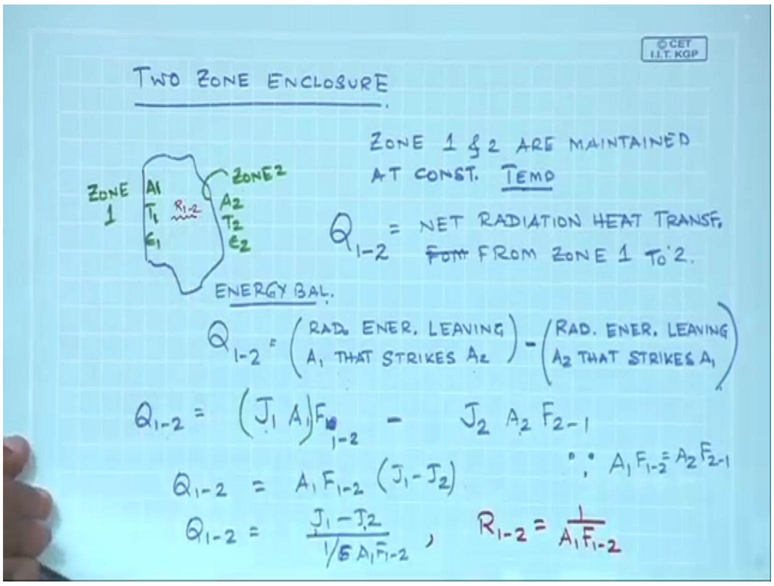
Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 57
Network Method – Two and Three Zone Enclosure

In the previous class, what we have seen is that there exist something called the surface resistance radiation which connects between the radiative potential of the surface had this been a blackbody and the radiosity of the surface which is the radiative flux just which is can be observed just outside of the surface. Based on these concepts we are going to start our first network method for a very simple case which is a two zone enclosure.

So, you have two surfaces which are forming an enclosure and we would like to find out what is the net heat exchange between these two surfaces which are forming the enclosure.

(Refer Slide Time: 01:03)



So, let us look at how this would look like. So, it is a two zone enclosure and this is my zone – 1, where the area is A i it is at a temperature of T A 1 temperature is at T 1 and the value of epsilon is epsilon 1. The rest is zone 2 which again has area A 2 the temperature T 2 and the emissivity is epsilon 2. So, zone – 1 and 2 are maintained at constant temperatures and we would like to find out what is the net radiative heat exchange between the two. So, let us called the net head radiative heat exchanged between 1 and 2 as Q 1-2 which is the net radiation heat transfer from zone 1 to 2.

So, the amount of heat transfer from 1 to 2 is called as Q 1-2, since we make an energy balance. So, from an energy balance from an energy balance I can write that Q 1 to 2 is the algebraic sum of radiation energy which is leaving A 1 that strikes A 2 this is one which is leaving this leaving this 1 minus the radiation energy living A 2 that strikes this must be the amount of net radiation heat transfer from 1 to 2. So, some amount of radiation radiative energy is coming out of A 1 and it is going to strike A 2. So, it is the radiative energy which is leaving A 1 that strikes A 2 and the other one is this is also the 2 is also emitting energy and a fraction of that energy may strike zone 1.

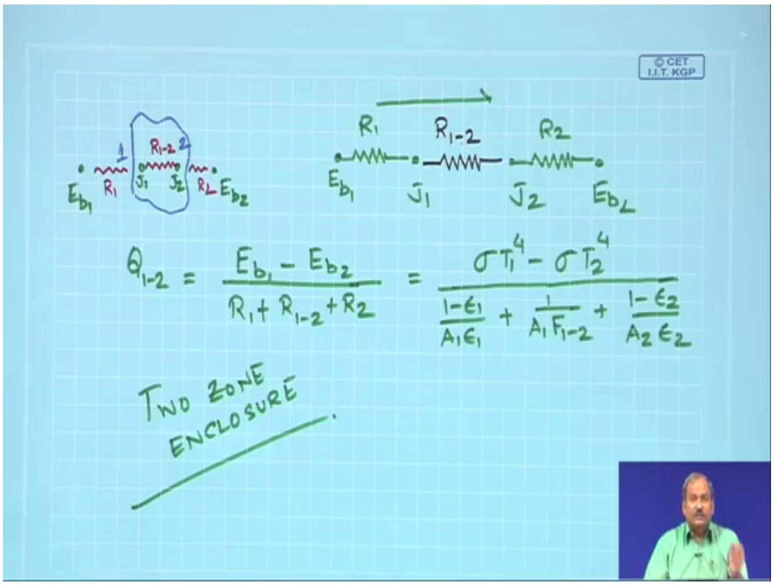
So, Q 1-2 which is the net radiative heat transfer from 1 to 2 must be equal to this. So, let us try to fill up the try to express this in terms of known quantities. So, what is the radiation energy leaving A 1? This is the radiosity; because radiosity is the potential of zone one which is seen by an observer who is standing just outside of A 1. So, therefore, the surface one has potential which the radiation energy which is coming out of surface one must be equal to J 1 now a fraction of that is going to strike the second surface. So, what is the fraction of that surface? It is going to be A 1 F 1 to 2 that is the view factor. So, A 1 F 1 to 2 multiplied by J 1 would give you the radiation energy leaving A 1 that strikes A 2.

Similarly, the radiation energy leaving A 2 is J 2 and what a fraction of that is going to strike surface 1. So, this must be equal to A 2 F 2 to 1, I will go through it once again. This is the net radiation heat transfer from 1 to 2. So, 1 to 2 must be equal to whatever be the radiation energy leaving A 1 that strikes A 2. The radiation energy which is leaving A 1 is this which is watt per meter square times meter square. So, this is the total radiation energy which is leaving surface 1. A fraction of that is going to strike 2, what would that fraction be? From the definition of view factor we know that F 1 2 which is from 1 to 2 would give us the amount of the fraction of energy emitted by 1 that strikes 2.

So, the amount of energy is J 1 A 1. So, the fraction that strikes 2 is J 1 A 1 times F 1-2 similarly the radiation energy which is leaving A 2 is J 2 multiplied by A 2. So, of this amount of energy this fraction is going to strike 1. So, the radiative energy leaving A 2 that strikes A 1 would simply be J 2 A 2 J 2 A 2 times F 2 to 1. So, we can we can we would we would also we can we can then take it a little bit further to be as A 1 F 1 to 2 times J 1 minus J 2 and we are simply used the reciprocity relation that A 1 F 1 to 2 should be equal to A 2 F 2 to 1. So, using the reciprocity relation I can express it in this way.

So, therefore, this is Q 1 to 2 is simply equal to J 1 minus J 2 by 1 by F A 1 F 1 to 2. So, this A 1 F 1 to 2 is nothing, but the resistance which is 1 by A 1 F 1 to 2. So, J 1 and J 2 are the radiosities of surface 1 and surface 2. These two are the potentials and from the fundamental equation we can see that these 2 potentials are to be divided by one by A 1 F 1-2 to obtain what is the net heat flow from 1 to 2. So, if this is the current, this is the potential difference then this denominator must be equal to some sort of a resistance which is defined as one by A 1 F 1-2 and this is the resistance between 1 and 2 is R 1 to 2.

(Refer Slide Time: 08:44)



So, when we think of the surface when we think of this surface. Once again this is my 1 and this is my 2 which in between I have R 1 to 2, the potential at this point is J 1 the potential at this point is J 2 and if I have assumed this to be blackbodies with potentials at E b 1 and E b 2 then the resistance in between these two is what we have what we have defined before. So, if this is R 1 and this is R 2 then I can write this draw the circuit diagram as E b 1, this is my J 1, this is my J 2 and this is E b 2. They are connected by a resistance which is the surface resistance to radiation for surface 1, this is R 2 which is the surface resistance to radiation for surface 2 and in between I have R 1-2 with and we already know what is what is R 1 and what is R 2.

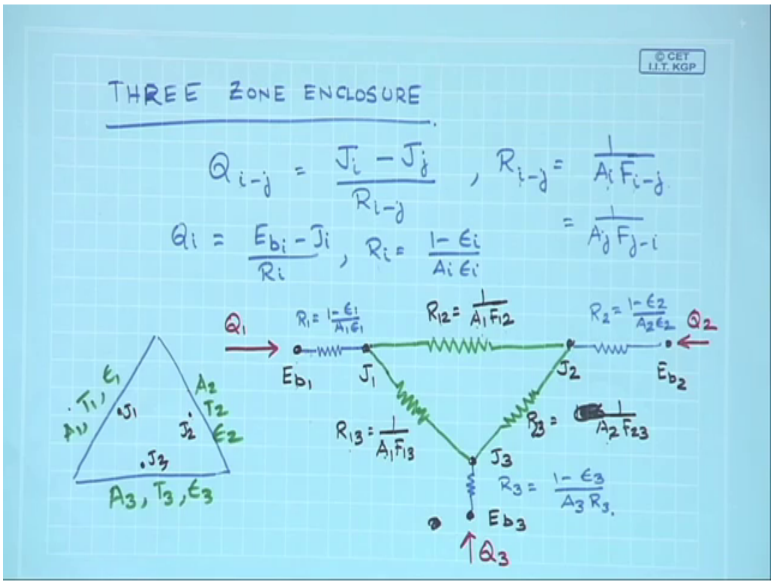
So, I can write this as E b 1 minus E b 2 these are the potential difference. So, the heat flow from 1 to 2 must be equal to the potential difference by the sum of these three resistances which are in series. So, R 1 plus R 1 to 2 plus R 2 and when you substitute the form for this is going to be sigma T 1 to the power 4. These are blackbody potentials minus sigma T 2 to the power 4 and when you plug in the expression for R 1 it is as we have done in the last class 1 minus epsilon 1 by A 1 epsilon 1 plus for the case of R 1 2 as we have seen in this case R 1 to 2 is simply A 1 F 1 2. So, this is going to be 1 by A 1 F 1 to 2 plus for the case of R 2 it is going to be 1 minus epsilon 2 by A 2 epsilon 2.

So, this gives you the amount of heat which is to be supplied from this is from 1 to 2 which is Q 1 to Q 2. This concept this is so, this is for the relation for a two zone enclosure. The concept of two zone enclosure which is straight forward can now be extended for more realistic situations like three zone enclosure. So, what is the three zone enclosure and how do I how do I express it in terms of the concepts that we have developed so far; so, instead of two surfaces, as was done previously, now I have three surfaces. So, previously 1 surface was interacting with only ones other surface in this case one surface for a three zone enclosure one surface would be interacting with two more two more surfaces.

So, there will be three radiosities to deal with J 1 J 2 and J 3 three blackbody radiation power is to be is to be considered which is E b 1, E b 2, E b 3. E b 1 and J 1 are connected E b 2 and J 2 are connected E b 3 and J 3 are connected by the relation 1 minus epsilon i by A i epsilon i that is the resistance in between. But, what happens in between let us quickly take a look at it and then we will be in a position to do all these furnace calculations and so on.

So, our next topic which we are going to consider now is how to formulate the circuit diagram for a three zone enclosure. The enclosure is being formed by three zones.

(Refer Slide Time: 13:18)



So, let us see what three zone enclosure look like and in three zone enclosure the Q i to i the heat flow from i to i would be J i radiosity of i minus radiosity of J divided by R of i to j, where we understand that R of i to j would be 1 by A i F i to i which due to reciprocity would be equal to A j F j to i. So, this is the background ready background one which we need to find out. So, this is exchange between 1 and 2 and for Q i which is which is which should be equal to E b i minus J i by R I, where R i would simply be 1 minus epsilon i by A i epsilon i. So, these are the relations which we are going to use for this case.

So, let us think of a three zone enclosure like this where this is A 1 maintained at a temperature T 1 with an emissivity equal to epsilon 1, this is A 2 maintained at a temperature T 2 epsilon 2 and A 3 T 3 epsilon 3. I would like to find out what is the net radiative heat exchange between the two.

So, the first thing that I would do is for 1 my potential is going to be E b 1 which is inside the surface, but it is a blackbody radiative potential. So, it is it is a real surface it is not a blackbody. So, it is potential over here is going to be J 1 here it is going to be J 2 sorry J 3 and here it is going to be J 2. So, this is going to be my J 1. On the other hand over here I have this as E b 2 which is this surface it is connected to J 2 in here I have E b 3 with the radiative potential being J 3.

So, these are the three points or the six points that I have done for each of the surface the potential if it is a blackbody the radiosity. Since it is not a blackbody it is the radiative amount of a it is a amount of radiation energy which is coming out of the surface as observed by an observer standing outside of the surface. Similarly, E b 2 J to E b 3 and J 3; these are connected by radiation this resistance which is this. So, what this resistance should be? This R 1 would simply be equal to 1 minus epsilon 1 by A 1 epsilon 1. This one and these two are R 2 which is 1 minus epsilon 2 by A 2 epsilon 2 and this is R 3 which is 1 minus epsilon 3 by a 3 R 3. So, that is straight forward from our previous discussion.

Now, this is now going to interact with 2 as well as it is going to interact with 1. These two are this is the potential of 1 which is going to interact with 2 and it is also interact with 3. So, let us say it is interacting with 3 and of course, there is a resistance in here and J 1 is interacting with J 2. So, there would be resistance in here as well. So, what is the formula for interaction between J 1 and J 2? It is R 1 to 2. So, it should be 1 by A 1 F 1 to 2,.

So, the resistance R 1 2 would simply be equal to 1 by A 1 F 1 2. You can see clearly that it can also be equal to A 2 F 2 1, because of the reciprocity relation. So, whether it is R 1 2 is expressed as 1 by A 1 F 1 2 or 1 by A 2 F 2 1 they mean the same thing, right. F 1 2 is the view factor of 2 from 1 in this case this resistance is going to be R 1 3 which would be only be equal to 1 by A 1 F 1 3 and this can as I said this can also be written as A 3 F 3 1.

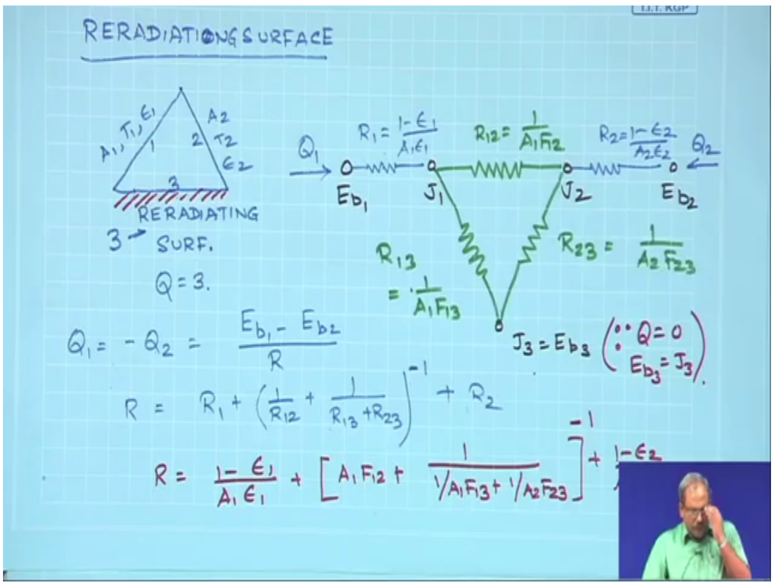
Now, one thing that is remaining in this to complete this circuit is between J 3 and J 2. So, what is in between J 3 and J 2? There must be some resistances in here as well whereas, where this R 2 to 3 or rather this R 3 can be expressed as 1 minus sorry, R the R 2 3 this is between 2 and 3 R 2 3 can be expressed as 1 by A 2 F 2 3. So, the connection between J 2 and J 3 the resistance R 1 2, R 1 3, R 2 3 would simply be equal to 1 by A 2 F 2 3 or it can also be written as 1 by A 3 F 3 2 using the reciprocity relation.

So, this essentially takes into account all the entire circuit. And because of the difference in values between E b 1 and J 1 E b 3 and J 3 E b 2 and J 2 some amount of heat is entering over here which is which I call it as Q 1 some is entering which we call it as Q 2 and some is entering over here which is Q 3. Now, the potential difference between E b 1 and J 1 arises due to the non black nature of the body. Had this been a blackbody E b 1 would simply be equal to J 1. And therefore, Q 1 would be equal to 0.

In our in the most general case we do not assume that the body from where radiation is coming or the radiation is taking place is not a blackbody. So, there is a difference between the blackbody emissive power and the radiosity. So, a current that is an equivalent heat must flow from E b 1 or J 1 E b 1 to J 1 or J 1 to E b 1. So, Q 1 can be positive or negative depending on whether g J 1 is less than E b 1 or J 1 is more than E b 1. Another point which I would like you to remember is that the value of J 1 is affected by the presence of other surfaces emitting radiation. So, whatever is the value of J 1 is going to be dependent is going to be going to be dependent on whatever radiation it receives from the other surfaces. So, 1 2 and 3 these three surfaces are there for connected any change in the value of Q 3 or Q 2 would affect the value of J 1 as well.

So, this is the network method for three zone enclosures and you would be able to find out what is the current at each of these nodes, what is the amount of heat that is to be supplied or to be extracted from node 1. The node 1 is surface 1 from this surface to maintain it is temperature at a constant value. So, that is extremely important and we will solve a number of problems on these. But, let us think of a surface which is perfectly insulated. So, this if one of the surface is perfectly insulated that is known as the re-radiating surface.

(Refer Slide Time: 22:51)



And, a simplification can be made for the case of a reradiating surface. So, in many in many engineering applications a zone can be thermally insulated and in such a case if in such a case the net radiative heat flux in that particular zone is 0 and such a surface is called a reradiating surface.

So, let us say this is a reradiating surface and in a reradiating surface obviously, the value of Q 3 is equal to 0, whatever it gets it radiates the same amount. So, we have A 1, T 1, epsilon 1, A 2, T 2, epsilon 2, but this surface 3. So, this is 1, 2 and 3 surface 3 is a reradiating surface. So, how would the how would the circuit look like in that case? You have E b 1 you have J 1 this is J 2 this is E b 2 over here.However, J 3 is equal to E b 3 because it is a reradiating surface. So, if it is a reradiating surface then there is no flow of heat through this surface and therefore, it in such a case J 3 is equal to E b 3.

Let us complete the circuit over here. So, I have resistance which is 1 minus epsilon 1 by A 1 epsilon 1, this is R 2 is 1 minus epsilon 2 by A 2 epsilon 2. So, in this case R 1 3, 1 2 3 should be equal to 1 by A 1 F 1 3, this is R 2 3 would be 1 by A 2 F 2 3 and R 1 2 would be 1 by A 1 F 1 2. So, for this reradiating surface since Q is 0 there is no heat that is to be extracted or to be supplied in for a reradiating surface. So, it is perfectly insulated let us say in which case since Q is equal to 0 E b 3 is equal to J 3. So, for a perfectly insulated perfectly insulated surface is a closed approximation to a reradiating surface reradiating surface and Q would be equal 0. So, E b 3 is equal to J 3.

So, if I now consider between E b 1 and E b 2, if I am trying to find out what is the value of the heat to be supplied to Q 1 the same heat in order to maintain thermal equilibrium in order to maintain equilibrium the same heat will have to be extracted from 2. So, if this is Q 1 and you have Q 2 in here, so, Q 1 must be equal to Q 2, and this is simply going to be E b 1 minus E b 2 by the effective resistance between this and this.

So, what is the effective resistance between these two points? This resistance R 1 and R 2 are going to be in series and the equivalent resistance of these three where these two are in series and which this resistances in parallel with the sum of these two. The equivalent resistance between E b 1 and E b 2 can be substituted and this would simply be where the effective resistance R this effective resistance R would simply be equal to R 1 plus R 2.

So, these two are in series and the equivalent of this which is in series with R 1 and R 2 can simply be written as 1 by R 1 2 plus 1 by R 1 3 plus R 2 3 to the power minus 1. So, this is the equivalent resistance of these three of this entire resistive network and when you take it further this R would simply be 1 minus epsilon 1 by A 1 epsilon 1 plus R 1 2 is 1 by R 1 2 is 1 by 1 F 1 1 by F 1 2 to this A 1 F 1 2 plus.

So, we have covered a number of interesting important observations in this class starting with the concept of radiosity and the concept of surface resistance to radiation. We went ahead and to we found out what is the resistance to heat transfer between two surfaces which we found to be equal to 1 by A 1 F 1 2 or 1 by A i F F i j. Through the use of this surface resistance to radiation and the resistance to radiation between surfaces which are which are functions of the view factor I could create a circuit the same way we have done it in electrical technology so to say.

So, a three zone enclosure forming by three surfaces would have 6 resistances, three resistances each for each of the surfaces. So, 1 minus epsilon 1 by A 1 epsilon 1, 1 minus epsilon 2 by A 2 epsilon 2 and so on would form the surface resistance to radiation for these three surfaces. Each of this surfaces will have will interact with the other surface other two surfaces and the resistance in that case would simply be equal to 1 by F 1 2 or 1 by A i F i j. So, the circuit which we have developed then on one side I have E b 1 surface resistance to resistance the surface resistance to radiation J 1 E b 2 surface resistance to radiation J 2 similarly for E b 3.

So, I have the three nodes of a triangle where the potentials are J 1, J 2 and J 3. J 1, J 2, J 3 are connected by 1 by A 1 F 1 2 1 by A 2 F 2 3 and 1 by A 1 F 1 3 that completes the circuit. So, I should be able to find out what is the net heat which is entering 1, which is entering through 2 and which is entering through 3. In order to maintain the thermal equilibrium the algebraic sum of Q 1, Q 2 and Q 3 must be equal to 0. We took it one step further we define what is the reradiating surface where it emits whatever it gets. So, it is an perfect insulated surface is an example of a reradiating surface.

And, I have shown you how what a reradiating surface a three zone enclosure circuit diagram can be drawn, where there is going to be two surface resistances in series and the third resistance which when you convert it gives a simple expression for the net heat flow between these two surfaces. So, we are going to do number of problems on this to clarify any doubts that you may have, and it these are very interesting applications; real applications in furnace calculations.

So, we will take those up in the next two classes.