# Heat Transfer

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## Lecture – 52
## Tutorial on Emissivity, Absorptivity and Blackbody Radiation Functions

We were discussing about the Emissivity and Absorptivity. So, when these two can be taken to be equal. So, in order to do that, we started with the spectral directional emissivity and spectral directional absorptivity and we understand that from statistical thermodynamics, these two will always be equal. So, therefore, epsilon lambda theta, lambda denotes the spectral nature of emissivity; whereas, theta denotes the directional dependence of the emissivity should be equal to alpha lambda theta. So, the spectral directional quantities emissivity and absorptivity are equal.

Starting with that, we wanted to get rid of the directional dependence. So, in order to get the directional dependence part, I mean in order to remove the directional dependence we had to make certain assumptions. So, that is once we do that we get the form that epsilon lambda; that means the spectral emissivity is equal to alpha lambda the spectral absorptivity. Through another set of assumptions, we arrive at the overall hemispherical emissivity epsilon is equal to the overall absorptivity alpha.

So, this is the most restrictive form of Kirchhoff's Law, but which is also the most useful as well. However, one thing one has to keep in mind is that the epsilon depends on the emission from the surface itself; whereas, alpha depends on the irradiation falling on to the surface so, these two are different things. So, even so, it is not easy to say that the alpha is going to be equal to epsilon, since they refer to emission and irradiation from different sources. So, I will quickly go through whatever we have done in the previous class and then, try to solve the problem which would clarify your doubts.

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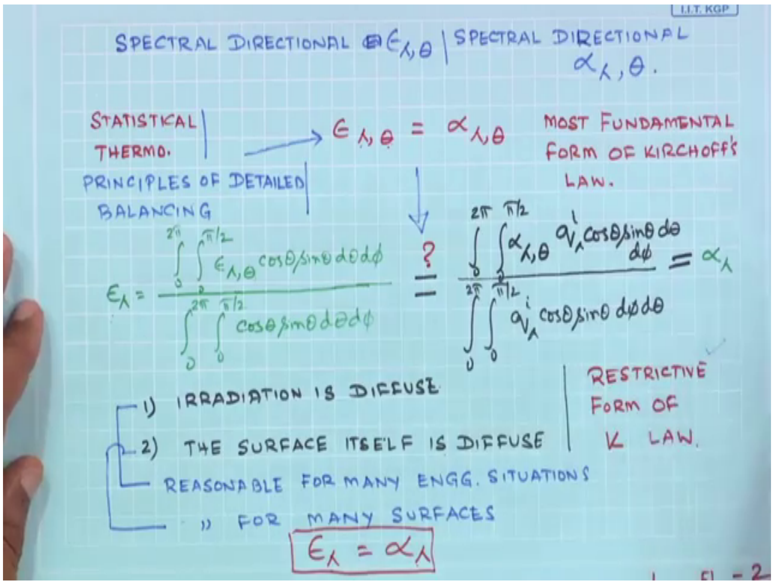


So, we start first with this one that we have done in the last class that epsilon lambda theta is equal to alpha lambda theta, see this is the most fundamental form of Kirchhoff's Law. In order to obtained obtain the spectral emissivity; we need to integrate it over all possible direction in a hemispherical space. So, we already understand that this is going to be the form of that.

And the absorptivity, if you would like to find out what is the spectral absorptivity from the spectral directional absorptivity, we have to again integrate it over the space. However, here I am going to have the incident spectral, incident radiation on the object in the numerator as well as in the denominator. So, the question is when these two are going to be equal. So, if the irradiation is diffuse; that means, if this is a diffused irradiation then this q i lambda is going to be independent of direction.

So, both can be taken outside from numerator and denominator and they will simply cancel out. Now, since alpha lambda theta is equal to epsilon lambda theta; so, if this 2 can be cancelled out, then these are going to be equal. So, that is the first assumption that is if the irradiation is diffused, I get rid of qi lambda and qi lambda both from the numerator and denominator. Then, obviously, alpha lambda is going to be equal to epsilon lambda.

Secondly, if the surface itself is diffused; then, these epsilon lambda theta and alpha lambda theta are independent of the direction. So, they can be taken outside and since epsilon lambda theta and alpha lambda theta are same; what you would see is that the numerator and denominator will simply cancelled, will cancel out leaving epsilon lambda theta in the outside and alpha lambda theta on the outside and they will they will simply cancel out.

So, that is also a case in which we would get alpha lambda to be equal to epsilon lambda. So, this relation is true, epsilon lambda is equal to alpha lambda is to if the irradiation is diffused and if the surface itself is diffused. Now, these irradiation is diffused for many surfaces; whereas, the surface to be defuse is reasonable for many engineering situations. So, it is customary it is not unusual to take this relation, slightly restricted form of Kirchhoff's law that epsilon lambda is equal to alpha lambda.

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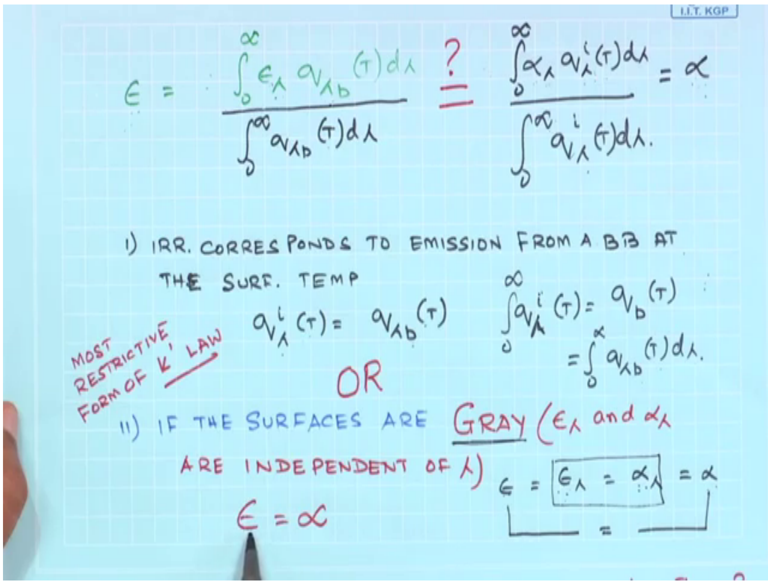


The question becomes more critical if you would like to go from the spectral emissivity to overall emissivity or spectral absorptivity to the absorptivity itself. By definition, the overall emissivity is simply this is the irradiation coming from a blackbody. This is also irradiation from a blackbody, this is the incident radiation and this also is the spectral incident radiation.

So, we would like to see when these two are going to be equal. So, if it so happens that the incident radiation that is falling on the surface is coming from a blackbody at the same temperature as that of the surface.

Then, qi lambda T would be equal to q lambda by T so, the incident radiation is coming from a black body at the same temperature. So, therefore, this would be q lambda b T this is also going to be q lambda b T and if you now see both sides of the equation since epsilon lambda is equal to alpha lambda.

So, therefore, alpha would be equal to epsilon, since q lambda b is equal to q so, you are going to have q lambda b T on both sides. So, that is the very restrictive most restrictive form of Kirchhoff's Law which assumes that the surface from where the irradiation is coming from is a black body and it is temperature is equal to the surface on which the radiation is falling.

So, that is the most restrictive form of Kirchhoff's Law. Secondly, if we can see that if the surfaces are gray that is a new kind of surface in which epsilon lambda and alpha lambda are independent of lambda; that means, the spectral and the spectral emissivity and spectral absorptivity are independent of alpha.

Then, they can simply be taken outside and once you take the take outside, then the numerator and the denominator will simply cancel out leaving epsilon lambda on this side and alpha lambda on this side. And since alpha lambda is equal to epsilon lambda so, therefore, we can say that epsilon is equal to lambda; that means the overall emissivity of a surface is equal to the overall absorptivity of the surface.

So, this is this was what I was talking about and therefore, these two are equal so, this is another form of Kirchhoff's Law. This is the most useful form of Kirchhoff's Law, but it defines a new kind of surface; new class of surfaces which is known as gray surfaces. Now, for many surfaces the assumption of gray behaviour is justified so, what it says is that the properties the irradiative properties, mostly the emissivity and absorptivity are independent of wavelength.

So, that means, spectral emissivity is equal to the overall emissivity and this gives us the desired relation that the emissivity is equal to absorptivity. So, if you know one, then you know the other and this simplifies many of the calculations specially radiation between surfaces in a closed container.

So, if we can assume great behaviour for such surfaces, then our calculations of radiation exchange between surfaces in an enclosure becomes a lot simpler. So, we would always try to see if this is the case and proceed from there. However, one must use caution while using the relation alpha is equal to epsilon because, we still have to keep in mind is that the epsilon depends on emission from the surface; whereas, alpha depends on irradiation from a source.

So, we need to see that whether the emission and the incident radiation, they are in the same spectral range at which alpha lambda is equal to epsilon lambda. So, whether the spectral emissivities and spectral absorptivities are same in the range, in the operating range that we need to carefully observe.

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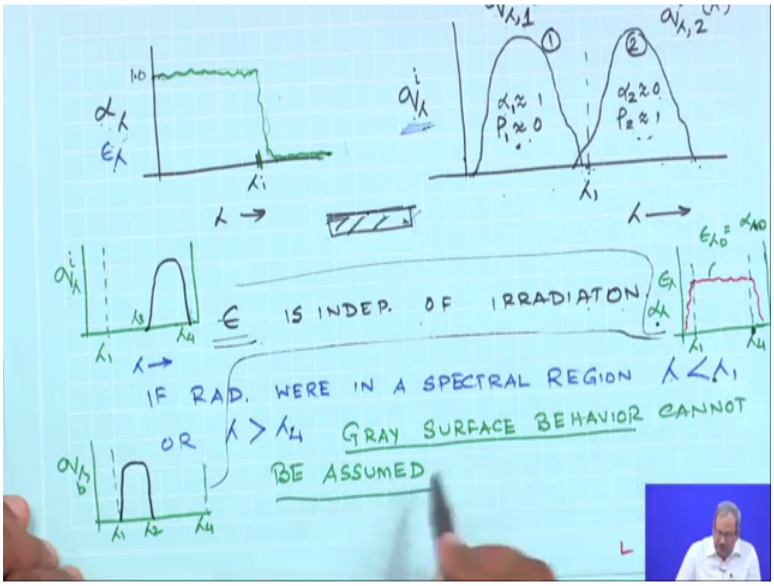


So, the I am I have given you an example which I will show you once again. So, this is a case in which you have the absorptivity which is roughly about 1 up to a wavelength of lambda 1 and then, it falls to a very low value equal to 0. And we have 2 irradiations; the first is irradiation is like this. It is between 0 to lambda 1 and the second irradiation is this where it is lambda 1 and beyond. So, if you consider this case, this irradiation between 0 to lambda 1 alpha is about 1.

So, the reflectivity is 0, we are considering OPEC substrates. However, on the if the irradiation in the second part, then irradiation if it irradiation falls in this wavelength range. Then, alpha to the absorptivity is 0 as suggested by this figure in the reflectivity is about 1.

So, therefore, the value of alpha, the amount of energy absorbed by the surface strongly depends on the spectral distribution of the incident radiation as well as the value of alpha in that region. So, you can see that the, but; however, the epsilon is independent of q lambda i, epsilon the is does not depend on the irradiation rather it depends on what is the temperature and what is the nature of the of what is the nature of this substrate? So, even though epsilon alpha lambda is equal to, this is also epsilon lambda alpha lambda is equal to epsilon lambda depending on the irradiation, you get a value of alpha equal to 1 or alpha equal to 2.

However, your value of epsilon will remain unchanged and therefore, it is not safe to say that alpha is always going to be to be equal to alpha is going to be equal to epsilon. So, if radiation where in a spectral region which is which the lambda is less than equal to lambda 1 or lambda is greater than lambda 4, I will draw these figures once again.

So, I have figures like this, let say this is lambda 1 and this is q lambda i and I have lambda over here. Let us say I have a I have an irradiation which looks like this is at lambda 3 and this is at lambda 4. Second case is I have another range in which I simply have q lambda which is from a blackbody, this is as before lambda 1 and my distribution is in here.

So, this is lambda 2 and here, I have lambda 4. So, if the radiation is in a spectral region lambda less then lambda 1 or lambda greater than lambda 4, gray surface behaviour cannot be assumed. I will go through this figures once again, this is a case in which the values are close to 1 and then, it falls sharply beyond lambda 1. We have two incident radiation, one is within lambda 1, the two is beyond lambda 1 so, if it is within lambda 1; if it is within lambda 1, then alpha is going to be equal to 1, the reflectivity is going to be 0.

However, if the irradiation is beyond lambda 1, then alpha 2 this value is going to be about 0 and the value of the reflectivity is going to be equal to 1. So, the value of alpha strongly depends on the irradiation, on the spectral nature of irradiation; whereas, epsilon is independent of radiation that is one case.

Let us say we have a we have another situation in which the q lambda i falls over here and it is also falling in this region and the corresponding the corresponding distribution is shown as epsilon lambda and alpha lambda. This is lambda 1 and this is lambda 4 so, the curve looks something like this and this is epsilon lambda 0 is equal to alpha lambda 0.

So, as you can see over this wavelength range lambda 1 to lambda 4, the values of emissivity is equal to the values of absorptivity. So, if you have radiation within lambda 1 and lambda 4, if it is within that; then, we can say that overall in overall emissivity and overall absorptivity are going to be the same.

However, if you have irradiation that is before alpha 1 or after alpha 4; that means, in this or in this region, then the gray surface behaviour cannot be assumed. So, this is something which we have to keep in mind what I think the issue would be more clear when we solve a problem and therefore, you where we would see that even the epsilon lambda is equal to alpha lambda.

That means the spectral emissivity and the spectral absorptivity are the same, but the overall values will not be the same; that means, gray surface behaviour cannot be assumed principally, primarily because the irradiation is from a source which has a different condition as the as compared to that of the substrate. So, let us solve that problem and see how we can arrive at such a such a case.

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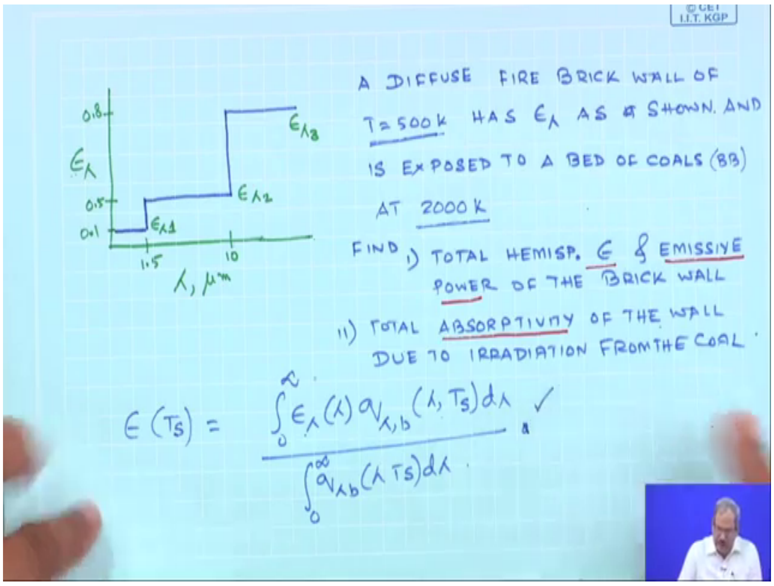


So, the problem that we are going to deal with, first of all let me draw the emissivity curve as a function of lambda in micron. So, this is about 0.5, this is about 0.8, 1.5

micron and 10 microns obviously, it is not at not as per scale in the value and this is 0.1. So, if value of emissivity would be like this was all the way to 0.5. Up to 10, its value is going to be 0.5 and then, it goes all the way up to 0.8 and it remains at 0.8. So, this is the distribution the spectral distribution of emissivity as a function of wavelength.

So, let us call this is E lambda 1; this is E lambda 2 and this is E lambda 3. So, this is what is provided in this. The problem that we have is a diffuse fire brick wall of T equals 500 degree Kelvin has epsilon lambda as shown. So, this is the figure of epsilon lambda for this lambda and is exposed to a bed of coals at 2000 Kelvin.

So, we have a surface whose spectral absorptivity is shown, but it is also exposed to a bed of coals at 2000 Kelvin. So, at the onset you can see that the emission is coming from a bed of coals which is which can be treated as a blackbody at 2000 Kelvin. So, the temperature from where the emission is coming from is different from the temperature of the substrate itself.

So, we have we cannot use the condition the restrictive most restrictive condition of Kirchhoff's Law. So, we do not know whether this epsilon is going to be equal to alpha. So, what you have to find is total hemispherical emissivity and emissive power of the brick wall and second is total absorptivity of the wall due to irradiation from the coal.

So, these are the two, I need to find out epsilon, I need to find out the emissive power of the brick wall and total absorptivity of the brick wall due to irradiation from coal. So, we first start with the definition of epsilon; where, T s is the temperature of the surface. So, by definition it is going to be 0 to infinity epsilon lambda which is a function of lambda, q lambda b the emission from a blackbody which is spectral and its temperature is T s d lambda divided by integration q lambda b lambda T s d infinity from 0 to infinity ok.

So, this can now be simplified, this is nothing, but this one is nothing but this q b, the total emissive power of a black body which is sigma T to the power 4. So, this what is known, but I can break this into 3 parts from 0 to 1.5 from 1.5 to 10 and from 10 to infinity because between 0 to 1.5, the value of epsilon lambda is known and is a constant. Between 1.5 and 10 is also known and between 10 and infinity, it is also known to us.

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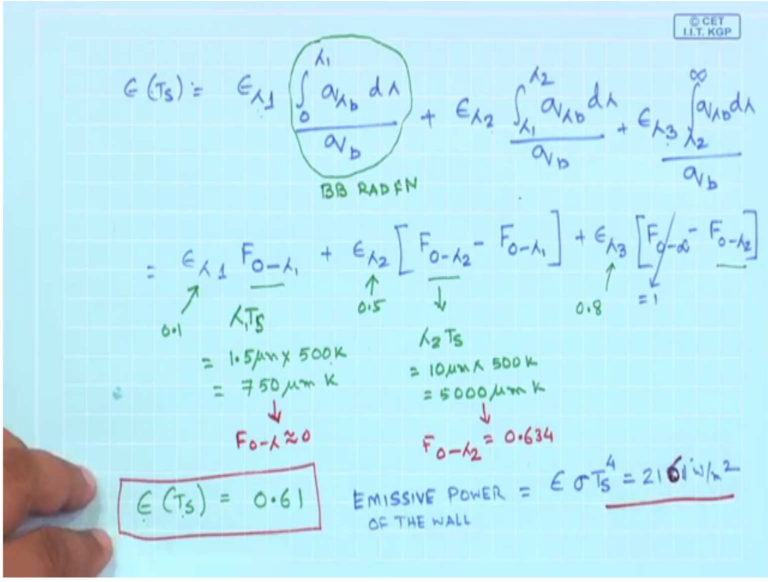


So, I am going to break the numerator into 3 parts in the following way that my epsilon which is at T s is going to be epsilon lambda 1; epsilon lambda 1 being the value the constant value of epsilon between 0 to 1.5; epsilon lambda 2 between 1.5 to 10 and so on.

So, this can be divided into 0 to lambda 1 q lambda b d lambda by q b; where, q b is the radiation emissive power of a blackbody over the entire wavelength plus the next one is going to be q lambda 2 from lambda 1 to lambda 2 and this is in q lambda b same as before by q b plus epsilon lambda 3. This is going to be from lambda 2 to infinity q lambda b d lambda by q b. So, this is simply dividing it into 3 parts in adding them together now, if you look at this part only it is nothing but the black body radiation function.

So, it simply tells you that what fraction of energy is going to be contained between 0 to lambda 1 for emission from a blackbody which is the definition of blackbody radiation function. So, this can be written as epsilon lambda 1. The f of blackbody radiation function from 0 to lambda 1 plus epsilon lambda 2 which is this, this can be divided into 2, from 0 to lambda 2 and 0 to lambda 1. So, the first one is going to be the blackbody radiation function from 0 to lambda 2 and from that I subtract the fraction from 0 to lambda 1. So, this gives me fraction of energy which is contained between lambda 1 and lambda 2.

The third one is going to be epsilon lambda 3 and it is going to be F from 0 to infinity minus F from minus F from 0 to lambda 2. We understand that f 0 to infinity is simply equal to 1, since that is that fraction would be equal to 1, if I say 0 to infinity that is equal to q lambda b. So, the fraction of energy contained between 0 to lambda is simply going to be equal to 1. Now, I have to evaluate all this, in order to evaluate these, I need to first find out what is lambda T s; what is lambda 1 T s? So, the lambda 1 in this case the is 1.5 micron, if you look at this the lambda 1 is 1.5 micron, lambda 2 is 10 micron and so on.

So, lambda 1 is 1.5 micron into T s is 500 Kelvin. So, this total is 750 micron Kelvin and the corresponding value of s which you can see from the table from these tables that we have provided in the last class for epsilon T s to be equal to 750, the value of F 0 to lambda is approximately equal to 0. On the other hand, when you do this; this is going to be lambda 2 T s. So, lambda 2 T s is 10 micron, T s is 500 Kelvin. So, this is 5000 micron Kelvin and the corresponding value of F 0 to lambda 2 from the same table is about 0.634. So, you are going to and this epsilon lambda 1 from here is 0.1, epsilon lambda 2 is 0.5 and epsilon lambda 3 is 0.8.

So, I am going to write this as 0.1, this as 0.5 and this is equal to 0.8. So, when you do this the value of epsilon T s would simply be equal to 0.61. So, the hemispherical emissivity of the surface at T equals T s that means, at 506 500 degree Kelvin would simply be equal to 0.61. So, given the spectral distribution of epsilon using the black body radiation function, one can obtain the overall emissivity from the spectral emissivities which are provided in the experimental results and what is going to be its emissive power? The emissive power of the wall is simply epsilon times the emissive power epsilon times the emissive power of the wall if had it been a black body. So, that is what is that is what is the definition of emissive power.

If this is a blackbody, then it is emissive power would simply be equal to sigma T to the power 4 or in this case sigma T s to the power 4, but it is not a black body. So, therefore, its emissive power is going to be epsilon times sigma T s to the power 4 and we have already evaluated epsilon, the hemispherical emissivity of the surface over all the wavelengths to be equal to 0.61. So, 0.61 times sigma T to the power 4 would be the emissive power of the of this wall which is at 500 Kelvin. So, when you plug in the values the value that you are going to get is going to be 218, 2161 watt per meter square.

So, that is the emissive power of the wall, 2161 watt per meter square is the emissive power of the wall. Now, comes the calculation of the absorptivity.

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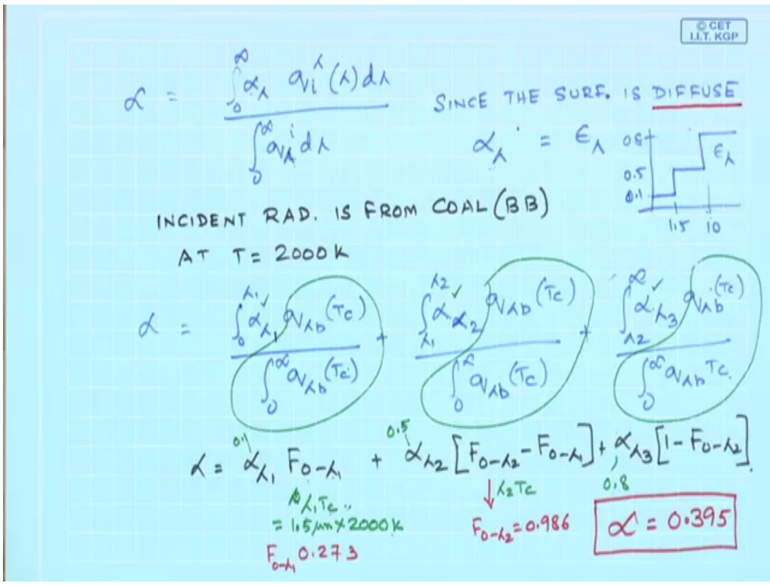


So, how do I calculate alpha? The alpha is defined as 0 to infinity; alpha the spectral value times q i lambda which is a function of lambda times d lambda divided by 0 to infinity q i lambda times d lambda. So, this is the incident radiation; this is the incident radiation spectral incident radiation and this is a spectral absorptivity. Now, since the surface is diffused, this is was surface is diffused, it was mentioned in the problem itself and the keyword is diffused if it is diffuse; then, alpha lambda must be equal to epsilon lambda. So, whatever figure that we had for epsilon would be the same thing for this case as well.

So, this was the value of epsilon lambda which was provided. Since, the surface is diffused the same figure; the same values can be taken as that of the spectral absorptivity. The question that remains is what am I going to do with this remember that this incident radiation is from coal which is approximated at as a at a blackbody at T equals 2000 Kelvin.

So, therefore, the value of alpha can simply be used as before 0 to lambda 1 alpha lambda times q lambda b at the value of T coal divided by 0 to infinity q lambda b. This is T of the coal plus from lambda 1 to lambda 2. So, this is alpha lambda 1, this is alpha lambda 2 q lambda b T c 0 to infinity plus from lambda 2 to infinity alpha lambda. This is lambda 1, lambda 2 and this is lambda 3; 0 to infinity q lambda b T c.

Now, these are constants; we know the values to be equal to 0.1. This value is equal to 0.2 and this value is equal to 0.8. However, this part is simply the black body radiation function. So, this alpha then can be written as before alpha lambda 1 F 0 to lambda 1 plus alpha lambda 2 F 0 to lambda 2 minus F 0 to lambda 1 plus alpha lambda 3 1 minus F0 to lambda 2.

What remains is then to calculate 0 to lambda 1, but we have to keep that in mind is that in this case alpha this F 0 to 1 in this case alpha 1 T c is equal to 1.5 micron into 2000 Kelvin. This is what we are different from the previous one, while we are calculating alpha 1 T s; now we are calculating alpha 1 T c. T s was 500 so, when we were evaluating the emissivity, it is the temperature of the substrate which is important.

When we are calculating the absorptivity, it is the temperature of the source which is important. So, this is epsilon is for the surface substrate and alpha is the temperature of the source which is 2000 Kelvin. So, when you do that the value of F 0 to lambda this would turn out to this you can calculate.

So, this would turn out to be 0.273. So, F 0 to lambda 1 would be 0.273and the value of 0 to lambda 2; so, 0 to lambda 2, this would come out to be 0.986 and so on. And again, it is going to be lambda 2 times T c; lambda 2 is going to be equal to equal to 10 and T c is going to be equal T c is going to be equal to 2000. So, you should be able to find out what is the value of this.

So, once you do this, the value of alpha; the final value of alpha after you put these things. So, this is equal to if I look at the profile again, alpha lambda 1 alpha lambda 1 is 0.1, lambda 2 is 0.5 and this is 0.8 so, this is 0.1, this is 0.5 and this is 0.8. So, when you plug in all these numbers, the value of alpha that you are going to get is 0.395. So, the value of the hemispherical absorptivity is equal to 0.395 ok compare that with the value of the hemispherical emissivity which is 0.61.

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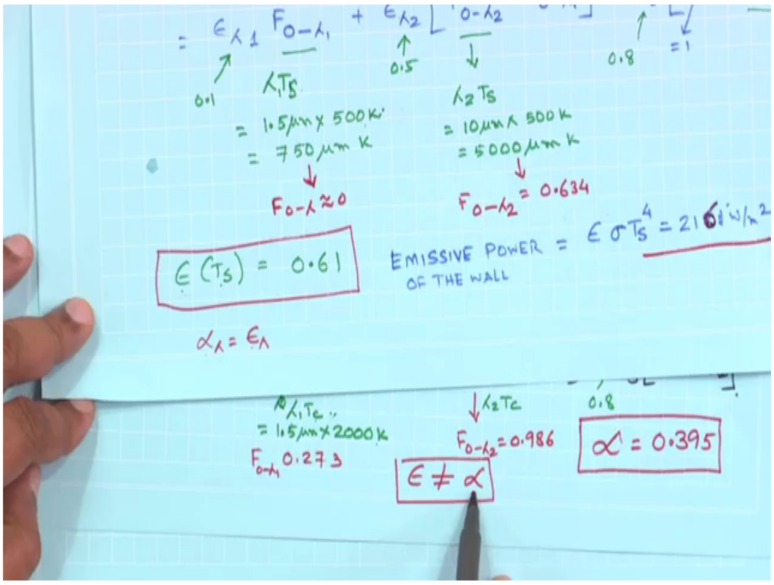


So, even though alpha lambda is equal to epsilon lambda; the epsilon is not equal to alpha that is the message which I would like you to take from this class. So, the spectral values of emissivity and absorptivity maybe the same, but the hemispherical absorptivity may not be equal to hemispherical emissivity. The primary reason for that is the emission is calculated based on the temperature of the substrate itself which is at 500 degree Kelvin; whereas, the alpha the absorptivity is calculated based on the temperature of the source from where the emission is coming.

So, the surface therefore, we can say that the surface is not gray; epsilon is not equal to alpha. The emission was associated with 500 degree Kelvin; whereas the whereas the absorption is related to 2000 degree Kelvin. So, this is a nice exercise which would show you how to evaluate the overall emissivity and the overall absorptivity with the help of black body radiation functions. And to appreciate that even though the spectral values maybe same, the overall values may not be equal; stressing the fact that it is not a gray body.

So, this I considered to be very useful example and there are many such examples both solved and unsolved in the textbook and I would advise you to take a look at them and see how these quantities are calculated using the table of blackbody radiation functions.