Heat Transfer
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Lecture – 28
Tutorial Problem on External Flow and Behavior of Heat Transfer Coefficient

The next two classes would be tutorial ones where I am going to solve some problems on convective heat transfer when the flow is taking over a flat plate. In the last class, we solve one problem in which the flow was taking place over an extremely rough surface and we use the analogy to obtain what is the value of the heat transfer coefficient and so on. But this class is going to be more on problems in which we would directly identify the flow pattern, whether it is laminar or turbulent and calculate the value of the heat transfer coefficient, the local value of the heat transfer coefficient and for this specific problem, we would see what is going to be the heat transfer between two streams of air at different temperature but separated by a thin metal wall of high conductivity.

So, it is a flat sheet of metal, thin with high thermal conductivity and flow is taking place over the plate as well as under the plate and the two streams of air which are following on the top at and through and at the bottom of the plate, they are at different temperatures. So, it which we have to find out what is the value of the local heat transfer in between the two air streams through the thin plate which is the separating plate in between the two types of flows.

So, it is more or less straight forward problem, where we need to find out what is the local value of heat transfer coefficient at the specified location and we know that the inverse of the heat transfer coefficient would give us the resistance to convective heat transfer.

So, the total amount of heat transfer, the heat flux, the heat flux between two streams would be provided by the temperature difference between the two streams of air which are flowing at the top and below the plate divided by the sum of the resistances, sum of the convective resistances. Since the plate is thin, we may not have to consider the conductive heat transfer the conductive heat transfer resistance for heat flow through the plate.

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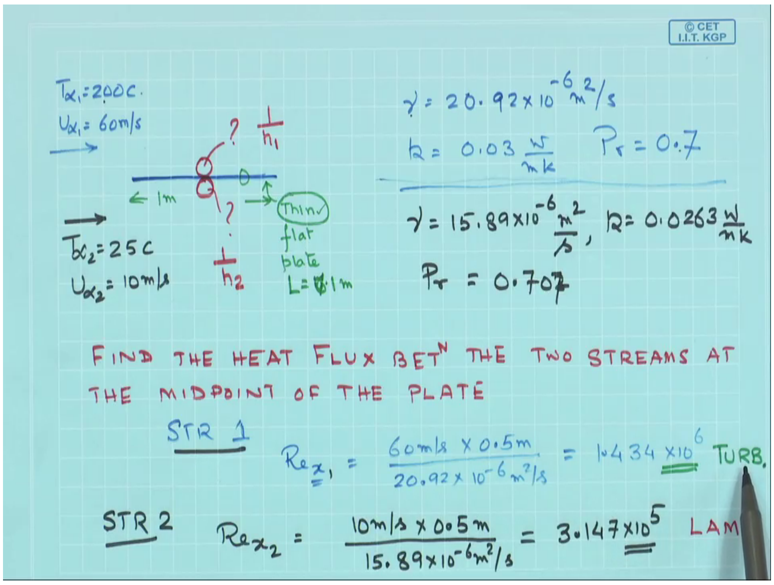

So, let us look at the diagram which depicts the flow situation. So, this is a thin flat plate and we have an air flow at 200 degree centigrade, but at 60 meter per second which is flowing over the plate and a temperature of 25 another air flow, at a temperature of 25 degree centigrade and velocity of 10 meter per second which is flowing below the plate.

So, this plate separates the two streams of air flowing at different temperatures. And since it is thin, so the key word here is thin, since it is thin we do not have to consider the resistance of the plate, two heat transfer from the hot stream to the relatively cooler stream. And the heat transfer, there is going to be two heat transfer resistances; one is here which is the convective heat transfer resistance from the hot air to the plate and again convective heat transfer resistance which is from the plate to the cooler air.

So, we need to find out what are these two resistances. And we also know that the heat transfer resistance for convective heat transfer is simply 1 by h 1, if I call it is for the stream one and this is going to be 1 by h 2. So, if we are, so this is when I we are obtaining the heat transfer, the total amount the heat transfer in terms of the heat flux. So, is the area is the same. So, I can safely drop area and express everything in terms of flux.

So, the entire problem boils down to finding out the values of h 1 and h 2 from the available correlation for heat transfer that we have in a that we have studied so far. And some of the properties of the air which are flowing is given as the kinematic viscosity is 20.92 into 10 power minus 6 meter square per second. The thermal conductivity of air is

0.03 Watt meter Kelvin and the Prandtl number for air both the cases can be assumed to be prefer for this is 0.07.

So, this is for stream 1. For stream 2, it is at a different temperatures, so, it is kinematic viscosity is going definitely going to be different. So, it is at this is going to be it is kinematic viscosity, the thermal conductivity is slightly different Watt per meter Kelvin, the Prandtl number though it will remain more or less the same. So, this is one of the reasons why for flow for a heat transfer involving air, the Prandtl number is taken to be equal to 0.7 in most of the cases. As you can see, a difference in temperature between 200 to 25, substantially alter the value of the kinematic viscosity, it changes the value of the thermal conductivity as well; however, the Prandtl number more or less remains some sort of a constant.

So, therefore, it is common practice to take the Prandtl number for situations involving air to be equal to 0.7. And what is the, statement of the problem is find the heat flux between the two surfaces, the two streams of air at the midpoint of the plate and it is given that the length of the plate is equal to 1, 1 meter. So, from here to here is 1 meter. So, we need to find out the heat flux between the two streams at the midpoint of the plate. Wherever you come across such a problem, the first thing one needs to find out is what is the type of flow that is taking place in this cases; in other words, you find out what is the value of the local value of the local Reynolds number to evaluate whether you have turbulent flow, laminar flow or maybe the occurrence of a mixed flow.

So, in one case you seen this problem the velocity is substantial higher than the other situation. So, we need to ensure that the flow types are evaluated a priory before choosing the appropriate correlation for that type of flow. So, the first step is to evaluate what is the value of the Reynolds number at the specified location for both the streams. So, that is what we are going to do next, so what we do is will take the fluid stream 1. So, this is we call as stream 1, in which case, you can find out that the Reynolds number based on location, the local value of the Reynolds number is 60 meter per second times 0.5 meter which is the length divided by the kinematic viscosity which is provided as 20.92 into 10 to the power minus 6 meter square per second which would give you a value of 1.434 into 10 to the power 6.

And the movement it is 10 to the power 6, greater than 5 into 10 to the power 5, so we know that this is going to be a turbulent flow turbulent boundary layer which is which exists for the flow over the flat plate. Now, next we are going to find out what is the situation for stream 2. We again find out the Reynolds number which in this case is the velocity is significantly lower, the location remains fixed at 0.5 meter and the kinematic viscosity is 15.89 into 10 to the power minus 6 meter square per second which would give rise to a Reynolds number of 3.147 into 10 to the power 5.

So, it is 10 to the power 5, but it is less than 5. So, therefore, this one is going to be in laminar flow condition. So, the by looking at the values of the Reynolds number, local Reynolds number we can see that one of the stream is in turbulent flow, the other stream is laminar flow. Once we ascertain what is the flow of region that is that exists over these two plates.

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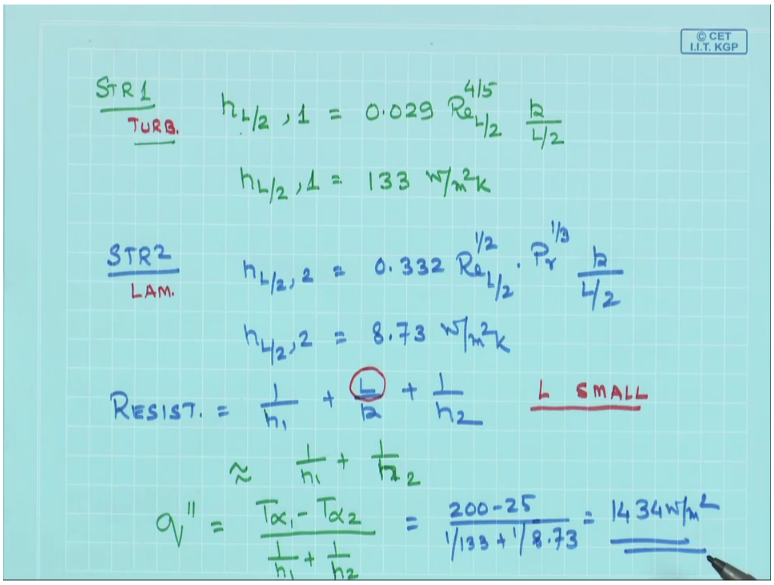

The next one would be again if I calculate what is stream 1, we would see that for the case of stream 1, the value of the heat transfer efficient at a location of L by 2 which is at the midpoint of the plates and this 1 strength force the stream 1 would be use if we use the correlation Reynolds number L by 2 to the power 4 by 5. Note that this is the turbulent condition times k L by 2, when you put the values in here you would see that h the heat transfer coefficient at location L by 2 for stream 1 which is under turbulent condition would turn out to be 133 Watt per meter square Kelvin.

Similarly, for stream 2 which is in laminar condition, you can choose the appropriate expression for corresponding to laminar flow and this is 0.332 Reynolds to the power Reynolds at L by 2 to the power half into Prandtl to the power one-third k L by 2. So, this is the relation which we have obtained for laminar flow over a flat laminar flow over flat plate and you can find out this value of h L by 2 for stream 2 to be equals 8.73 Watt per meter square Kelvin. Now, you may ask that why did I choose the relation corresponding to turbulent flow when I know that the flow is definitely going to be laminar in the beginning and then it is going to change to turbulent.

So, the correct expression to be used in this case when we calculate the flow Reynolds number to be more than 5 into 10 to the power 5, the correct relation to be used should be the mixed flow relations, absolutely right. I should have used the mixed correlation; however, if you find out if you evaluate what is the transition length for the flow to change from laminar to turbulent, you would probably find out that the transition length is small as compared to L by 2.

So, if the transition length for change over from laminar to turbulent that length is small very small less than 10 percent in fact, correct figure is 0.95. If the length of the transition region, where the flow changes from laminar to turbulent to the total length if these two refuse are about 0.95, then you can use the turbulent flow relations for to evaluate what is the local value of heat transfer coefficient.

So, that you can definitely do; moreover at this for this problem, we are trying to find out what is the local value of heat transfer coefficient not what is the average value of heat transfer coefficient. So, if you look at them mixed flow relation, the mixed flow relation would give you the value of the average heat length average heat transfer coefficient. So, considering these two cases, these two constraints I am compelled here to use the expression for the local value of heat transfer coefficient based on the flow being entirely turbulent from the beginning.

So, this is an approximation and as long as you identify that it is an approximation, your understanding to the entire process would be complete. So, I am using an approximation to find the local value of heat transfer coefficient assuming the flow to be turbulent from the very beginning which let us me use the expression corresponding to turbulent flow to find out what is the value of h.

So, what I have got now is the value of the heat transfer coefficient both at the same location; one for above the plate and the other for below the plate. Now, when you consider the heat transfer which is taking place, the three resistances are in series here; one is the convective heat transfer resistance for flow above the plate which is given by 1 by h 1; the second one what is below the plate convective resistance what is 1 by h 2 and the third is the resistance corresponding to the solid plate itself which is the conduction of resistance in which is given by L by k, since I am expressing everything in terms of flux. So, I am using it is L by k.

So, the three resistances in series are one at what the total resistance is going to be 1 by h 1 plus L by k plus 1 by h 2. Now, it has been mentioned that this is a thin plate. So, the value of L is very small, if the value of L is small, then this resistance can be equated as 1 by h 1 plus 1 by h 2; that means, I am neglecting the conductive heat transfer resistance since the solid plate which presumably would have a high value of thermal conductivity to be small in comparison to in comparison to h 1 and h 2.

So, therefore, this is the formula for conductive, the total heat transfer resistance and the heat flux in between the two streams separated by the solid plate would simply be T infinity 1 minus T infinity 2 by the sum of resistance is which is 1 by h 1 plus 1 by h 2. And when you plug in these numbers, they are going to be 200 minus 25, 1 by 133 plus 1 by 8.73 and the answer would be 1434 Watt per meter square that is the flux.

So, this problem provides you within a within with an idea of how to calculate the heat transfer coefficient based on the regime which is in which the flow is what is the flow regime, choose the right value of the heat relation between the Nusselt number, Reynolds number and Prandtl number and from that calculate the value of the local heat transfer coefficient.

Next, I am going to discuss something slightly different is how would the heat transfer co-efficient behave when we have a flow which starts as laminar and slowly it changes from laminar to turbulent. So, how would the heat transfer coefficient would look like and how what is going to be the to the heat transfer, the total amount of heat transfer for flow where a plate? And for that, I invoke an interesting experiment which probably some of you are going to do letter on in natural convection.

So, what we what is natural convection? A natural convection is the case in which you have let us say a rod which is suspended in steel air stagnant air the rod is heated electrically such that it is going to lose some amount of heat is going to be generated if this is the rod, some amount of heat is going to be generated in this hot in the in this rod and the rod at steady state whatever be the heat generated in this rod by electrical means has to be dissipated to the outside air. And since the air is not moving, the only way it can dissipate the heat is by natural convection.

So, what is going to happen is that the air which is surrounding the rod, its temperature is going to rise and as the temperature rises, its density will decrease as compared to the to the air which is as compared to the bulk air. So, there is going to be difference in density in between the two and the light air would start to rise up to be replaced by cold air from the surrounding. So, a flow like this would start a circulatory flow like this would start for situations involving natural convection. Now initially, at the tip where it is in contact with the cold air from where the flow starts it is definitely going to be in laminar flow, but as you move upwards the situation would come such that the laminar flow will change to turbulent flow.

So, for part of the rod where the flow is parallel to the rod, this is going to be in laminar flow and some portion is going to be in turbulent flow. The question is how are you going to what is going to happen to the value of the heat transfer coefficient? So, if you can plot the value of the heat transfer coefficient as the function of axial length in the direction of flow, you would see some interesting trend. So, in the remaining few minutes, I am going to talk about that trend and try to explain it that what you would expect for the heat transfer coefficient for a situation in which either you have flow along a vertical plate whose temperature vertical cylinder whose temperature is more than that of the air such that a flow current will setting. So, this circulatory flow or the case of a flat plate over which you have flow of air and let us say this plate is electrically heated, so that you can calculate what is the amount of heat transfer at every place and calculate experimentally what is the heat transfer coefficient.

So, how would heat transfer coefficient vary with position that would strongly depend on what kind of flow regime you have for such a situation? So, let us look at how it would look like.

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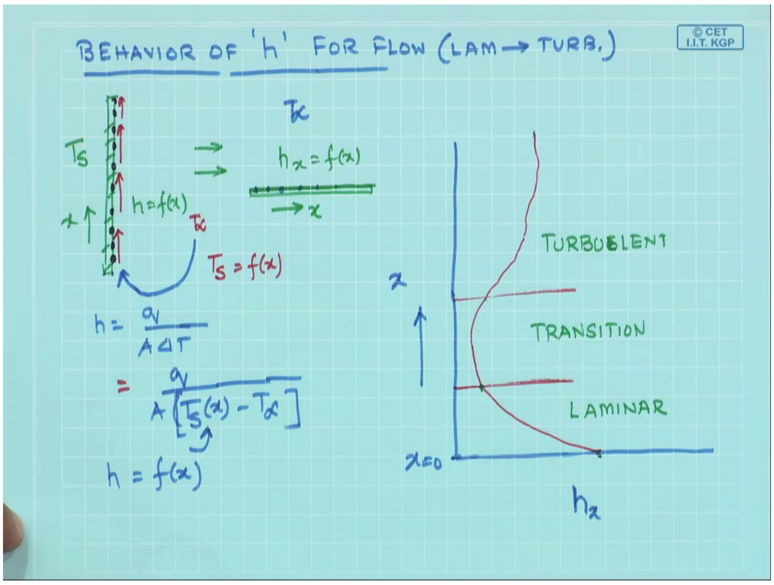

So, the example that I have discussed is a solid rod whose temperature, let us it is being maintained at a constant temperature of T s. So, this is heat transfer coefficient the behavior of heat transfer coefficient for flow when it changes from laminar to turbulent and so on.

So, what would what is going to happen is air in here is going to get heated and it will start to rise towards the top and cold air is going to come like this and replacing the air which is moving up along the plate. The same situation would you would see if you have just a flat plate and you have flow of air over it and you are trying to find out how does h vary as a function of x. So, how does h vary as a function of x where this direction is x and in this case this is a x direction.

So, when you do the look at the experimental results where h can simply be calculated as h would simply be q by A times delta T ok. So, that is a definition of q. Now if you have thermocouples which are embedded on the plate, so these thermocouples would give you the temperature s as a function of x right. So, these thermocouples would give you what is the local temperature along this.

So, this delta T would simply then be equal to q divided by a times T s which is the function of x minus T infinity where T infinity is the temperature, the constant temperature of the fluid surrounding it. So, this is T infinity and this T s is a function of x which is experimentally measured by a number of thermocouples which are embedded on it, on the horizontal cylinder or on this plate itself. So, one you one would be obtained to by putting in the experimental value of T s and by knowing the heat that it is dissipating, you should be able to obtain h as a function of x. So, h as a function of x can be obtained by measuring the value of T s as a function of x through the use of all these thermocouples. When you do that, you see an interesting trend. Let us say this is the location at x equal 0 and my x keeps on increasing.

So, this is x equal 0 and this is x equals L. So, this is the direction in which x is changing. And I am plotting the value of h on this axis ok, what you would see the behavior of the heat transfer coefficient would look something like this. So, what does it say that at this point over here the value of heat transfer coefficient is a maximum, but as you move in this direction, the heat transfer coefficient decreases rapidly, the convective heat transfer coefficient decreases rapidly. It reaches the plateau and then it starts to increase slowly it starts to increase and which is more or less some sort of an asymptotic value in over here.

So, what is happening in here? You can clearly mark three regions in this case, the first region where the flow is essentially laminar through in which the heat transfer coefficient monotonically decreases with length and till it reaches a value like this, value over here. In from this point onwards the rate of decrease of h is somewhat arrested and you get a plateau where it is more or less remaining some sort of a constant. So, do where the transition from laminar to turbulent is going to take place and once the flow heats turbulent, the heat transfer coefficient starts to increased because of the formation of eddies which are going to be responsible for most of the heat transfer in turbulent flow situations.

So, this is a very useful curve which would be of which we are going to use in our subsequent problem solving and for our understanding as well. So, what I have then is, initially the flow is going to be laminar, the as the flow takes place the thickness of the laminar boundary layer will keep on increasing. And the thickness of the boundary layer is essentially inversely proportional to is directly proportional to the resistance of heat transfer in such a case. So, as the thickness of the boundary layer increases, the resistance to heat transfer increases and since a resistance to heat transfer is inversely proportional to the heat transfer coefficient, the value of heat transfer coefficient start to decrease.

So, the heat transfer coefficient will start to decrease as the thickness of the film under laminar flow will keep on increasing in the actual direction. Now, we say that there is a sharp transition from laminar to turbulent, but that does not happen. The change over from laminar to turbulent would take place over a significant length in which first small eddies are going to form and the heat transfer is going to be due to convection, due to the laminar flow as well as the eddy imposed additional convection which is a hallmark of turbulent flow.

As the length increases, the Reynolds number keeps on increasing and more and more eddies are found. So, slowly the reduction in heat transfer coefficient in laminar flow will be more than of set by the formation of more and more eddies. So, the reduction in the value of h is arrested and it will start to show the reverse strain that is with increase in distance with increase in axial location, the value of h is stabilizing and then it will start increasing.

So, that region where the growth over the rate of decrease of h is arrested and reversed is known as the transition region. Once the eddies take over as the principal mode of heat transfer, convective heat transfer, any increase in length will increase the number of eddy, the size of the eddies, there will be more turbulence more convection and the value of h will start to increase. But this increase cannot go indefinitely as the thickness of the boundary layer turbulent boundary layer is also increasing very rapidly.

So, again after a period of sharp increase in the value of h when turbulence sets in a plateau will again be reached where the increase in the formation of eddies will be offset by the increase in the boundary layer thickness which would give us a profile like the one I have drawn over here. The value of the heat transfer coefficient therefore, starts at the high value since the thickness is the least here, decreases as thickness increases, eddies start to form eddies take over the increase the value of the heat transfer coefficient, the turbulent sets in and then it increases and it reaches somewhat of a constant value.

So, this picture is important which shows the behavior of the heat transfer convective heat transfer coefficient for flow when you have a laminar to turbulent transition. And this is an experiment which many of you are going to do in the heat transfer laboratory where yours you have to calculate the value of the heat transfer coefficient by noting the value of the actual location dependence of T s and generate a curve like this, verify that this is the behavior of the heat transfer coefficient. So, we will solve few more problems on convective heat transfer for external flow when the flow is taking place over a plate under different situations and that should clarify any doubts that you may have.