Heat Transfer
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Lecture – 50
Weins's Law, Stephen Boltzmann Law, Blackbody Radiation Function, Tutorial Problem

In the previous class, we spoke about the Black body Emissive Power and we wanted to see what is the relation between the Black Body Emissive Power commonly denoted by E b lambda at a given temperature with the black body radiation intensity. And the relation which we have obtained considering a black body of unit area placed at the centre of a hemisphere and measuring what is the total energy emitted per unit time per unit area per unit wavelength by a black body into the hemispherical space. We have obtained the relation between the emissive power and the intensity of blackbody radiation.

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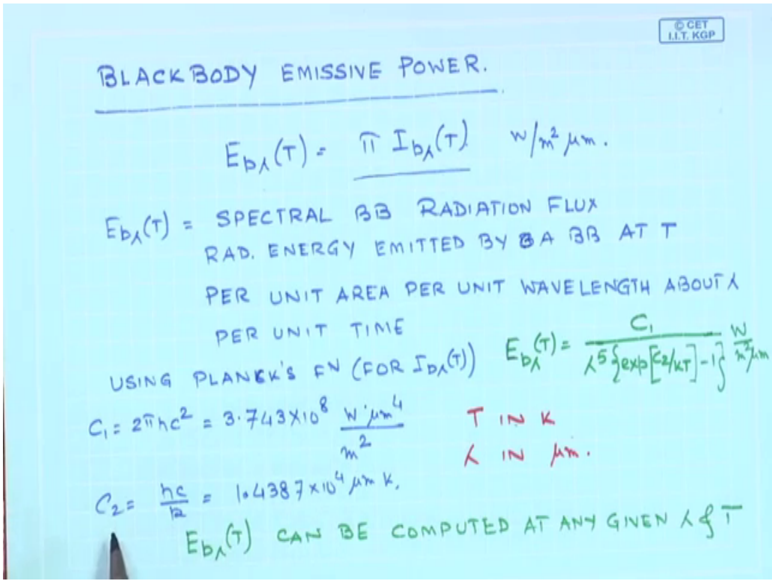

So, the relation that we have obtained is the following where the E b lambda T is the spectral black body radiation flux which is essentially the energy emitted by a black body at any given temperature T. We need to remember that this T, the temperature in radiation is always does always refer to the absolute temperature per unit area per unit wavelength about that wavelength lambda per unit time.

So, this relation between the spectral black body radiation flux and the radiation intensity are simply this pi times radiation intensity is the black body emissive power, which has units of what? Per meter square per micron, where this micron refers to the wavelength of the radiation, that is wavelength of the radiation.

If you remember the units of I b lambda T that has units of watt per meter square per micron watt is the radian, but since we have integrated this over the entire hemispherical space taking into account the variation of the angle. So, therefore, this E b lambda T does not contain any still an any solid angle. So, if this is the formula and we already know what is going to what is through use of Planck's function; what is the expression for I b lambda T.

So, using the Planck's function for I b lambda T, 1 can write E b lambda T as in this form. So, this is in watt per meter square per micron again. The only thing that we have done in this, these expression is substituted the expression for the intensity from Planck's function which we have discussed in the previous class.

So, when you do that, then you get this expression for the black body radiation flux and the C 1 and C 2 are constants. So, C 1 is simply twice by h Planck's constant C. The velocity applied and you calculate the value of this constant and similarly C 2 is h c by k all of them have the usual meaning. So, this is the value of that constant.

So, what this tells us what this expression tells us is that at any given temperature and at any given wavelength E b lambda T can be computed. So, not only you can and so, E b lambda T is a function both of temperature and of wavelength and therefore, if you know the wavelength and the temperature, the black body radiation flux can be computed.

So, provided we know what is the temperature, then the spectral blackbody emissive power, I use the word spectral here purposefully to stress that the black body radiation intensity is will also depend on the wavelength at which we are measuring the radiation flux.

So, all these are strongly dependent on the wavelength. Therefore, since we know what is what is now we know what is the relation between E b lambda T, which is the black body radiation flux with lambda and T, we can plot the black body radiation function as a function of lambda, the wavelength at different values of temperature.

Once again, E B lambda T is a function of wavelength and is a function of temperature. So, keeping one constant in this case the temperature constant I can compute the variation of E b lambda T with lambda and then, I can choose another value of temperature and again compute the same variation of E b lambda T as a function of lambda.

So, I get a series of curves, the series of curves at different values of temperature which shows the variation of the black body radiation flux as a function of wavelength given by the expression which is shown over here. So, a fix lambda I sorry I fix T and plot E b lambda as a function of lambda and then, I choose a different value of temperature and plot this again. Therefore, this would result in a family of curves which would look something like this.

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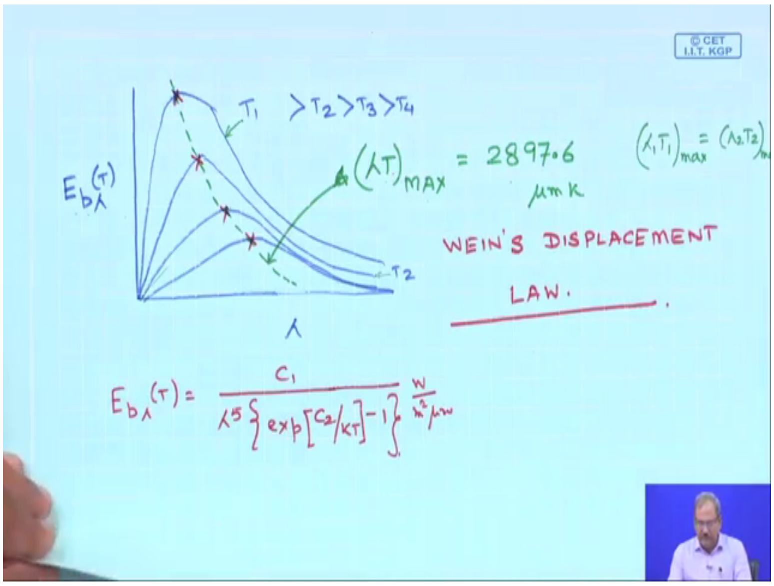

So, the variation of E b lambda T as a function of wavelength will show variations like this. These are at different values of temperature T 1, T 2, T 3 etcetera in this T 1 is greater than T 2 greater than T 3 greater than T 4 and so on. So, the top one, top curve corresponds to the highest temperature, the bottom curve corresponds to the lowest temperature.

So, if these maxima the point at which the E b lambda is a maximum; that means, the blackboard emissive power is maximum, if these points are joined together, the locus of this line is very interesting. It is lambda T corresponding to the maximum value of E b lambda T is simply a constant 2897.6 micron times Kelvin. The always in radiation the wavelength is referred to as a wavelength is expressed in microns and T is an Kelvin's. So, I will go through it once again.

Using the formulation that what we have written in the previous slide, let me write it once again here. Some constant C 1 by lambda to the power 5 exponential this is the formulation, it is in watt per meter square per micron. Using this formulation, I fix temperature. Let us say I have taken the temperature to be equal to T 1 here and this is T 2 T 3 and T 4.

So, if I fix T 1, I can plot E b lambda T as a function of lambda. This is its variation. Then, I fix the temperature at T 2 slightly lower than T 1 and compute this function once again and this is what I am going to get corresponding to T 2. Then, T 3 which is less than T 2 this is the distribution of E b lambda T as a function of lambda. Interestingly, when you look at the maximum of these curves, what you see is that the product of lambda times T corresponding to the max of E b lambda T is a constant.

So, if you find out what is the corresponding lambda and what is the value of T 4; if you multiply them together and you find out what is the corresponding lambda and what is the corresponding temperature T 1; so, essentially it will tell you that lambda 1 T 1 corresponding to the maximum of this must be equal to lambda 2 times T 2 corresponding to the maximum of E b and so on. In this relation that lambda corresponding to maximum E b lambda T is a constant is known as Wein's Displacement Law.

So, what it states is that if you know the temperature, if you know the temperature, then the corresponding value of the wavelength at which E b lambda T is a maximum can be simply obtained by this relation. We will see its use in subsequent our subsequent analysis, but let us do something which is even more which we know even better.

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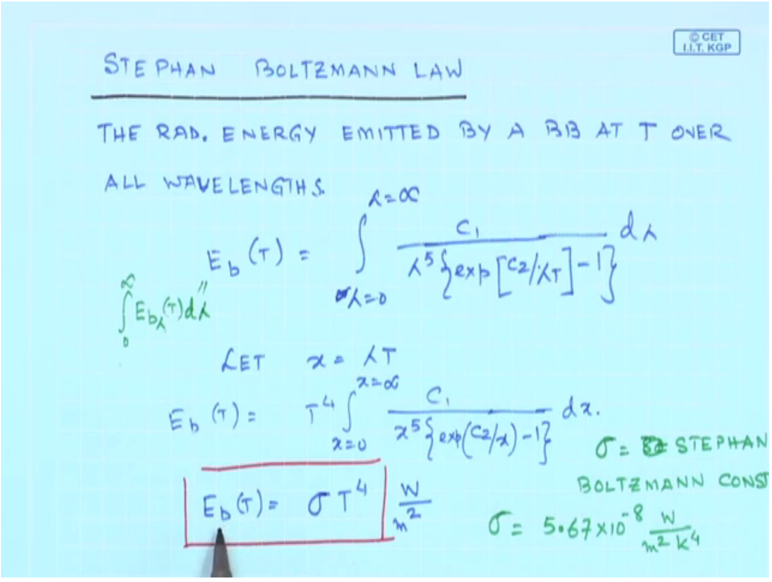

So, the Radiation energy that is emitted by a black body, by a black body at T over all wavelengths so, the expression that I have this is the emission radiation emission by a black body at a specific wavelength.

But we probably have more interested in to finding out what is the total energy emitted by the black body over all possible wavelengths. So, mathematical speaking, I would like to know what is the total energy emitted by the black body over a wavelength range of 0 to infinity. So, if I could find out the total if I would like to find out what is the total energy emitted by the black body.

Then, the spectral black body radiation must be integrated over all possible values of wavelength. So, to convert from E b lambda T to E b T where which does not have any lambda; that means I have must have taken care of all the lambdas as possible. So, in order to obtain E b T from E b lambda T, I need to integrate E b lambda T overall wavelength. So, that is how you obtain the total amount of energy emitted by a black body at an absolute temperature of T, which is what we are going to do next.

So, if you do that if you are trying to find out at all wavelengths, then E b T is from lambda equals to 0 to lambda equals to infinity and the expression for E b lambda T which is this times exponential. Since, what we are just saying that this must be equal to integration from 0 to infinity. E b lambda T times d lambda that is a standard definition and instead of E b lambda, I have put the expression incorporating Planck’s distribution.

So, this integration can simply be evaluated by saying that let some quantity x to be equal to lambda times T. So, your E b T is going to be equal to T to the power 4 x equals to 0 to x equals infinity and this is C 1 by x to the power 5 exponential C 2 by x minus 1 times d x in the when you do this, when you perform this integration. Since it is a definite integral the once you evaluate the integral, it simply going to be going to be a constant and this E b T this constant is expressed a sigma and you have T to the power 4. So, this is probably the most common radiation expression that you have seen before and this is watt per meter square.

So, the expression the unit for E b lambda T is watt per meter square per unit per unit wavelength which is in micron. Since, you have integrated over all possible wavelength. So, therefore, this one will not have this micron over here it simply watt per meter square. And this sigma, you are well aware of this is known as the Stephen Boltzmann Constant. Stephen Boltzmann Constant and the value of this sigma is 5.67 into 10 to the power minus 8 watt per meter square Kelvin to the power 4. In this relation obviously, then is known as the Stephen Boltzmann Law.

So, this is one of the unique laws of radiation which tells you that the total energy emitted by a black body at an absolute temperature T per unit area per unit time is a function of temperature absolute temperature to the power 4 and the constant of proportionality is simply sigma is which is a constant which can be evaluated by evaluating this definite integral and it has a numerical value of 5.67 into 10 to the power minus 8 and this sigma is known as the Stephen Boltzmann Constant.

So, what we then see is there 2 interesting observations from here. The first of all that the black body radiation function, the black body that emissive power of a black body is a function of wavelength and is a function of temperature. Since it is a function of wavelength so that is why the emissive power of a blackbody, we use the adjective spectral before it to underscore the importance of wavelength while specifying the emissive power of a black body.

Now, when you integrated over all possible wavelengths, what you get is the total emissive power of the black body at that given temperature. So, it is the amount of energy emitted by a black body per unit area per unit time overall wavelengths possible. So, that is why its total and I can drop the spectral from the it from the from the a description of the emissive power. The moment I integrate over it and the result of disintegration is the Stephen Boltzmann Law.

In some applications, you are interested to know what is not what is the total power emitted by the black body. You are more interested to find out what is the power emitted by the black body within a certain wavelength range let us say you would like to find out between 2 to 4 micron, how much of the energy is going to be concentrated.

So, the entire blackbody emits radiation starting at lambda equals 0 to lambda equals infinity, but you would you do not want that; you want within a specific range of lambda how much of energy is going to be released, how much energy is going to be emitted by the black body as a fraction of the total energy emitted by the black body.

So, this is an end this is an important parameters which we will keep on using in our subsequent discussion. That means, the fraction of energy emitted by a black body within a specific wavelength range. So, that is that is known as the Black Body Radiation Function.

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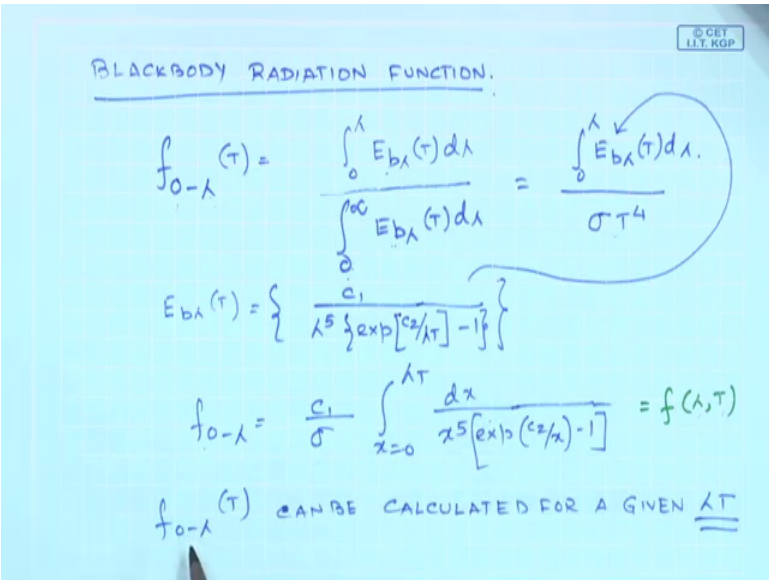

So, we would see what is a Blackbody Radiation Function? So, Blackbody Radiation Function as I have express to you; so, it is between and wavelength from 0 to lambda at a given temperature is the amount of energy emitted by the black body between 0 to lambda divided by 0 to infinity; that means, the total energy emitted by the black body at that a given temperature.

So, once again, the fraction the black body radiation function, the fraction of energy which is emitted by the black body within a wavelength range from 0 to lambda is nothing but the amount of energy emitted by the black body between 0 to delta divided by the amount of energy emitted by the black body over 0 to infinity. So, this is the total energy and this is the amount of energy within a fixed wavelength range.

So, from Stephen Boltzmann's Law, we know that this is simply going to be sigma T to the power 4 and the numerator would be like this T times d lambda. So, if you can introduce the expression of E b lambda T which was as before this expression that we have obtained, C 1 by lambda to the power five exponential C 2 by lambda T minus 1. So, when you put this one in here and you do this do the analysis, what you would get is f 0 to lambda is C 1 by sigma x equals 0 to lambda T dx by x to the power 5; the same way we have done before exponential C 2 by x minus 1.

So, again I have used that x equals lambda times T in the way we have done it for the Stephen Boltzmann equation, this is what you are going to get. Of course, here also you would see that it is a function this fraction is a function of lambda and a function of T as well. So, this integral can be evaluated and f 0 lambda T can be calculated for, can be calculated for a given T given lambda T. So, you in your text you would I guess the in the idea behind this is very clear to you. I am trying to find out the fraction of energy which is emitted by the black body that is going to be concentrated within a wavelength range of 0 to lambda.

So, if this is a fraction, then this is going to be the energy which is going to be within 0 to lambda by the total energy emitted by the black body. Total energy emitted by the black body is nothing but is sigma T to the nothing but sigma T to the power 4 and this can be substituted by the Planck's function as we have done before. When you incorporate that you get the fraction in this form. This is again and integral equation which can be evaluated provided you specify lambda and T. So, once you specify lambda and T, this fraction can be evaluated.

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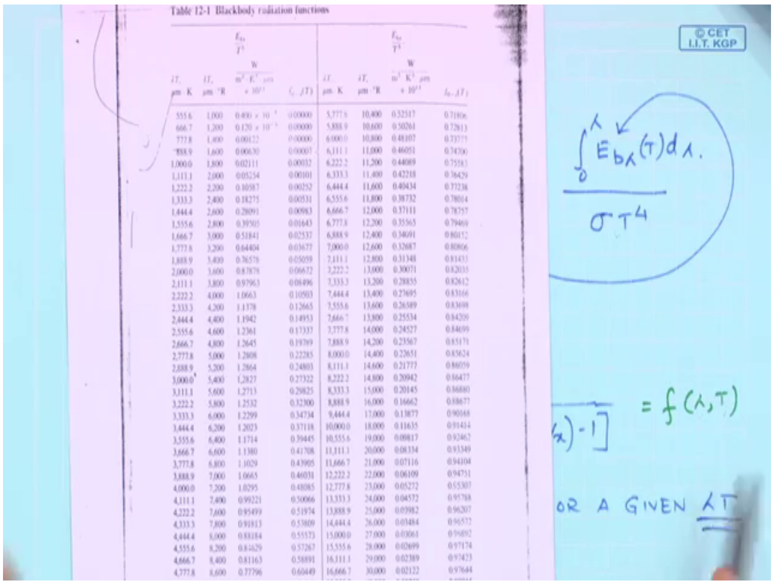

So, this fraction is calculated for a given lambda times T and you would see that the results of the fraction is provided in a table.

So, in that table the total emissive power of a black body within which is contained within the wavelength range is provided as a function of T and as a function of lambda. So, one you must keep in mind that this T is always going to be in Kelvin and lambda is going to be in micron.

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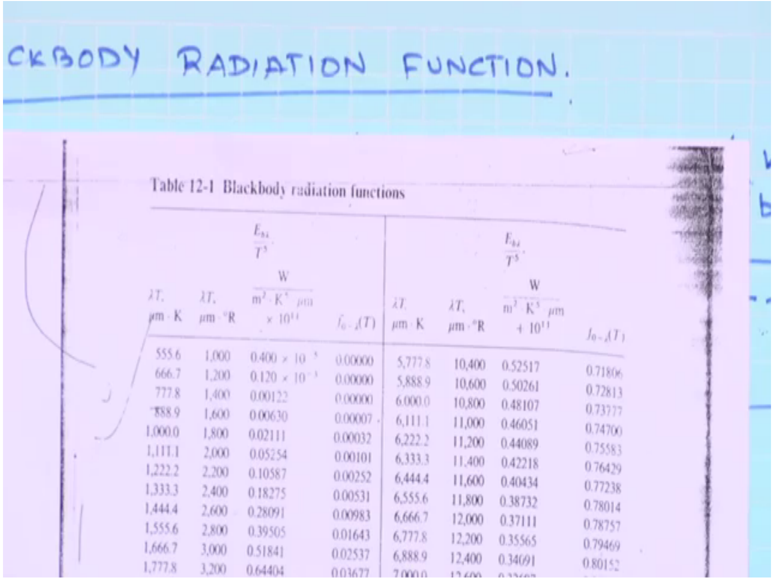

Table 12-1 Blackbody radiation functions

λT, μm · K	λT, μm · °R	$\frac{E_{b\lambda}}{T^5}$, $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$f_{0-\lambda}(T)$	λT, μm · K	λT, μm · °R	$\frac{E_{b\lambda}}{T^5}$, $\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$f_{0-\lambda}(T)$
555.6	1,000	0.400×10^{-5}	0.00000	5,777.8	10,400	0.52517	0.71806
666.7	1,200	0.120×10^{-3}	0.00000	5,888.9	10,600	0.50261	0.72813
777.8	1,400	0.00122	0.00000	6,000.0	10,800	0.48107	0.73777
888.9	1,600	0.00630	0.00007	6,111.1	11,000	0.46051	0.74700
1,000.0	1,800	0.02111	0.00032	6,222.2	11,200	0.44089	0.75583
1,111.1	2,000	0.05254	0.00101	6,333.3	11,400	0.42218	0.76429
1,222.2	2,200	0.10587	0.00252	6,444.4	11,600	0.40434	0.77238
1,333.3	2,400	0.18275	0.00531	6,555.6	11,800	0.38732	0.78014
1,444.4	2,600	0.28091	0.00983	6,666.7	12,000	0.37111	0.78757
1,555.6	2,800	0.39505	0.01643	6,777.8	12,200	0.35565	0.79469
1,666.7	3,000	0.51841	0.02537	6,888.9	12,400	0.34091	0.80152
1,777.8	3,200	0.64404	0.03677	7,000.0	12,600	[illegible]	[illegible]

So, if you look at the table now, the table here shows you; the table here shows you the value of lambda T in micron and this is E b lambda divided by T to the power 5. This is how the fraction is this is how it is shown. So, you know what is lambda T. Let us say you pick a value of lambda T to be over here. Lambda T is 2777.8. So, any combination of lambda and T which gives the products to be about 2777, it would give you a value of the f as 0.22.

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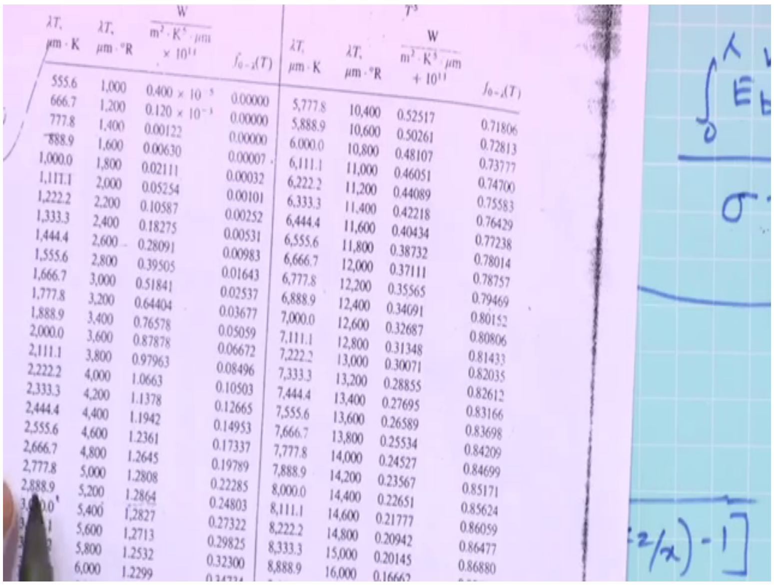

λT, μm·K	λT, μm·°R	$\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$f_{0-\lambda}(T)$	λT, μm·K	λT, μm·°R	$\frac{W}{m^2 \cdot K^5 \cdot \mu m} \times 10^{11}$	$f_{0-\lambda}(T)$
555.6	1,000	0.400×10^{-5}	0.00000	5,777.8	10,400	0.52517	0.71806
666.7	1,200	0.120×10^{-3}	0.00000	5,888.9	10,600	0.50261	0.72813
777.8	1,400	0.00122	0.00000	6,000.0	10,800	0.48107	0.73777
888.9	1,600	0.00630	0.00007	6,111.1	11,000	0.46051	0.74700
1,000.0	1,800	0.02111	0.00032	6,222.2	11,200	0.44089	0.75583
1,111.1	2,000	0.05254	0.00101	6,333.3	11,400	0.42218	0.76429
1,222.2	2,200	0.10587	0.00252	6,444.4	11,600	0.40434	0.77238
1,333.3	2,400	0.18275	0.00531	6,555.6	11,800	0.38732	0.78014
1,444.4	2,600	0.28091	0.00983	6,666.7	12,000	0.37111	0.78757
1,555.6	2,800	0.39505	0.01643	6,777.8	12,200	0.35565	0.79469
1,666.7	3,000	0.51841	0.02537	6,888.9	12,400	0.34091	0.80152
1,777.8	3,200	0.64404	0.03677	7,000.0	12,600	0.32687	0.80806
1,888.9	3,400	0.76578	0.05059	7,111.1	12,800	0.31348	0.81433
2,000.0	3,600	0.87878	0.06672	7,222.2	13,000	0.30071	0.82035
2,111.1	3,800	0.97963	0.08496	7,333.3	13,200	0.28855	0.82612
2,222.2	4,000	1.0663	0.10503	7,444.4	13,400	0.27695	0.83166
2,333.3	4,200	1.1378	0.12665	7,555.6	13,600	0.26589	0.83698
2,444.4	4,400	1.1942	0.14953	7,666.7	13,800	0.25534	0.84209
2,555.6	4,600	1.2361	0.17337	7,777.8	14,000	0.24527	0.84699
2,666.7	4,800	1.2645	0.19789	7,888.9	14,200	0.23567	0.85171
2,777.8	5,000	1.2808	0.22285	8,000.0	14,400	0.22651	0.85624
2,888.9	5,200	1.2864	0.24803	8,111.1	14,600	0.21777	0.86059
3,000.0	5,400	1.2827	0.27322	8,222.2	14,800	0.20942	0.86477
[illegible]	5,600	1.2713	0.29825	8,333.3	15,000	0.20145	0.86880
[illegible]	5,800	1.2532	0.32300	8,888.9	16,000	0.16662	[illegible]
[illegible]	6,000	1.2299	[illegible]				

So, let us write it over here if I write it in my in this so, for a value of lambda T equal to 2777, the value of the fraction is 0.2285. So, for lambda T equals 2777.8, the value of f is going to be equal 0.22. So, which it means which means is that you choose some value of lambda and T, the product comes to be comes to comes to close to 2777.8. So, if that is the case, then within that wavelength range at that given temperature, you can find out what is the fraction of energy that is emitted by the black body is contained within the wavelength.

And I think it would be more clear to you once we solve the problem using the table and this f the black body radiation function, the table of blackbody radiation function you can refer to and find out that given the value of lambda and T what is the fraction that is going to be, what is a black body radiation function. So, you will quickly solve 1 problem and I think that would clarify any remaining doubts that you may have. But fundamentally, we are finding out the fraction of energy which is contained within 0 to some lambda at a given temperature for a for the blackboard emissions. So, let us quickly solve 1 problem and then we will move on.

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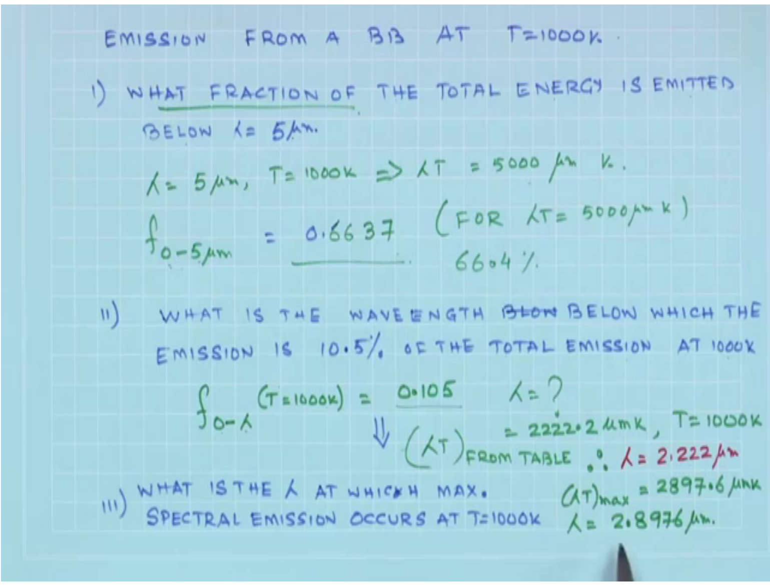

So, what the problem says is that the emission is from a surface, from a black body at T equals 1000 Kelvin. The first part is what fraction of the total energy is emitted below lambda equals 5 micron. So, what we need to do then is this my lambda is 5 micron, T is 1000 Kelvin. So, lambda T is 5000 micron times Kelvin.

So, the when would sees fraction. So, I would like to find out 0 to 5 micron. The value of the black body radiation function between 0 to 5 micron, fundamentally what this tells us is f 0 to 5 is the fraction of the total energy that is emitted by a black body at a given temperature between the range 0 to 5 micron. So, from the adjoining table, I need to find out what is the value of f of 0 to 5; 0 to 5 micron.

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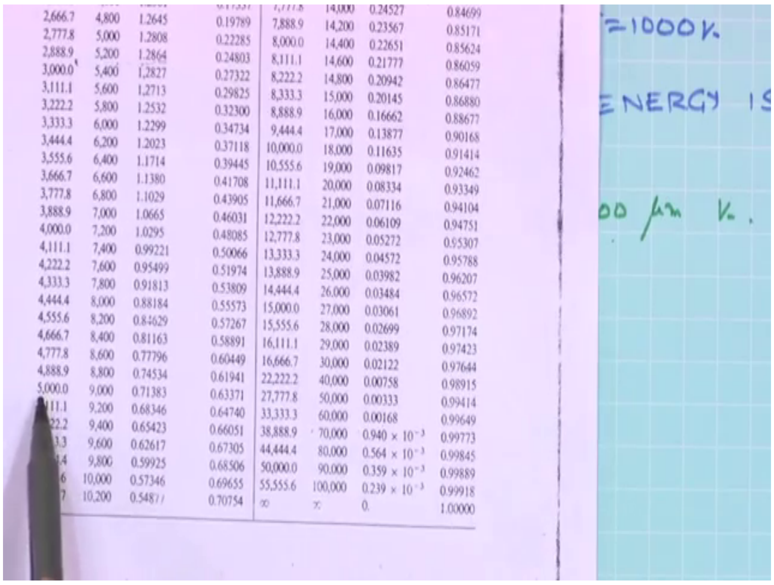

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14,000	0.24527	0.84699
2,666.7	4,800	1.2645	0.19789	7,888.9	14,200	0.23567	0.85171
2,777.8	5,000	1.2808	0.22285	8,000.0	14,400	0.22651	0.85624
2,888.9	5,200	1.2864	0.24803	8,111.1	14,600	0.21777	0.86059
3,000.0	5,400	1.2827	0.27322	8,222.2	14,800	0.20942	0.86477
3,111.1	5,600	1.2713	0.29825	8,333.3	15,000	0.20145	0.86880
3,222.2	5,800	1.2532	0.32300	8,888.9	16,000	0.16662	0.88677
3,333.3	6,000	1.2299	0.34734	9,444.4	17,000	0.13877	0.90168
3,444.4	6,200	1.2023	0.37118	10,000.0	18,000	0.11635	0.91414
3,555.6	6,400	1.1714	0.39445	10,555.6	19,000	0.09817	0.92462
3,666.7	6,600	1.1380	0.41708	11,111.1	20,000	0.08334	0.93349
3,777.8	6,800	1.1029	0.43905	11,666.7	21,000	0.07116	0.94104
3,888.9	7,000	1.0665	0.46031	12,222.2	22,000	0.06109	0.94751
4,000.0	7,200	1.0295	0.48085	12,777.8	23,000	0.05272	0.95307
4,111.1	7,400	0.99221	0.50066	13,333.3	24,000	0.04572	0.95788
4,222.2	7,600	0.95499	0.51974	13,888.9	25,000	0.03982	0.96207
4,333.3	7,800	0.91813	0.53809	14,444.4	26,000	0.03484	0.96572
4,444.4	8,000	0.88184	0.55573	15,000.0	27,000	0.03061	0.96892
4,555.6	8,200	0.84629	0.57267	15,555.6	28,000	0.02699	0.97174
4,666.7	8,400	0.81163	0.58891	16,111.1	29,000	0.02389	0.97423
4,777.8	8,600	0.77796	0.60449	16,666.7	30,000	0.02122	0.97644
4,888.9	8,800	0.74534	0.61941	22,222.2	40,000	0.00758	0.98915
5,000.0	9,000	0.71383	0.63371	27,777.8	50,000	0.00333	0.99414
[illegible]	9,200	0.68346	0.64740	33,333.3	60,000	0.00168	0.99649
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[illegible]	9,600	0.62617	0.67305	44,444.4	80,000	0.564×10^{-3}	0.99845
[illegible]	9,800	0.59925	0.68506	50,000.0	90,000	0.359×10^{-3}	0.99889
[illegible]	10,000	0.57346	0.69655	55,555.6	100,000	0.239×10^{-3}	0.99918
[illegible]	10,200	0.5487	0.70754	∞	∞	0.	1.00000

So, for which case lambda T is 5000 micron Kelvin. So, if you look at the value over here for a value of lambda T to be equal to 5000 micron Kelvin, the value of f is 0.6671. So, from the table, I read that this is equal to 0.6637 for lambda T equals 5000 micron Kelvin which simply tells me that 66.4 percent or 0.664; the fraction 66.4 percent of the total energy is emitted by the black body at 1000 Kelvin, below lambda equals 5 micron; that means, this is the value of the of the fraction.

Second is, second question is what is the wavelength below which the emission is 10.5 percent of the total emission at 1000 Kelvin? So, it is reverse of the previous problem. In this case f 0 to the unknown lambda at a given temperature of 1000 Kelvin is provided to be 0.105. We need to find out what is the value of lambda. So, once again the emission is 10.5 percent of the total emission at 1000 Kelvin. So the fraction which of energy which is contained between 0 to the unknown lambda at 1000 degree Kelvin is specified to be 0.105. We need to find out what is the value of lambda for such a case.

Once again, we look at the table and when we look at the table, now we have to look at lambda T to be lambda T to be the fraction to be 0.105. So, when you look at the fraction to be 0.105, you read the value of lambda T to be 2222; four 2's point 2 micron Kelvin. So, this refers to the value of lambda T. This you read from the table to be equals four 2's point 2 micron Kelvin and since, you know the temperature to be equals 1000 Kelvin. So, therefore, your lambda is simply going to be 2.22 micron.

So, at 2000 degree Kelvin at 2, sorry at 1000 degree Kelvin, the emission is going to be 10.5 percent of the total emission at and wavelength of 2.2 micron and the third one is. So, what is the wavelength? What is the lambda at which the maximum spectral emission occurs at T equals 1000 Kelvin?

So, the third question is what is the wavelength at which you are going to get the maximum spectral emission? The moment I say spectral that means, its wavelength depended so, the value of the temperature is provided to you; you have to find out what is the wavelength at which the emission is going to be maximum.

So, of course the law that his that we need to use for this is Wein's Displacement Law. The Wein's Displacement Law simply states that lambda times T is a constant when we are talking about the maximum spectral radiation intensity at a given temperature. Since the temperature over here is provided we need to find out what is the lambda in this case. So, this problem can simply be solved using Wein's Law as lambda times T corresponding to the maximum emission is 2897.6 micron Kelvin. So, for T equals 1000 degree, this lambda would simply be equals to 2.8976 micron. So, therefore, if your temperature is at 1000 Kelvin, then the maximum the spectral maximum of emission from a black body at 1000 degree Kelvin will take place at a wavelength of 2.8 micron.

So, what we have discussed, if I would like to do a summary of this is that looking at the black body black body emission when incorporating the Planck's formula in it, we have derived the Wein's Displacement Law which shows that the product of lambda and T corresponding to maximum emission maximum spectral emission from a black body is a constant which is 2897.6 micron Kelvin.

Secondly, the spectral emission if you would like to convert it to the total emission from a black body over all possible wavelengths that would give rise to the well known Stephen Boltzmann Law where the proportionality between the emission and the temperature is E equals E is proportional to T to the power 4 and the proportionality constant is Planck's constant.

Next, we wanted to know the fraction of energy which is emitted between a certain wavelength range as compared to the total energy emitted by the black body overall wavelength range. So, this black body radiation function is evaluated for different values of the product of mu and T, lambda and T different values of the product of the wavelength and temperature.

So, these black body radiation functions provide us with the knowledge of how much of energy is going to be emitted by a black body at a given temperature within a specific range of wavelengths. And we have solved the problem numerical problem which I think would clarify any doubts that you may have in the use of Wein's Displacement Law, the Stephen Boltzmann Law and the Blackbody Radiation Function.