Heat Transfer
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Lecture - 58
Tutorial Problem on Radiation Exchange using the Network Method

We have introduced the concept of network method for Radiation Exchange in an enclosure. And there we have seen that there are two resistances to deal with when we are considering the radiative Heat Transfer from a surface. One is the inherent resistance to radiation of the subject of the surface itself, which gives us the difference between the radiosity of the surface and the emissive potential had this surface been a blackbody.

So, the emissive power of the surface if it were a black body, and the radiosity which is the radiation flux which would be observed by a person whose standing just outside of the surface; these two are connected by the emissivity of the surface itself. And after that from that read from the radiosity, the energy is going to be exchanged with the surrounding surfaces, with the surfaces which other surfaces which form the enclosure. And there the fraction of energy emitted by the surface which is intercepted by the other surfaces, namely the concept of view factor came into play.

So, we had 2 resistances to consider, one is the surface resistance to radiative emission and the second is the exchange between two surfaces. So, one was 1 minus epsilon 1 by a 1 epsilon 1, where epsilon 1 is the emissivity of surface 1 and the other one the a 1 is the area of the surface. And in between two surfaces it is going to be 1 by a 1 f 1 2, which is the resistance which gives you the resistance of radiative heat transferred between two surfaces, where the view factor for the surface 2 from 1 is denoted by f 1 2.

And we can use the summation rule and the reciprocity rule to obtain the unknown values of the view factors for these enclosures. So, with that then we have proceeded to write, to draw, what would be the circuit diagram for 3 surfaces forming an enclosure. 2 surfaces forming an enclosure is straightforward, but the 3 surface one we have to think of 3 potentials at 3 ends, which are the black body, which are the emissive potential of these 3 surfaces if they were black bodies, then the radiosity at the 3 epics of the triangle which forms the circuit.

So, the radiosity of surface 1 that of 2 and of 3 and then these radio \cities are connected through the radiation exchange resistance between 1 and 2, which is 1 by a 1 f 1 2 between 1 and 3, which is 1 by a 1 f 1 3 and between 2 and 3, which is 1 by a 2 f 2 3. So, that completes the circuit and one should be able to able to obtain the heat flow rate into this enclosure by the standard techniques which I have discussed in the previous class.

In some cases we have something called a reradiating surface. So, whatever it gets, it radiates back the same amount, which would be true at steady state for the; if a surface is perfectly insulated. So, if the back side of the surface is perfectly insulated then, whatever it gets by the by the means of radiation from other surfaces it must radiate it back in order to maintain the constant temperature of the surface. Such a surface is known as the reradiating surface.

For such a surface, the surface emitting emissivity does not play any role. That means, the resistance between the black body emissive potential of the surface, if it is a black body and the radiosity the these two are equal and therefore, eb 3 if the surface 3 is the reradiating surface, then eb 3 would be equal to j 3.

So, the radiosity and the black body emissive potential for a radiating surface are equal moreover, if you can calculate what is j 3, which is equal to eb 3, then through the use of Stefan Boltzmann equation that is eb 3 is equal to sigma t 3 to the power 4, you can obtain what is the value of the unknown temperature of the surface.

So, for a reradiating surface, epsilon is equal to 1. There is no resistance, no surface resistance to radiation and the unknown temperature of the surface can be obtained simply through the use of Stefan Boltzmann equation and the substantial simplification of the entire assembly can be obtained. So, I think we have covered this much and if we now solve a problem then it would be even more clear to all of us.

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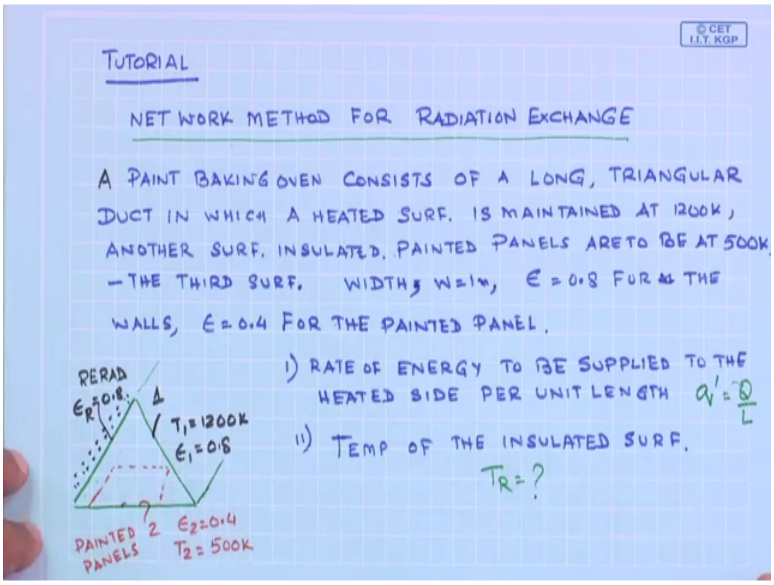

So, the problem that I have chosen is this for solving the network method radiation exchange. We have a paint baking oven, which consists of a long triangular duct. So, it looks something like this. It is an equilateral triangle. So, these three sides are equal and it is extended in this direction. So, this is a paint baking oven and this side one of the side a heated surface is maintained at 1200 degree Kelvin.

So, let us call this as surface 1. So, t this is 1; so t 1 is 1200 Kelvin and another surface is insulated. So, this surface is insulated, in let us call this as; and obviously if it is insulated as per our previous discussion, this has to be a reradiating surface. So, this is the re radiating surface and the epsilon value for this is provided to be equal to 0.8.

The value of epsilon for the other surface which is at 1200 degree Kelvin is also mentioned at 0.8. So, epsilon equal to.08 for both the walls and the paint baking, the paint panels, the panels are kept like this. So, this is my other surface, this is the surface 2 which is the painted panels are kept in here. In the epsilon 2 for this case is 0.4 and the temperature that needs to be maintained for proper baking of the paint t 2 must be kept at a 500 Kelvin.

So, as you can clearly see in order to maintain the temperature over here at 500 Kelvin also temperature over here at 1200 Kelvin, some heat must be supplied to the to the surface 1. So, this is a an equilateral paint baking paint baking oven, where the painted panels are kept at the bottom that I call as surface 2, the value of epsilon 2 t 2 are known.

This is the heated surface which is 1 surface 1 at 1200 degree epsilon is 0.8. This is perfectly insulated. So, as per our understanding, this is a reradiating surface. So, whatever it gets, it is must emit the same thing back. And therefore, the formula or the concept of radiating surface is applicable here.

So, what we need to know, the problem asks us to find out what is the rate at we heat must be supplied to surface 1 to maintain its temperature at 1200 degree Kelvin. So, that is the first part of the problem. So, what is the amount of heat that needs to be supplied to the surface at 1200 degree 1200 degree Kelvin to maintain its temperature at that value. So, that is the first part of the problem.

The second part of the problem tells us to find out, what is going to be the temperature of the insulated surface; that means, what is going to be t r. So, therefore, I here I need to find out what is q prime, which is the heat and the rate of energy to be supplied to the heated site per unit length. So, this must be Q by L, where Q l is the total energy supplied and L is the length of it. So, amount of heat to be supplied per unit length, let us call it as q prime and we would also like to find out what is going to be the value of TR.

So, the first thing that we need to do is convert this to the resistance diagram which we have shown before.

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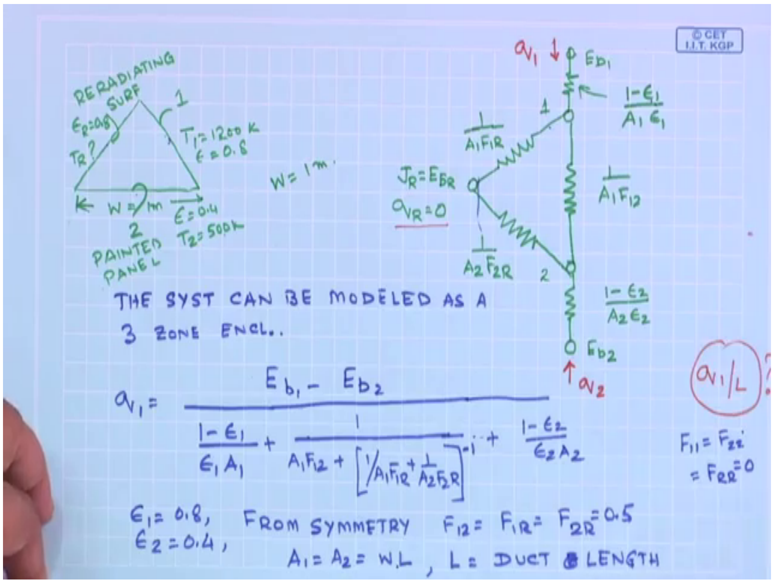

So, once again I will quickly draw this oven, where this is my surface 1. This is surface 2, which is the painted panel and this is the reradiating surface. The temperature here is given as 1200, the epsilon is 0.8, here epsilon is 0.4 and T 2 is 500 Kelvin, let us call the epsilon R over here is also 0.8, I do not know what is the value of TR in this.

So, let us convert this to the circuit diagram that we that we generally use. So, over here, this is the reradiating surface. Since it is a reradiating surface, JR must be equal to EBR that is the definition of reradiating surface. And since it is insulated, QR would be equal to 0, so we start from here and we realize that, these 2 relations are going to be valid. Then this is going to exchange radiation with 1, this is also going to exchange radiation with surface 2.

So, I have 2 resistances to consider here ok. So, let us call it as 1 and this is my 2, which is the painted panel. So, as per our previous discussion, this is simply going to be 1 by A 1 F 1 R and between 1 and 2, I also have radiation resistance. So, this is going to be 1 by A 1 F 1 2 between 1 and 2, and this 1 is going to be 1 by A 2 F 2 R. So, these are the free resistances inside the enclosure, but both 1 and 2 are not reradiating surfaces. So, they must have surface resistance to radiation.

So, this is going to be E b 2. The emissive potential of surface 2 had this been a blackbody at the same temperature and this the resistance over here, the surface resistance to radiation is simply going to be 1 by epsilon 2 by A 2 epsilon 2, in a similar fashion this is going to be E b 1 and the resistance over here is going to be 1 minus epsilon 1 by A 1 epsilon 1.

So, that completes the circuit. And let us assume, let some amount of heat is entering through this which is q 2, the amount of heat which is to be supplied or extracted. I do not know at this point what whether it is extracted or it is going to be supplied. But if you look at the 3, will look at the figure then obviously, 2 is going to receive some amount of energy from one. Since the temperature of 1 is significantly more than that of 2. So, obviously heat is to be extracted, but that then the way we have done it, if we are going to calculate the value of q 2, this should turn out to be negative such that heat is going to be taken out.

So, we are doing it in this way, it still does not matter because, we will get a negative value of q 2. And let us say the amount of heat which is to be supplied at 1 is q 1. So, at any point of time in order to maintain steady state, the algebraic sum of these 2 would be 0 and we understand that q R is therefore 0.

So, the system that we have over here, the system can be modeled as a 3 zone enclosure ok. So, the 1 that is to be supplied over here we said q 1, the heat that is supplied to surface one q 1 would simply be this potential, this is a potential difference. I have these 2 resistances in series. That means this and this resistance in series and these 2 are going to be in series with the equivalent of this resistance.

So, when we try to find out the equivalent of this resistance, so these 2 are in series, which in turn are parallel to this one. So, this has to be this has to be replaced by the equivalent resistance of this circuit, which is going to be in series with these 2. So, if you do a little bit of if you do it on your own, what you would see that the heat flow which is similar to that of current flow is going to be 1 minus epsilon 1 by epsilon 1 A 1, which is this plus in then what you have over here is this one, 1 minus epsilon 2 by epsilon 2 A 2 and the equivalent resistance of this, which is going to be 1 by A 1 F 1 2, which is this 1 plus 1 by A 1 F 1 R plus 1 by A 2 F 2 R whole to the power minus 1.

So, that is the standard formula. Then you are going to use epsilon 1 to be equal to 0.8 and this we would also be able to and epsilon 2 is equal to 0.4. And from symmetry, one can write that F12 is equal to F1R is equal to F2R is 0.5. So, that is what, so whatever heat since it is an equilateral triangle, whatever heat comes out of this is half of it the view factor for this one is going to be 0.5 and the view factor is going to be 0.5 over here as well, the view factor 2 itself since it is a plain surface is 0.

So therefore, F11 is equal to F22 is equal to F R, these are 0, since there plane surfaces. So, by symmetry F12 and F1R and F2R all are going to be equal to 0.5 and we also can say that A1 is equal to A2 is equal to W times L where, AW is the width and the width of this the W, the width of this is equal to 1 meter ok. So, W the width of this is equal to 1 meter. So, this is 1 meter, this one is 1 meter as well as this one. So, this is W times L, where L is equal to the duct length. And we have to find out what is q 1 by L? This is the one which we have to find out.

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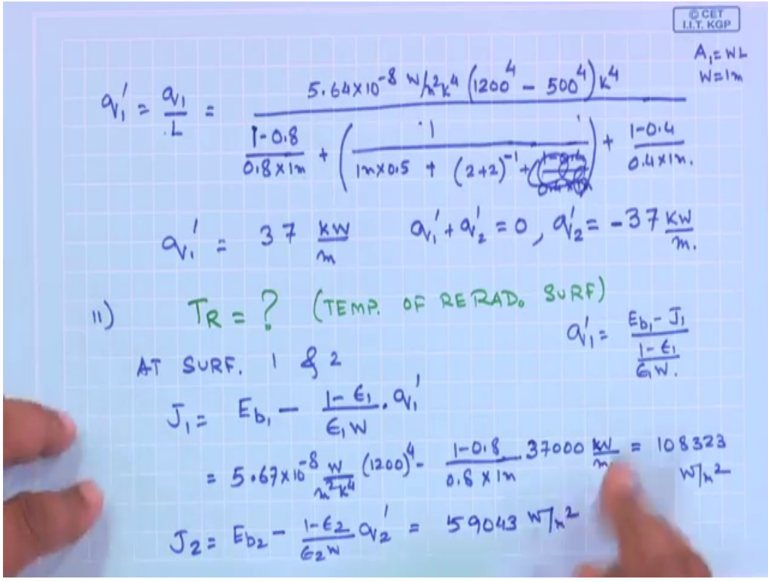

So, from this expression which is straightforward. We can proceed to obtain that q 1 prime, which is q 1 by L would simply be equal to when you put the values in here, the Boltzmann constant 10 to the power minus 8, so this Eb 1 is sigma t 1 to the power 4.

So therefore, this is 10 to the power minus 8 watt per meter squared Kelvin to the power 4 times 1200, that is the temperature in Kelvin minus 500 to the power 4 Kelvin to the power 4. So, what you have then is watt per meter square. And in the new in the denominator you have 1 minus epsilon 1 which is 0.8 by epsilon 1A1 So, this is 1 meter in the ln here has been brought over here. So, since this is A1 is equal to W times L and W is equal to 1 meter.

So, L has been brought on this side and we are left with W which is 1 meter plus the equivalent resistance which is 1 by A1 F1, so 1 meter into 0.5. So, A1F12 and when I take L out of this it is going to be W times F12 plus the equivalent resistance of the middle 1, which is this. And again A1 is simply equal to 1 meter. F1R is 0.5. So, 1 by 0.5 is equal to 2. So, 2 plus 2 to the power minus 1, and that is what I have written over here, 2 plus 2 to the power minus 1 plus 1 minus 0.4 that is the emissivity of the painted panels in the emissivity 0.4 into 1 meter. That is sorry, this ends over here plus this one comes 1 minus 0.4 by 0.4 into 1 meter ok.

So, this is the equivalent resistance of this 3. This is 1 minus epsilon 1A1 epsilon 1 and this is 1 minus epsilon 2A2 epsilon 2 and when you look at that I have written all these figures. So, q 1 prime is going to be, if you calculate these values 37 kilowatt per meter. And of course, if q 1 in this figure to maintain steady state, if q 1 is 37 kilowatt per meter, that amount of heat is to be supplied at surface 1. And since this is insulated, the same amount of heat is to be extracted out of the out of the surface which is forming the panels or surface 2.

So, q 1 plus q 2 should be equal to 0. And therefore, since q 1 prime plus q 2 prime at steady state must be equal to 0. q 2 prime, the heat the heat that is to be extracted would be equal to 37 kilowatt per meter with a minus sign then the second part. So, what is the second part tells us how to find out what is the value of the TR, the temperature of the reradiating surface. This has to be evaluated. Now in order to do that, what you have to do is first of all you have to find out what is the value of J1, the value of J2, then at this node, the total heat flow the sum of the algebraic sum of the heat flow must be equal to 0.

This is a same way we have we have done it in electrical technology. So, if I could find what is J1 and what is J 2, then the heat flow from here to here must be equal to the heat flow from here to here in the opposite direction. So, the heat flow from 1 to R would be would be J 1 minus JR divided by the resistance which is this. The heat flow from 2 to JR would be J 2 minus JR divided by this resistance.

So, these 2 must be equal and opposite in order to, in order to ensure that there is no net heat flow at this point. So, the first step for the second part of the problem is to find out the unknown values of J1 and J2. Now we know what is the heat flowing from here to here, what is the value of the resistance, what is the value of Eb1, so J1 can simply be calculated and that is what I am going to do next. So, for this case at surfaces 1 and 2, J1 would simply be equal to Eb 1 minus 1 minus epsilon 1 by epsilon 1 W times q 1 prime.

So, this is the heat flow. So, the heat flow would be the potential difference. So, what is I am saying is that q 1 prime must be equal to Eb 1 minus J 1 by the resistance, which is 1 minus epsilon 1 by epsilon 1 times W, this is per unit length. So, this is what we are going to do and when you put the value of Eb1, which is 5.67 into 10 to the power minus 8.

Putting the values 1 meter 37 kilo watt, so this is 37 watt per meter ok, so this you would see as 108323 watt per meter square. In a similar fashion, you should be able to find out

J 2 to be equal to Eb 2 minus which would be 59043 watt per meter square So, we have now evaluated what is the value of what is the value of J1, what is the value of J2. So, the unknown in this case is now, JR.

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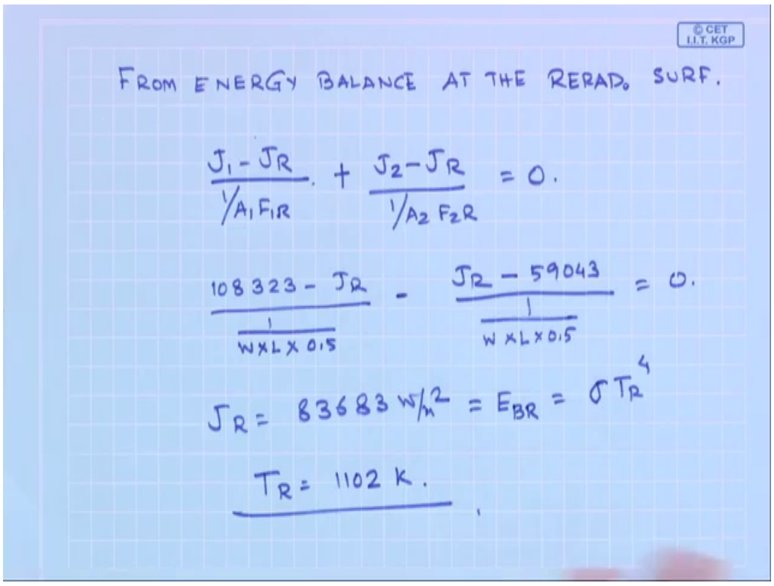

So, for that we are simply going to say that from energy balance at the reradiating surface J1 minus JR divided by the resistance. So it is 1 by F1R minus or other plus J2 minus JR by 1 by A2F2R must be equal to 0. Coming back to the figure once again, J 1 minus J 1 minus JR divided by the resistance plus J 2 minus JR divided by the resistance. The sum of these two in order to maintain steady state must be equal to 0.

So, when you when you expand we expand this what we are going to get is the value of J1 minus JR and this is 1 by W times L times 0.5 minus of JR, I just took the minus on this side, minus 59043 by 1 by W times L times 0.5 this is equal to 0. So, from this by to plugging in the value of all these WL etcetera. And this is just an equation in JR you should be able to obtain the value of jr to be 83683 watt per meter square. And for a reradiating surface JR is equal to EBR. So, this JR is equal to EBR.

And EBR is equal to sigma times TR to the power 4. It is a blackbody so obviously, there is no emissivity, emissivity is equal to 1. The only unknown in this is TR, which should be equal to 1102 Kelvin. So, this problem is a nice example of how to calculate the radiosity, how to calculate the overall resistance for heat transfer between surfaces.

What to do if we have a what simplification can we make if we have a real reradiating surface and if we can find out what is the J R radiosity of the reradiating surface which would be equal to the black body emissive potential for the for the if for the reradiating surface at the same temperature. So, if we have a blackbody at the same temperature as the reradiating surface. Then we from the since the radiosity and emissive power are equal for our reradiating surface. Therefore, Eb 3 in this case the Bb R that we have done for in this case would simply be equal to sigma times TR to the power 4. So, the unknown temperature of the reradiating surface can be quickly obtained following this method. So, this is one complete example tutorial problem on network method for radiative heat exchange in an enclosure.

In the next class I would solve one more problem. In that should and once you practice these problems from your text, it should clarify any doubts that you have. Otherwise I encourage you to contact me and the teaching assistants such that you have a clear idea of how to do, how to analyze the exchange of radiation between surfaces in an enclosure.