

**Heat Transfer**  
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**Lecture - 56**  
**Network Method for Radiation Exchange in an Enclosure**

We already know what radiosity is in the, what is the connection between the radiosity of a surface and that offer the emissive power of a black body. Now, this radiosity is going to play a major role when we are going to do calculations about radiative heat exchange in an enclosure. This is the practical application of whatever we have studied so far.

For example, if you are designing a furnace and you would like to maintain a surface exposed to a constant temperature or maybe a constant heat flux, so what is the power that you need to provide through the other surfaces which are forming the enclosure, so that the temperature of the test surface, temperature of the surface of interest can be maintained let us say at a constant temperature.

So, whatever we are going to going to study in the next few classes would tell us about the radiative heat exchange in an enclosure, how much power is required for such cases and we have to also think of the situation in which the walls of the enclosure are going to interact with each other in terms of radiative heat exchange. Not only they are going to emit some radiation, they are also going to absorb some incident radiation, which are falling on it. So, this kind of emission as well as absorption, a fraction of the radiation incident on the surface is going to be absorbed because of its inherent temperature it is going to emit.

So, the balance between these two whatever is being emitted and whatever is being incident or rather the fraction of the incident energy which is absorbed by the surface, a heat balance between the two should give us what is the amount of heat to be supplied to that surface to either maintain it at a constant, to maintain it at a constant temperature. So, the enclosure therefore takes into account the interaction between the surfaces which are forming the enclosure. So, you can clearly see, imagine the role the view factor is going to play in these calculations. So, we need to be concerned about what is the view factor of one surface to the other surfaces which are forming the enclosure because the

radiative energy which is emitted by the surface is going to be intercepted by the other surfaces and the view factor  $f_{12}$   $f_{13}$  and so on, they will play a vital role in it.

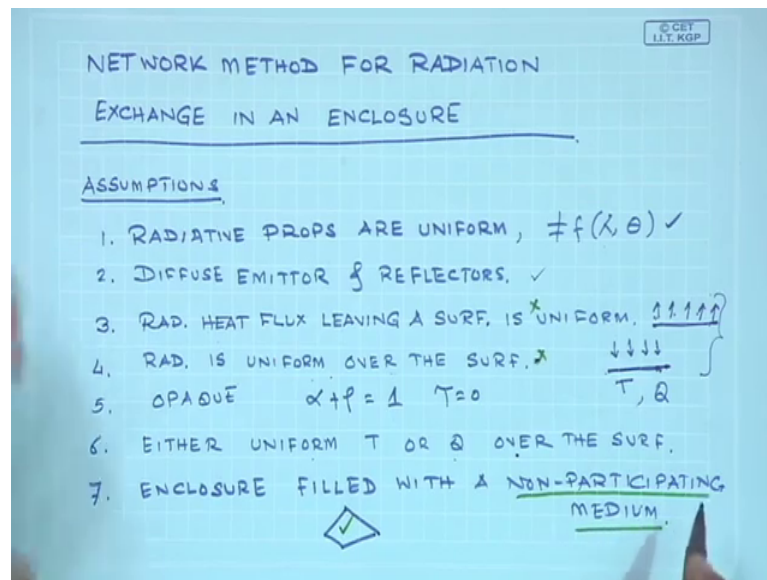
Therefore, these are all the real surfaces and they may not be black bodies. So, if they are real surfaces, what is the relation between the black body emissive power and the radiosity if I consider that we are already aware of. So what is going to be the emissive power of a real surface or real object to that of the radiosity of the surface. This is some relation that we are need to find out. So, there may be we can think of it as if there is maybe a resistance which is going to be in between the black body emissive power corresponding to the temperature of the substrate and the radiosity of the substrate.

So, these are the two potentials; the potential of the body in terms of its temperature had this been a black body and the radiosity, the actual heat flux which is being observed by someone standing just outside of the surface. What is the relation between the two because that is the amount of heat which is going to flow from one surface to just outside of the surface. So, what is the resistance that connects the two? We will we will discuss that in this class itself. So, there has been, it has been proposed that following network method like the same type of network method which you have done in electrical circuits something like that can be adopted for the case of radiative heat exchange between the surfaces in an enclosure.

But in order to do that we need to make certain assumptions. Some of these assumptions are obvious and they do not may not introduce much error to it, much error to the final results, but there are two assumptions which are slightly restrictive in nature. So, we have to be careful about what is the source of inaccuracy in our final calculation and what is the contribution of each of these assumptions to the final answer.

So, we start in this class with the calculation procedure for radiative exchange between surfaces forming an enclosure and these are the following assumptions we shall go through one by one.

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So, the first is we are going to start the network method for the radiation exchange in an enclosure. We would assume that the radiative properties are uniform. That means they are not a function of wavelength or of theta. So, this is something which is followed by many of the engineering materials. It is a diffuse emitter of the surface. The surface is forming the enclosure are diffuse emitters and diffuse reflectors.

This is also something which is followed in many realistic situations. Radiative heat flux leaving a surface is uniform. So, if you have the surface, then the radiation that is being emitted by the surface is irrespective of whether you are measuring it here, here, here or anywhere else, so its radiation is uniform over a surface. Secondly, if you have a surface and you have incident radiation falling on it, then the radiation is going, we are assuming that the radiation is uniform over the entire surface.

So, that radiation leaving the surface and the radiation which is incident on the surface, both are same. Both remain constant at all points on the surface. These two are the problematic cases. They may or may not be valid in all cases and therefore, these two are the source of errors, but in order to simplify our calculation, there is that we need to assume these two, but we have to understand that these two may not be followed in some situations. The other assumption which we need to make is it is an opaque surface. So, therefore alpha plus rho is equal to 1. That means, the reflectivity and absorptivity is equal to 1 and the transmittivity is equal to 0.

We would also assume that it is either an uniform temperature or an uniform heat flux is provided over the surface. So, we can either say that the temperature of the surface is known or the heat flux to be added to the surface or extracted from the surface is known to us. The last assumption is that the enclosure which is let us say the enclosure which is formed by this, the inside of this enclosure is filled with a non-participating medium. This is important because as we would see towards the end of this course is that there are certain, most of the gases observe and scatter some of the radiation which is which is passing through it.

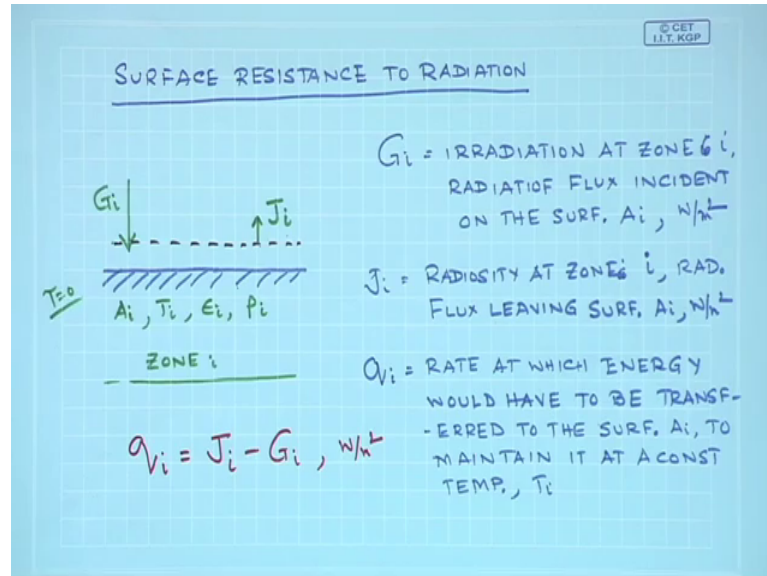
So, when that happens depending on what is the thickness of the gas layer through which this the radiation is propagating. The amount of radiation which you which you would get at the end of the gas film is obviously going to be different from the radiation which enters the gas film. So, these are known as non-participating medium. There are specific laws which would tell you what is the decrease in radiation intensity as you move along in the gas. So, we will take, we will consider that later, but for the time being network exchange method what we are assuming is that the enclosure is filled with a non-participating medium. So, radiation coming out of one surface is going to reach the other surface without any loss due to absorption in the part in the gases or air which is present inside the surface.

So, all these are some of the assumptions which are made to calculate, but using radiation using the network method what is the radiation exchange between surfaces, but before we proceed to that as I said before we need to find out what is the relation between the black body emissive power, let us say the temperature of the surface is  $T$ . So, the corresponding black body emissive power would be  $\sigma T^4$ , however it is not a blackbody.

The surface is a real surface; it is not a blackbody. So, therefore the emissive power of the surface had this been a black body and the radiosity of this radiosity which is the radiative flux that is that is perceived just outside of the surface, what is the relation between the two? So, if these two are different and these two obviously are different since the surface is not a black body. So, the difference in the potential between these two points must be connected by some sort of a resistance and this resistance would account for the drop in potential between the surface which is not a black body and the

radiosity of the surface. So, our next job is to find out what is the relation between radiosity and emissive power of a surface. So, let us try to find that out.

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So, the concept that we are going to introduce here is known as the surface resistance to radiation and you can imagine I think you can follow that this surface resistance would obviously be 0 for the case of the imaginary black body, but we would come to that later. Think of the surface and the properties of these are this it is  $A_i$ , the area  $T_i$  being the temperature,  $\epsilon_i$  being the emissivity and  $\rho_i$  being the reflectivity and as I have said that  $\tau$  is equal to 0, it is an opaque surface. This is my zone  $i$ , and let us assume this is the thin layer at which we are going to measure the radiosity.

So, the flux which is coming out of it is the radiosity and if some amount of incident radiation which is given as  $G_i$ . So,  $G_i$  in this case is the irradiation at zone  $i$  which is nothing, but the radiative radiation flux which is incident on the surface and the surface has area equal to  $A_i$  and this is the unit of this will be in watt per meter square.  $J_i$  is the radiosity at zone  $i$ . So, this is nothing, but the radiation flux which is leaving surface  $A_i$  unit also would be in watt per meter square.

Now, the amount of heat, amount of radiative energy which is leaving the surface is  $J_i$  in watt per meter square, the amount of incident energy which is falling on it is  $G_i$ . So, therefore, the net amount of heat if I call  $q_i$  as the rate at which the energy would have to be transferred to the surface  $A_i$  to maintain it at a constant temperature of  $T_i$ . So, these

are the three things as you can see from here.  $G_i$  is the irradiation at zone  $i$ ,  $J_i$  is the radiosity at zone  $i$ . So, it is a net, it is a radiative flux leaving the surface and  $q_i$  is the rate at which energy would have to be transferred to the surface because something is coming, something is leaving. So, in order to maintain a thermal equilibrium, some amount of energy will have to be added to this or extract it depending on which one is higher.

But if I say that  $q_i$  is the rate at which energy would have to be transferred to the surface or from the surface, it all depends on the values of  $G_i$  and  $J_i$ , then  $q_i$  is the rate at which energy would have to be transferred to the surface  $A_i$  to maintain it at a constant temperature  $T_i$ . Then, you can clearly see this  $q_i$  must be equal to  $J_i$  minus  $G_i$  in watt per meter square. So, this  $J_i$  is the radiative energy which is leaving the surface as observed by an observer just outside of the surface and  $q_i$  is essentially based on the net energy balance in the interior of the surface.

So, I guess this is a fairly straightforward explanation of the radiosity, the irradiation and the net amount of heat which is to be added or extracted from the surface to maintain it at a constant temperature. So,  $q_i$ , the relation between  $q_i$  the net energy,  $J_i$  the radiosity which is the radiation flux observed just outside of the surface and  $G_i$  which is the radiative heat flux incident on the surface. The balance between the two would give you the relation which I have just written as  $q_i$  equals  $J_i$  minus  $G_i$ . So, let us expand this a little further and see what we would get out of this.

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$J_i = (\text{RAD. EMITTED BY THE SURF}) + (\text{RAD. REFLECTED BY THE SURF})$   
 $J_i = \epsilon_i E_{bi} + \rho_i G_i$   
 $\rho_i + \alpha_i = 1 \quad (T_i = 0)$   
 $\epsilon_i + \rho_i = 1$   
 $J_i = \epsilon_i E_{bi} + \rho_i G_i \rightarrow G_i = \frac{J_i - \epsilon_i E_{bi}}{\rho_i}$   
 $G_i = \frac{J_i - \epsilon_i E_{bi}}{1 - \epsilon_i}$   
 $q_{net} = J_i - G_i$   
 $q_{net} = J_i - \frac{J_i - \epsilon_i E_{bi}}{1 - \epsilon_i} \Rightarrow$

First of all this  $J_i$  is radiation which is emitted by the surface. The definition of  $J_i$  by the surface, this is the first contribution plus the second contribution is what is the radiation which is reflected by the surface. So, this is the definition of radiosity which is two parts; radiation emitted by the surface and the radiation reflected by the surface. So,  $J_i$  the radiosity for a real surface which is not a black body must be equal to  $\epsilon_i E_{bi}$ , where  $\epsilon_i$  is the emissivity times  $E_{bi}$  which is the black body emissive power corresponding to the temperature of the surface.

So, had this been a black body, then the radiosity and  $J_i$  radiosity and  $E_{bi}$  could be same, but we understand that the real surfaces will have an  $\epsilon_i$ . So, therefore, these two may not be equal plus some amount of energy is incident on it, incident on the surface a fraction of it denoted by the reflectivity is going to be reflected out of this. So, this is the emission which is coming out and this is part of the reflection which is coming which is being reflected by the surface  $J_i$  is a sum of these two. So, the amount of emission which is coming out is  $\epsilon_i E_{bi}$ , the amount of reflection which you would get is  $\rho_i G_i$ .

So, if you are the person standing over here to you, the total amount of energy which is coming out of this surface would be the sum of these two and we also understand that for this specific surface  $\epsilon_i + \rho_i + \alpha_i = 1$ . Since your  $\tau_i$  is equal to 0, there is no transmittivity, but it is also a grey surface. It is also a grey

surface. Therefore, what you can write is  $\epsilon_i + \rho_i = 1$ . So, these two are equal absorptivity and emissivity are equal. Therefore, the relation  $J_i$  is equal to  $\epsilon_i E_{bi} + \rho_i G_i$  can be written as because my aim to get rid of this  $G_i$  from this relation and try to see if I can connect  $J_i$  with  $E_{bi}$ .

So, in order to do that first I find out what is  $G_i$  which is from this equation is  $J_i - \epsilon_i E_{bi}$  by  $\rho_i$ . So, this can be therefore  $G_i$  can still be written as putting the value of  $\rho_i$  from here as  $1 - \epsilon_i$ . So, if you now see  $q$  which we have obtained as  $J_i - G_i$  from our previous page, this is the fundamental balance equation which we have written I am now going to get rid of this  $G_i$ . So, my total heat, the heat which gets transferred heat which has to be applied is  $J_i - G_i$ . For this  $G_i$ , I put the expression  $\epsilon_i E_{bi}$  divided by  $1 - \epsilon_i$ .

So, the two relations which are we are using is one is this one and the other is this one and this one, the aim of this is using these 3 equations. We are trying to get rid of  $G_i$  and express the  $q$  in terms of  $J_i$  and  $E_{bi}$ . So, we would continue with this, this relation which I am going to write once again.

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The image shows a handwritten derivation on a grid background. At the top right, there is a small logo: © CET I.I.T. KGP. The derivation starts with the equation  $q_i = J_i - \frac{J_i - \epsilon_i E_{bi}}{1 - \epsilon_i}$ . This is simplified to  $q_i = \frac{\epsilon_i}{1 - \epsilon_i} [E_{bi} - J_i]$ , which is boxed in red. To the left of this box, there is a note in a cloud shape:  $J_i = \epsilon_i E_{bi} + \rho_i G_i$ ,  $\text{For A/B/B}$ ,  $\epsilon_i = 1$ ,  $\rho_i = 0$ ,  $J_i = E_{bi}$ . Below the boxed equation, the net heat transfer is given as  $Q_i = q_i A_i = \frac{A_i \epsilon_i}{1 - \epsilon_i} [E_{bi} - J_i]$ . This is further simplified to  $Q_i = \frac{E_{bi} - J_i}{\frac{1 - \epsilon_i}{A_i \epsilon_i}} = \frac{E_{bi} - J_i}{R_i}$ , where  $\frac{1 - \epsilon_i}{A_i \epsilon_i}$  is circled in green. At the bottom left,  $R_i = \frac{1 - \epsilon_i}{A_i \epsilon_i}$  is boxed in red, with the text "SURF. RESIS. TO RADIATION." written next to it. At the bottom right, there is a note: "For A/B/B  $E_{bi} = J_i$ ".

I get  $q_i$  as  $J_i - \frac{J_i - \epsilon_i E_{bi}}{1 - \epsilon_i}$ . So, when you do this, you are simply going to get  $q_i$  as  $\frac{\epsilon_i}{1 - \epsilon_i} E_{bi} - \frac{J_i}{1 - \epsilon_i}$ . So, this is the relation which would give you the amount of heat that needs to be transferred, net heat that needs to be transferred to a surface which is having irradiation as well as



emission. So, therefore this is something which one has to find out and if you also look carefully, this  $E_{bi}$  and  $J_i$  can be treated as something of a potential. Let us expand this little further and  $q_i$ , the total amount of heat would be the heat flux multiplied by the area and therefore, this is going to be  $A_i \epsilon_i (1 - \epsilon_i) E_{bi} - J_i$ , ok. So,  $q_i$  therefore is  $E_{bi} - J_i$  divided by  $1 - \epsilon_i$  by  $A_i \epsilon_i$ . So, this is some sort of a resistance. So, I can write this as  $E_{bi} - J_i$  divided by  $R_i$ , where  $R_i$  is  $1 - \epsilon_i$  by  $A_i \epsilon_i$ .

So, this is extremely important what we have obtained here. What we see in this is I have a potential which is the black body. The potential of the surface potential of the substrate if it was a black body,  $J_i$  is the radiosity of the surface and the radiosity of a surface real surface is different from that of a; from that of a black body. So, it is the potential that an observer standing just outside of the surface would have seen, would have sensed had this been a black body, these two would be equal. This is also evident from the definition of  $R_i$ , the resistance where  $\epsilon_i$  for a black body would be equal to 0 and therefore, this resistance would vanish and therefore, in for a black body  $E_{bi}$  would be equal to  $J_i$  for a black body ok, but for a real surface, this is some sort of some sort of looks like an Ohm's law.

The Ohm's law is basically potential difference divided by a resistance which gives you a current. So, in this case, the potential differences are the black body radiative potentials, potential minus the radiosity divided by some sort of a resistance which is equal to the current and the current in this case is equivalent to that of heat flow. So, this  $R_i$  is known as the surface resistance to radiation, ok.

So, we started with the expression that  $J_i$  is equal to  $\epsilon_i E_{bi} + \rho_i G_i$  that was my starting point. So, what would happen for a black body let us see once again. My  $\epsilon_i$  would be equal to 1 and my reflectivity because this since it is a black body, it is going to absorb everything and therefore,  $\rho_i$  would be 0 and therefore,  $J_i$  would be equal to  $E_{bi}$ . So, this is for a black body, this is what you would get. So, for a black body there is no resistance, no surface resistance to radiation, but for any real object the potential inside the body, the emissive potential inside the body had this been a black body and the potential measured just outside of the surface, the difference between these two potentials is the cause which makes the heat flow from one direction to the other.

So, we may lose some heat from the surface which has to be again supplied to main from outside to maintain it at a constant temperature or the surface may gain some heat because of this exchange process and again in order to maintain the constant temperature, that amount of heat will have to be extracted from the surface. So, the concept of surface resistance to radiation is extremely important as it would connect the black body radiation power of a surface to that of a real surface. So, what is the surface resistance to radiation which takes into account not only the emissive power, emissive nature of the surface, but also takes into account the reflectivity.

So, if a surface forms one part of the enclosure for radiative heat exchange, it is not only going to emit, it is also going to get some irradiation from other surfaces. So, the algebraic sum of the emission and that of absorption due to radiation coming from another object. The algebraic sum would tell you whether heat has to be extracted or heat is to be supplied to maintain the surface, to maintain the surface at a constant temperature.

The concept which relates these two potentials, one inside the object and the other just outside the surface of the object, the emissive power of a black body at the same temperature as that of the surface and the radiosity of the surface. These two are the potentials. The difference in these two was non-existent for the case of a black body. So, the difference between these two potentials causes heat to move either from the surface or to the surface.

So, in order to calculate these two resistances, I need to know these two potentials and these two potentials taking an analogy from Ohm's law must be connected by some sort of a resistance and that resistance which we have evaluated that resistance  $R_i$  is called the surface resistance to radiation. So, if I knew the two potentials in here and if I knew the resistance in there and as you could see the resistance depends only on emissivity, it is  $1 - \epsilon_i$  by  $A_i \epsilon_i$  which is which depends only on the emissivity. So, if the emissivity of the surface is known, then the surface resistance to radiation will also be known.

So, if the two potentials are known, then the total amount of heat flow to or from the surface that is required to maintain the temperature of the surface at a constant value can

easily be found out and while doing this, I am allowing the exchange of radiative energy with from which are coming from other surfaces forming the enclosure.

So, this is the first tip to realize to understand how the radiosity and the black body emissive power are connected by the concept of, by the concept of resistance by this resistance the very surface resistance to radiation. So, when we move in the next class to the network method, it would be really easy to draw this circuit with the blackbody radiative power and the radiosity as two nodes, two potentials connected by a resistance  $R_i$ . We will take this up in the next class.