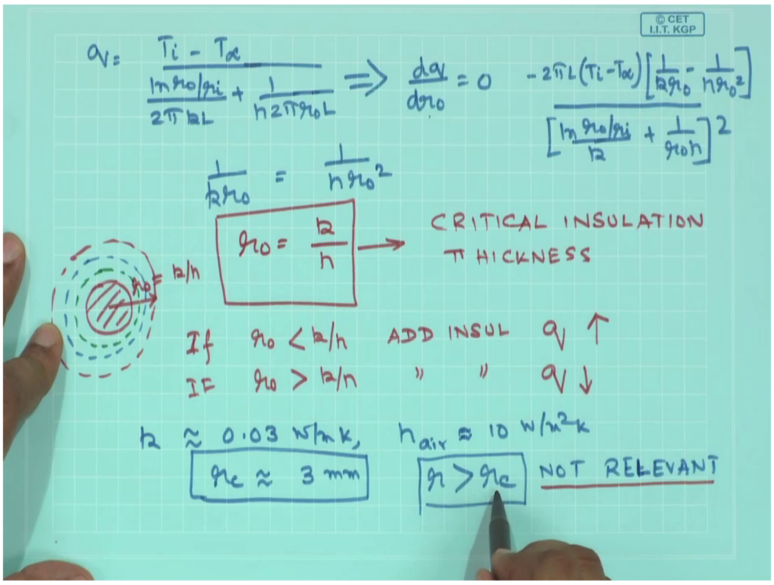

So, starting with this q which is I will write it again T i minus T infinity by ln r naught by ri twice pi k L plus 1 by h 2 pi r naught L I am going to differentiate this and set this to be equal to 0. So, I am trying to find out is there a radius r 0 where r 0 is the insulation thickness which would maximize the flow of heat which is denoted by q.

So, if do this then what I am going to get is, twice pi L T i minus T infinity 1 by k r 0 minus 1 by h r 0 square divided by this is straight forward maths it is nothing away. So, for this to be 0 1 by k r 0 must be equal to 1 by h r 0 square and what you get r 0 is equal to k by h. This r 0 the critical the thickness of the insulation which would maximize heat flow is known as the critical insulation thickness.

So, this critical insulation thickness therefore, it tells you that if you have a solid cylinder and you are putting insulations around it. So, this is your first level of insulation this is a and then at the next instant you increase the thickness of insulation some more and you are going to increase the thickness even more as long as your radius of this as you are adding. So, if this let say this is r 0 which is equal to k by h as long as you are below this value you are less than k 0 by h your q will increase. So, if r 0 is less than k by h add insulation q will increase, if r 0 is greater than k by h add insulation and q will decrease.

So, this is the concept of critical insulation thickness. So, there exists thickness when if you add insulation the heat should the heat flow will increase go beyond that and you add insulation and the expected things thing will have will take place that is the heat flow will decrease the why it should happen this can only happen when the thickness is very small.

So, when the thickness is very small by adding another layer of insulation you are increasing the resistance for flow of heat through that added layer of insulation; that means, you are increasing the conduction resistance by making the layer thicker; however, with putting the layer of insulation on the outside you are having you are making more area available for conduction because the area available for conduction is simply twice pi r multiplied by length, length is a constant. So, as r increases your area available for conviction increases. So, there is a there are and the, with increase in r the conduction resistance will increase ok.

So, the heater convective heat transfer is helped by adding insulation because of the additional area conduction heat transfer is going to be going to be reduced by adding the insulation. So, that these are 2 parallel mechanisms with compete with each other and for certain value of r as we has been previously equal to k by h if the result is you get the maximum heat transfer.

So, if you are below this value of k by h, increase the insulation thickness, increase the area available for convection and that more than offsets the additional conduction resistance that you put in the system that is what the concept of critical insulation thickness is. So, let us just workout with those since r is equal to k by h what are the typical values of k by h and that would tell us something about when do you expect this concept of critical insulation thickness to be when you are going to encounter search concepts.

So, let us write what are the typical values of k and the typical values of h and then we will know what this is. So, k is for an insulating k for insulation material is left about 0.0 3 watt per meter Kelvin and the h the convection over here is mostly going to be the convection in air which is the order of 10 watt per metre square per Kelvin. So, r c the critical thickness of insulation is going to be about 3 millimeters.

So, this your you are size has to be less than 3 millimeter for the concept for the occurrence of the phenomena associated with critical insulation thickness, most of the normal conditions you do not deal with 3 millimeter thin wire or 3 millimeter thin material. So, for most cases your r is going to be more than r c and therefore, the concept of critical thickness of insulation is not relevant.

So, concept of critical thickness you do not encounter the phenomenon associated with critical insulation thickness since the length scale involved is only about 3 millimeters. So, what we see what we say is that I increase insulation and I reduce heat transfer, but one example I can give you which would show the role critical thickness insulation thickness may play for system.

So, when you think of very thin words which conduct electricity ok, they can be less the diameter can be less than 3 millimeters. So, when current passes the heat is generated the ohmic heat is the joule, joule heat due to joule heat some are some amount of heat is generated and you want to dissipate that ok, but at the same time we do not want to live wear without any electrical insulation put on it.

So, what you do is on the thing where you put a layer of insulation, but what you get is something very interesting you are not only make it safe for the wire to be safe. Since you have put as electrical insulation this electrical insulation is going to act as a thermal insulation as well, because the heat that is generated is going to dissipate to the atmosphere by means of conduction through the electrical insulation and convection to the outside.

Now, if you radius of the electricity carry where is less than 3 millimeter, then you are enhancing the heat loss from the system by putting and insulation electrical insulation on top of it. So, you serve 2 purposes you cover the electrical wire with an installation an by doing. So, you are increasing the heat transfer and therefore, the wire itself now can be at a lower temperature which will probably prolong it is life and will be less hazardous.

So, critical thickness of insulation are most relevant in very thin electrical wires where you put an insulation on top of it, but for most of the practical systems since the dimension that we deal with are and the dimensions and more than 3 millimeters you do not get that in most of the applications, but concept wise it is very interesting that it is difficult to it is counter iterative when I say that by adding insulation I increase the flow of heat.

So, that is something which in this class we have seen that it is a peculiar nature of the phenomena which is a direct result of change in the heat transfer area with radius. As you go outward the area increases and therefore the heat rate remains the same, but heat flux will change.

So, this class we have seen what is radial system, how do we, what are the resistance is associated with radial systems, what is the peculiar phenomena of critical thickness of insulation. Then in next class we will very quickly go through what is going to be the equivalent form for spherical systems, and something which is known as the overall heat transfer coefficient. And then we will go into systems in which there is going to be generation of heat at steady state in what would be the form of temperature distribution for systems which are generating heat on their own.

So, that would be the topic of the next class.