Heat Transfer
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Lecture - 49
Spectral Blackbody Radiation Intesity and Emissive Power

We will discuss more about radiation in this class. In the last class we just introduced you to radiation, the requirement of not having any medium; that means, a radiation can take place without any medium being present. The variety of properties of a substrate in terms of reflectivity, absorptivity and transmitivity and the concept of an ideal body in terms of radiation, which is termed as a black body which can emit; whose emission at a given temperature is the maximum, it is going to absorb everything that falls into it. And, it is a diffuse emitter; that means, the emission coming out of a black body does not have any directional dependence.

We have also introduced the concept spectral; that means, anything which depends on the wavelength at which we are considering at which the emission is taking place. So, all these quantities, all these properties namely the reflectivity, the transmitivity and the absorptivity are spectral in nature; that means, the absorption of a radiation is going to be different at different values of the wavelength. So, the spectral nature of radiation concept of a black body and that the radiation can be treated in 2 ways, the one is based on the wave nature and the second is based on the quantum concept these were introduced in the last class. In this class we are going to know more about 2 fundamental properties of a black body.

That means, first is going to be: what is the black body radiation intensity, when a black body is at a given temperature. So, how much of energy per unit area per unit wavelength and per unit solid angle we will introduced the concept lateral that this black body is emitting. So, that is the spectral black body radiation intensity.

And secondly, we will see what is going to be the total emissive power of a black body at a given temperature; that means, if a black body having unit area is placed at the centre of a hemisphere, how much would be the total energy emitted by this black body of unit area at a given length and at a given temperature. So, these are the two main concepts which would cover in today's class.

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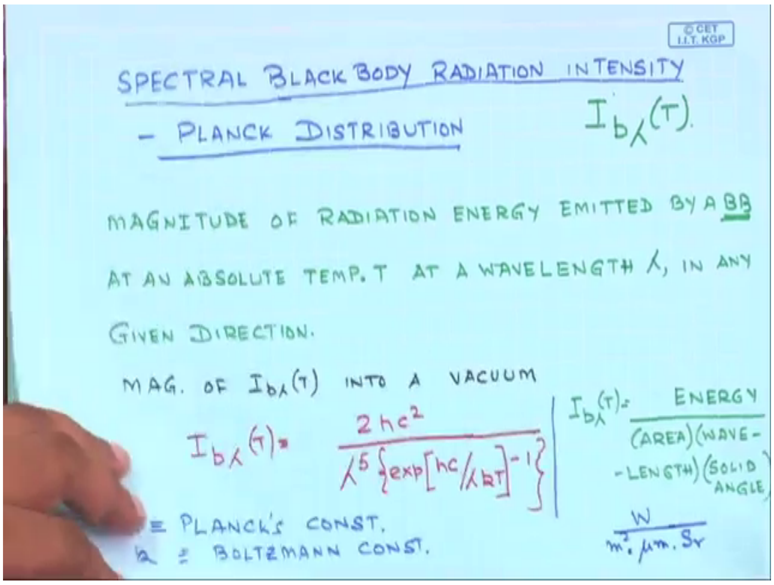

So, let us first start with black body radiation intensity in which is going to so, we denote that as I the intensity, b stands for the black body, lambda denotes the wavelength dependence and of course, it is going to be different and different temperature. So, the temperature has to be specified while we are discussing about the black body radiation intensity. And in this case we are going to rely on the Planck's distribution. So, what is black body radiation intensity? It is defined as the magnitude of radiation energy which is emitted by a black body at an absolute temperature T at any wavelength lambda in any given direction.

So, it gives you the idea the quantum of radiation energy which is emitted at in any given direction and at a specific wavelength. So, the radiation energy of course, would therefore, depend on the temperature, it is going to depend on the lambda, the wavelength and it will have a directional property.

So, using Planck's distribution the magnitude of the black body radiation intensity can be expressed for the case of vacuum. So, the magnitude of the black body radiation intensity, spectral black body radiation intensity at a given temperature into vacuum is expressed in this form. Where h is the well known Planck's constant. So, h is the Planck's constant, the value of which is available in any text book I am not writing it over here; c is the, if c is the velocity of light and the k the k that you see over here is the Boltzmann constant. So, that is also the value of which is also available in your text.

So, if you if you think about what would be the units of the black body radiation intensity from the definition we understand that it is going to be energy per unit area per unit wavelength per unit solid angle because, we are talking about in any given directions. So, I will tell you what this solid angle is all about, but a the definition got the units of the black body radiation intensity therefore, can be expressed in terms of energy which is watts, area, wavelength is customarily expressed in terms of micron. So, that is why the micron is there in the denominator and the solid angle is the unit of solid angle is steradian. So, steradian comes into the denominator.

So, the units of the black body radiation intensity I b lambda T, the spectral blackbody radiation intensity is watt per meter square per unit per micron per steradian. So, what is a solid angle let us since we are going to use the concept of solid angle.

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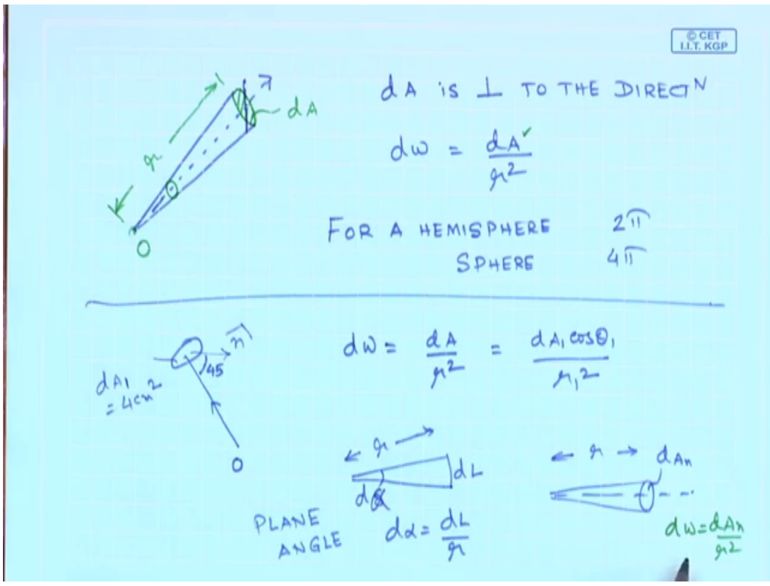

You are probably aware of this solid angle, but I will still do this one more time. So, let us see this is the over here therefore, it is sort of a cone which is my area of interest let us say the area is dA. The distance from the centre from this point which I denote it as O let it be r. So, this is the solid angle subtended by the area dA at the centre and this dA is normal perpendicular to the direction.

So, therefore, this dA perpendicular to the dotted line and the solid angle is defined as the area times r square. Had this area been at an angle with the line which comes out of this origin the specifying the direction then the projection of this area perpendicular to the direction has to be provided. That means, dA has to be substituted by dA cos theta if this is the area through which the radiation is passing through. So, therefore, the projection of this area on to this side has to be substituted for dA. So, dA is therefore, perpendicular to the direction and this has to be kept in mind.

So, if you extend this so, for a hemisphere this if the from the centre, from its centre the solid angle is simply going to be equal to twice pi. And when you consider as full sphere this is simply going to be equal to 4 pi. So, these are obvious; that means, what is going to be, what is going to be this value of the solid angle for the case of a hemisphere and for the case of a sphere. So, as before if let us say as I was telling you if this is the origin and I have some area which is not perpendicular to which is not perpendicular to this and therefore, let us see the area vector makes an angle of 45 degree with this direction, with this direction.

And let us say if this d area, the area is about 4 centimetre square. So, this is the area vector which is always perpendicular to this area. So, therefore, using the definition d omega the solid angle as dA by r square and this dA is simply going to be the area which is let us dA 1. So, this must be equal to dA 1 the projection of dA 1 in this direction. So, this is going to be dA 1 cos theta 1 divided by r 1 square. So, this is how the solid angle is evaluated.

So, if it is just part of a circle and if this is dL at a distance of r then the plane angle this alpha let us call it as d alpha. So, d alpha is simply going to be dL by by r, that is this standard relation when we were talking about a plane angle. And, when we are talking about a solid angle like this, if this is the distance is r and this area is dA n then in that case the d omega is simply going to be dA n by r square, where A n is the area normal to this direction. So, that is what we call is that plane angle and what is known as the solid angle. So, next is important because, next going to give us some more insights into the whole process.

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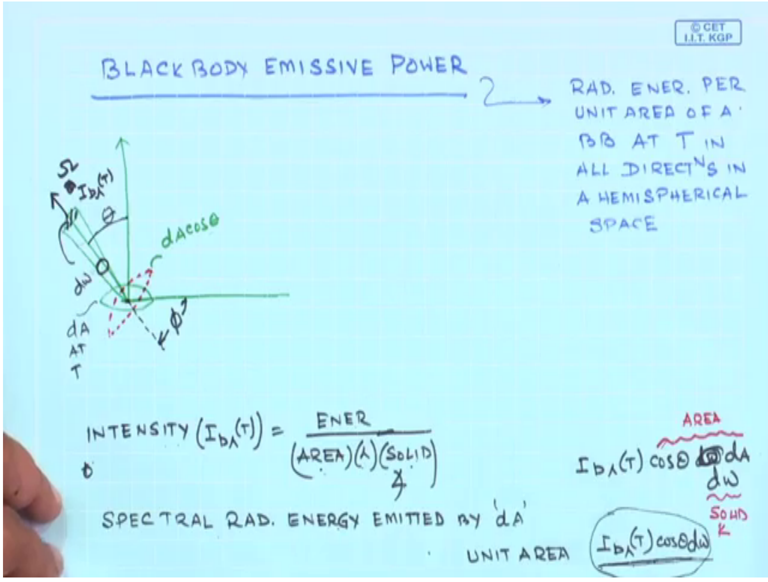

And it is a very important quantity which is known as the black body emissive power. So, what is a black body emissive power? The expression for which we need to find out so, what is black body emissive power? It is the radiation energy per unit area of a black body at T at an absolute temperature T in all directions in all directions in a hemispherical space. So, this is what is known as the black body emissive power and we need to find out what would be it is expression based on our knowledge of blackbody radiation intensity.

So, whereas, black body radiation intensity is in a given direction the black body emissive power is in all directions in a hemispherical space. So, the difference between these 2 must be kept in mind in one case the radiation intensity specifies a direction. So, it is the direction which is specified for the case of radiation intensity whereas, for the case of emissive power it is assumed that the black body is placed at the centre of a hemisphere.

And, we are trying to see what would be the total energy emitted by this black body of unit area which is placed at the centre of the hemisphere in all possible directions. So, we need to find out we understand that this I b lambda T which we have defined previously it has a directional dependence. So, the solid angle subtended by the area at a distance from the centre at a distance from the centre is used to obtain what is the black body radiation intensity. Now, what we have to do is we have to make this area which is subtending a solid angle over here travel in such a way that it defines a hemisphere.

So, if we can integrate the area in such a way that the entire hemispherical dome over this unit surface area can be specified then the total energy which passes in all directions in the hemispherical space at a given temperature will be known as the black body emissive power. And, that is what we are going to evaluate we are going to derive in this in this part of the class.

So, from the intensity now we are going towards emissive power. In one case the area is fixed it is dA 1 and we are trying to see what is the solid angle in the other case this area is essentially the area of the hemisphere. So; obviously, the solid angle will change depending on where this unit area is placed. So, this has to be taken into account while evaluating this. So, let us draw the draw this first and see what we get out of this. The first one let us say we have this is the area vector, this is the area, we would like to find out what is going to be the intensity in this specific direction.

So, this is the preferred direction let us call it is omega and in this direction the intensity is I b lambda T. The area over here is this is the area we are talking about and initially this is the area dA at a temperature T. But, as you can see it is not perpendicular it is not perpendicular to the direction of propagation of to the direction in which I would like to find out I b lambda T.

So, what I do is I try to see: what is the azimuth angle of this area over here and this being the angle theta. So, what I do is I will draw the projection of the green one dA and therefore, this area is simply going to be dA cos theta. So, my dA cos theta is now perpendicular to the direction in which I would like to find out what is I b lambda T. Once again I b lambda T is the black body radiation intensity. In fact, spectral black body radiation intensity at a given temperature in this at a specific direction which is a this direction.

The object from where this intensity is coming is dA. So, I am going to take a projection of these 2 make the area of perpendicular to the direction. So, therefore, this area is to going to be dA cos theta. So, if I write the intensity in this case which is I b lambda T should equal to energy per unit area per unit wavelength and per unit solid angle. So, this is my definition of the energy.

So, the spectral radiation energy let us the spectral radiation energy emitted by emitted by dA by dA the surface element dA in which which through an elemental solid angle d omega. So, this angle solid angle is d omega. So, this d omega I would like to find out how much of how much of spectral radiation energy emitted by dA passes through the solid angle d omega. So, this is the energy which is contained within this tube that I would like to find out ok.

So, this must be equal to I b lambda T then cos theta d omega times dA d omega. So, this energy; obviously, would be in order to obtain the energy I need to multiply the intensity with area. So, this is going to be my area, the solid angle is this. So, this is the solid angle, this is this is what is going to be the energy the spectral the spectral energy. So, if I looked like to find out the energy by unit if I want to do it with in terms of unit area, it should simply be equal to I b lambda T cos theta d omega.

I will go through it once again; my intensity is defined as energy per unit area per unit wavelength per unit solid angle ok. So, the area is dA which makes an angle of theta with the area vector. This d this dA and I would like to find out how much of energy is going through this tube which forms a solid angle dA at this point. So, in order to do that the first thing is I need to make sure that this area is placed in a direction perpendicular to this.

So, which is going to be dA cos theta and this is this is this is the azimuth angle which we will discuss later on. So, the energy per unit wavelength or in other words the spectral energy would therefore, be the product of intensity times area which is perpendicular to the direction times the solid angle. So, that in then it should be the spectral radiation energy emitted by dA would be the intensity times area which is dA cos theta times solid angle which is d omega. If you like to find out what is the spectral radiation energy emitted by an unit area I simply divided by dA and this is the expression of the spectral radiation energy emitted by and unit area instead of dA and unit area.

So, this is the quantity which I am going to use, but in order to effectively use this quantity I need another factor ok. So, another figure so, the figure that I am going to use going to draw is this one.

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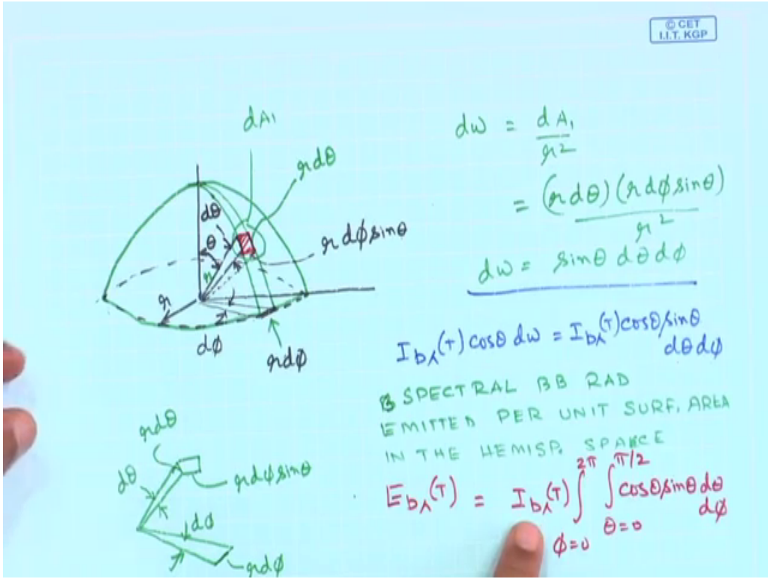

So, this is a hemisphere which I am drawing of base radius r and this is a slice of that which where the angle is d phi. And once I draw this it would be clear to clear to you. The angle over here is theta and this angle, this small angle is d theta ok. And this is d phi so, this one must be equal to r times d phi because this is a plane angle where the radius is r and the angle is d phi. So, the length of the chord must be equal to d phi. This when we go all the way up to 90 minus theta I get this line, if I go all the way to 90 I get a point. If I go up to this point this is simply going to be r d phi sin theta ok.

So, when sin theta becomes equal to 90 which is at the top then would this would be equal to 0. So, the length of the line will vanish and it will become a point. When you think of this one, this is d theta and the length the this thing the radius is r. So, therefore, this one is simply going simply going to be equal to r times d theta.

So, if I project it slightly in a better way this is what you get as r d phi. In when you go all the way up to this point so, this is d phi, this is d theta. So, what you get like this the area this length is r d theta and this length has to be r d phi sin theta. So, my area which is defined by the angles phi and theta is r d theta sin theta is 1 one dimension, the other dimension is r times d theta.

So, therefore, from this figure my d omega the solid angle is simply going to be equal to dA 1 divided by r square where dA 1 is this area. So, this area is dA 1, the area which is denoted by the red crosses and this dA 1 is simply the product of these 2 length scales.

So, it is going to be r d theta is one length one side the other side is r d phi sin theta divided by r square. So, this d omega is simply sin theta d theta d phi that is going to be that is going to be the solid angle.

So, when we go back to this figure once again I have my spectral radiation energy emitted by unit area is I b lambda T cos theta times d omega. So, I need to put instead of d omega in this expression the d value of d omega that I have obtain. So, therefore, I b lambda T cos theta d omega would simply be equal to I b lambda at constant temperature times cos theta sin theta d theta d phi. So, this is this is going to be the spectral radiation energy emitted by an unit surface area element through which substance a solid angle equal to phi over here now.

Now, I would like to make integrate this expression in such a way such that this area is going to represent the entire hemispherical area. So, if I can do the integration in such a way that this area is going to encompass the entire hemispherical area. So, I can see that in that integration my theta is going to vary from 0 to 90 degree, 0 to 90 degree and my phi is going to vary from 0 to 2 pi that is what the variation is going to be. Let us look at it to another way this is what I am trying to do.

So, in order to create the hemispherical space my phi is going to be from 0 to 2 pi whereas, my theta is going to be from 0 to phi. So, if I can let this area travel in terms of theta from 0 to 90 degree and in terms of phi from 0 to 2 pi then this area encompasses the entire hemispherical area available to exposed when the black body is placed at the centre.

So, the black body the spectral skill depends on the wavelength the spectral blackbody radiation which is emitted per unit surface area in the hemispherical space in the hemispherical space. Hemispherical space which is denoted as E b lambda T this is what is the spectral black body radiation emitted per unit surface area into the hemispherical space would simply be I b lambda T which is outside of the integration sin. And phi would be from 0 to 2 pi and theta would be from 0 to pi by 2 and inside would be cos theta sin theta d theta d phi.

One more time since it is black body radiation intensity so, it is independent of the direction, since the black body radiation is diffuse. So, I b lambda T does not depend on the direction and therefore, it can be taken out of the integration sin. So, what is left is cos theta sin theta d theta d phi and as I have explained the theta the phi is going to vary from 0 to 2 pi and theta is going to vary from 0 to pi by 2. If I perform this integration I have an expression of E b lambda T in terms of I b lambda T. So, what that expression would be? Once you perform this one once you perform this integration it is there in your text.

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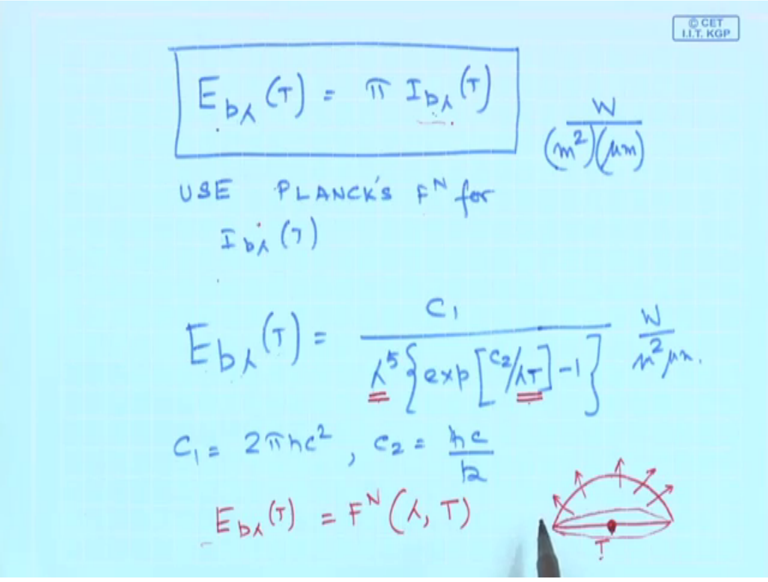

I am not going to do it over here it is a very simple integration. This is the expression for body radiation the spectral blackbody emissive power. So, since it is so, it is energy per unit area per unit wavelength that is going to be it is unit energy per unit area per unit wavelength. So, the spectral black body radiation intensity is related to spectral emissive power of an unit based on an unit area. In a hemispherical space is denoted by specific relation.

So, when you use function for I b lambda T E b lambda T would be c 1. So, the emissive power the spectral emissive power of a black body of unit area at a given temperature is provided as a function of wavelength and as a function of temperature. So, this is the important part.

So, E b lambda T is a function of lambda and is a function of temperature. This specific expression will be utilised in the next class to show how the emissive power of a blackbody spectral emissive power of a black body depends on the wavelength and depends on the temperature.

So, what we have done in this class is if we have placed a black body of unit area inside hemispherical dome. And, we have found out what is the total emissive power of this black body having unit area at a given temperature, what is the spectral power emissive power of black this black body into the hemispherical space.

So, this is what it looks like, in this hemispherical space how much of radioactive energy a black body of unit area at a given temperature is providing. So, this is related to the intensity of radiation and we know the intensity of radiation through the use of Planck's function.

And then we can find out E b lambda T as a function of the wavelength and as a function of temperature. So, this functional relation we will explore a bit further and then you would see that it is going to give rise to certain relations that we know of; for example, the most common example or relation of radiation the Stefan Boltzmann law can be directly derived from this blackbody emissive power.

So, we will do that in the next class.