Heat Transfer
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Lecture - 24
The Effects of Turbulence

Previously we have seen, how the velocity and thermal boundary layer, grows over a surface over a flat plate when there is a relative motion between the solid flat plate and the fluid in contact with it. So, the system that we have analyzed, which is the simplest possible situation is where we have a flat plate and the flow is taking place over it, where the temperature of the plate and that of the fluid are different. As a result of which we are going to have momentum transfer as well as heat transfer from the plate to the fluid.

Now, the momentum transfer is going to be principally governed by the viscosity of the fluid and the density of the fluid. Whereas, heat transfer being a combination of convection and conduction is going to be a function of k, the thermal conductivity of the liquid, rho the density C P, mu as well as the aim is the entire exercise was to connect, the heat transfer coefficient or in order to obtain an expression of heat transfer coefficient for simultaneous flow and heat transfer on a flat plate.

Now, if you recall we have said categorically that the analysis that we have done is restricted to laminar flow only. Now, when it is laminar flow then the entire transport, whether it is momentum or it is heat is going to be governed by the mostly by the molecular transport of momentum or molecular transport of heat.

So, we have used the approximations, which for flow inside the boundary layer both the hydrodynamic boundary layer, as well as the thermal boundary layer. And, we have seen that the left hand side of the energy equation or left hand side of the momentum equation, they contain only the advection terms. And, since based on an order of magnitude analysis, we have demonstrated that none of the terms for a 2 D flow inside the boundary layer can be cancelled.

So, I am going to have an u del T del x plus v del T del y is equal to the molecular transport of momentum. And, then molecular transport of momentum we have seen that due to the prevalence of a large temperature gradient across a thin boundary layer, most of the heat is going to get transferred by diffusion by thermal conduction in a direction perpendicular to the flow; that means, in the y direction. Whereas, since the since the temperature is not changing appreciably as the flow takes place in the x direction, the molecular transport of energy in the x direction, that is the conductive heat transfer can be neglected.

So, with that and with our knowledge of with our previous knowledge of the solution of the hydrodynamic boundary layer, we did obtain a closed form solution for the heat transfer coefficient or more specifically relation for Nusselt number, which is h d where d is the length scale. In this case it is the length of the plate it is a variable length of the plate the location of the plates. So, the local Nusselt number be h times x by k and we have seen that it is a function of Reynolds number, and it is a function of Prandtl number.

So, the compact expression which we have obtained for Nusselt number which is valid for a specific range of Prandtl number and which is also valid only for laminar flow characterized by a local value of Reynolds number less than 5 into 10 to the power 5. These to constraints on Reynolds number and Prandtl number give us a relation for heat transfer, the convective heat transfer from a flat plate when it is placed in parallel flow over a fluid stream, which has a temperature different from that of the plate.

So, after this point I think it is it is perfectly fine, we know how what is the genesis? How the, how the heat is getting transferred and so on? The situation gets more complicated as we move into turbulent flow. We understand that in laminar flow the streamline motion the platelet motion of adjacent fluid layers are mostly maintained. So, layers or fluid slip past one another and the string invisible string, which tries to slow down the faster moving fluid or tries to accelerate the slower moving fluid the viscosity, which is the molecular transport property, that controls the molecular transport of heat and momentum control everything.

But, in turbulent flow something else happens. The temperature at steady state at a given location at a given x and y in laminar flow is a constant. So, if you could measure the temperature at an actual location at a some value of y; that means, at a specific point in the flow stream inside the boundary layer, that temperature or that velocity will remain in variant with time.

So, we are going to read only one value of it. However, in turbulent flow the situation is more cerotic. So, most of the that is that going to flow cross flow everywhere this going to be well mixed behavior of the flow inside the boundary layer and everywhere else as well.

So, the momentum or heat gets of the transfer of momentum and heat they will get augmented by the presence of this inter mixing this cerotic motion, which is a hallmark of any turbulent flow. So, the entire fluid is going to be very well mixed and therefore, the temperature that you measure at a given point is going to be a function of time. In other words you should you would see local fluctuation in the value of the temperature that you measure, and the value of the velocity that you measure.

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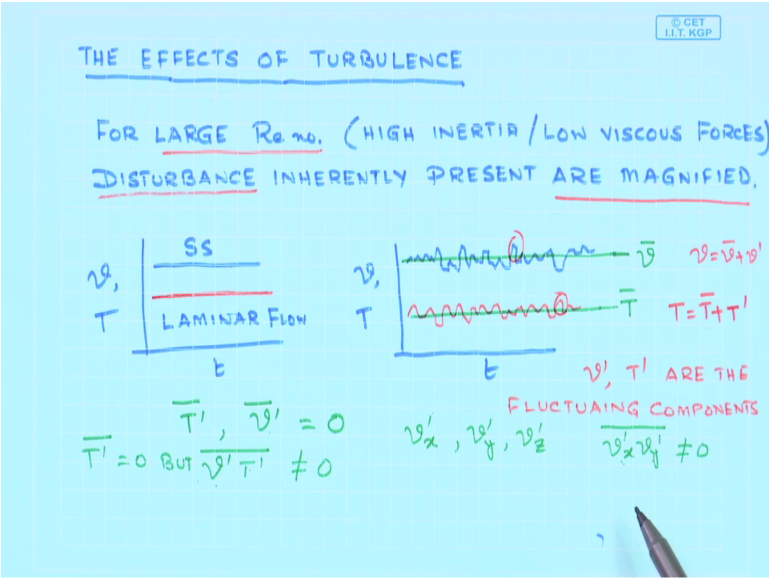

So, the velocity is not going to be like in laminar flow if you have if you measure the either the velocity or the temperature.

So, the velocity with this thing which time this it does not really change, it is a straight line and let us say the temperature is also going to be a straight line without any perceptible changes with time.

However so, this is the case for laminar flow at steady state of course, this is going to be at steady state. But, if you do not measure and if your measurements system is very accurate very sensitive and if you measure the velocity or the temperature what you would see is local variations like this? It is not just going to be a constant value. And, similarly for temperature you would you would see the value of the temperature is going to be something like this it is not just one value.

So, this is the time. However, if you take an average of that average of these values both for temperature and temperature as well as for velocity, what you would get this can be called as v bar or time average velocity this is what is v bar. And, this is time average temperature can be denoted as T star.

So, when we say that the velocity of the fluid in any in a specific situation is this much 5 meter per second. What essentially v c specially, for turbulent flow is this is the time average velocity or the time average temperature.

So, these are average temperatures and let us say I would like to know, what is the temperature? What is the velocity at this point and what is the temperature at this point?. So, the local value of temperature would simply be T bar plus minus plus or minus T prime and this v velocity is going to be v bar plus v prime.

So, this v prime and T prime are the fluctuating components of velocity or temperature. Of course, in laminar flow you do not have a fluctuating component and v the velocity is essentially same as the time average velocity. So, their constant so, that time average value would; obviously, be equal to the same velocity. Similarly, for temperature variation and in turbulent flow can be expressed as the sum of a time average temperature plus a fluctuating temperature.

So, these are the fluctuating components and there are certain properties, which one has to keep in mind is that this T bar the fluctuating component or v bar the fluctuating component of velocity, if you take the time average of that this is going to be equal to at 0.

So, you have up and down and it is completely an arbitrary function and it is it is so, if you take a large enough time and take the average the average time average of the fluctuating component individual component should be 0, but the same cannot be said about their product. So, this may or may not be equal to 0 ok. So, T bar is 0 T prime bar is 0, but this may not be equal to 0.

So, the fact that the time average of the fluctuating component is 0, but the time average of the product of the fluctuating components is not 0, has given rise to certain interesting change is in the equation of motion and the equation of energy. The temperature is a scalar so, I only have to deal with T prime only with T prime, but velocity is a vector. So, I may have 3 components of the flux the fluctuating components of velocity as v prime x v prime y and v prime z.

So, therefore, the time average of v prime x is 0, but v prime x times v prime y time average of that may not be equal to 0. So, whatever I said about velocity and time is also true for velocity component and velocity component product of 2 velocity components. So, the time average of v prime x is 0, but the time average of v prime x and v prime y may not be equal to 0. Now, this non 0 non 0 part v prime x v prime y is not equal to 0 has given rise to additional terms in the equation of energy and in the equation of motion.

So, what physically, what I mean here is that the presence of the fluctuating components essentially mean, additional transport of momentum or additional transport of heat. Over and above that specified by the laminar heat transfer, laminar heat transfer or laminar momentum transfer. We all understand that when you mix something mix a hot fluid and if you increase the velocity, if you increase carouse in the system, disorder in the system, then there is going to be more heat transfer, which is taking place.

So, this additional heat transfer or additional momentum transfer they are direct results of the non-zero nature of the fluctuating components of velocity and temperature or velocity component into directions. Another going to very much details of it, but you would probably know that the equation of motion can be can be expressed in terms of the not for just the velocity.

But, the instantaneous velocity by instantaneous velocity I mean this v. So, the instantaneous velocity is a sum of the time average velocity and the fluctuating component instantaneous temperature similarly is the time average temperature and the fluctuating temperature.

So, when you use this expression of velocity in equation of motion or this expression of temperature in equation of energy 2 identical 2 equations, which are fundamentally, conceptually same as that of the equation of motion.

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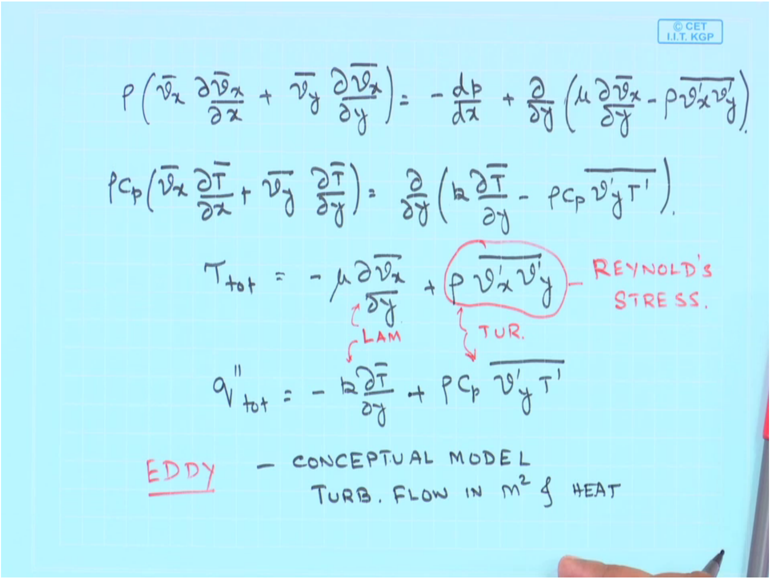

But being different because of the presence of this they appear and I simply write them as, let us I am writing it for 2 dimensional flow which is the flow inside a boundary layer, everything now is in terms of the times smooth value. So, that is why the bars should appear over here.

So, this is the x component of equation of motion is minus d p d x, which is the pressure gradient plus del y of mu del v bar x by del y minus rho v prime x v prime y and time average of that which is not 0 which may not be 0.

So, this is the equation of motion and similarly the equation of energy would be v x del T the time average value by del T x plus v bar y del T bar by del y equals del y of k, time average temperature gradient minus rho C P v prime y times T prime, and the time average of that. So, the extra terms these 2 extra terms on the right hand side, the account for the effect of turbulent fluctuations in momentum and turbulent fluctuations in temperature.

So, if you look at only this part mu times velocity gradient this is nothing, but the shear stress this is nothing, but the shear stress in laminar flow and this is nothing, but the conductive heat transfer in laminar flow. So, this tau total when I express this entire thing and call it some sort of a shear stress, this tau total is mu del v x del y this term which is for laminar plus an additional term v prime x v prime y. So, this is the total shear stress exerted by a moving fluid which as 2 components one is due to laminar. So, this is due to laminar part of it which with which we are concern so, far and this is the turbulent part of it ok.

Similarly, the total amount of heat transfer is going to be minus k del T by del y plus rho C P v prime y T prime ok. So, this is again this is a laminar one this is the laminar part this is the turbulent part and this is the turbulent heat transfer. And, this term has a special name which accounts for the shear stress experienced by a turbulent fluid by a fluid moving in turbulent flow it is called as the Reynolds stress ok.

So, Reynolds stress accounts for the additional amount of additional amount of moment momentum transferred, that one would expect in turbulent flow and the similar term for the case of heat transfer is given by the this term. So, these 2 are the laminar counter parts of momentum and heat transfer these are the turbulent contributions to the momentum transfer and to the heat transfer.

Now, when it is expressed in this way sometimes it is more convenient to express it in terms of something which we the term that we use very commonly, we call it as eddies. So, what are eddies you can visualize you can conceptually say that eddies are nothing, but packets of fluid, which has a very short length before which it disintegrates and mixes with the fluid stream once again. So, eddy is hypothetically a packet of fluid which flows cross stream to carry with it more momentum and more energy than that prescribed in laminar flow. So, the more the eddies more would be the heat transfer, and more would be the momentum transfer.

So, eddy is something which transports energy and momentum in turbulent flow. So, in many times the turbulent flow is characterized by the presence of eddies, the turbulent flow gets enhanced momentum transfer heat transfer as well as mass transfer, because of the formation of eddies. Now, there has been a considerable atom to define to characterize or to put a correlation of the eddy transport of heat eddy transport of momentum and that of mass. So, we would quickly see what how eddy can be combined with our traditional understanding of laminar transport and get some idea of the enhanced transport encountered in turbulent flow.

So, let us look at what this eddy concept is and a we are going to look at eddy. So, these are this eddy the model it is it is it is a conceptual model so, for turbulent flow in momentum and in heat. So, the turbulent flow in momentum and heat they are expressed in terms of this the presence of these eddies.

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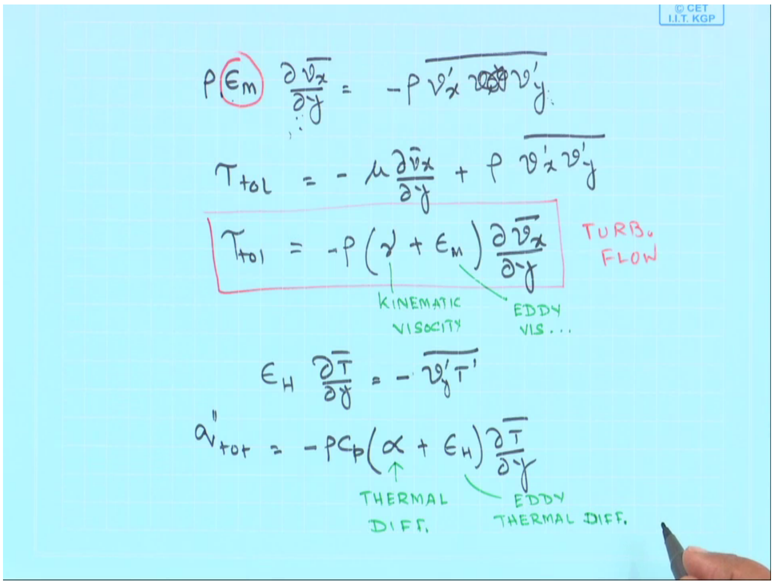

And, generally they are expressed as I will write the defining equation of the eddy rho epsilon m where epsilon m is the eddy times velocity gradient.

So, this if you look at this equation and then you would see why we are trying to express the eddy in this specific form. I have a k over here or I have a rho mu over here, which is expressed in terms of velocity gradient. So, I have the additional transport of momentum over here. So, if I could express this additional term in terms of something that we know, that is mu times d v x d y. So, what mu does in laminar transport in transport of momentum in laminar flow, this epsilon m if you look at the similarities between these 2 equations epsilon m plays apparently the same part as that of mu by rho, in the case of laminar transport laminar transport of momentum.

So, this entire term is expressed in terms of rho times epsilon m d v x by d v x by d y. So, therefore, the total shear stress the tau total which we have written before as minus mu times del v x this is what I have written in the previous page plus rho v prime x v prime y the time average of that this. Now, therefore, can be expressed as minus rho and once I takes the rho outside this mu by rho becomes the kinematic viscosity nu plus if I substituted over here it becomes epsilon m, and then del v x by del y.

And, similarly one can write so, this is the new expression for tau in turbulent flow and you can clearly see the additional transport of momentum due to the presence of this epsilon m.

So, if this is called the kinematic viscosity, the other one is called the eddy viscosity. So, the same way we have done it for momentum transfer we can also do it in the case of heat transfer as del T by del y is minus v prime T prime bar of over the time smooth part of it. So, I am simply trying to replace this part v y v y prime v y prime T prime this part by an equivalent form of Fourier's law of conduction in laminar flow, except I am replacing k I do I am not going to use k instead I use epsilon H, which is the eddy transport coefficient for turbulent flow heat transfer.

So, therefore, my q totaled q double prime totaled as from the previous equation would simply be minus if I take them rho C P come then this becomes k by rho C P and we understand that k by rho C P is nothing, but the thermal diffusivity plus epsilon H times del T bar by del y. So, these 2 equations when you see you can clearly see so, this is the thermal diffusivity and this is known as the eddy thermal diffusivity.

So, these 2 the value of the thermal diffusivity tells you what would be the laminar flow contribution of heat transfer, and this is the eddy transport of heat transfer for the case of turbulent flow.

Now, the situation is apparently simple, if we know what are the values of the epsilon m for momentum and epsilon H for heat transfer then my job is complete. Then, I should be able to find out what is the total amount of momentum transfer in laminar in turbulent flow, or the total heat transfer in turbulent flow. But, it is easier side than done even the people have tried to express the epsilons, the eddy diffusivities (Refer Time: 26:46) mass momentum or heat. In terms of known fluctuations in terms of fluctuations present, which are also a function of the degree of turbulence present in the system it is extremely difficult.

So, far no generalize expression for these eddy diffusivities are available, which is going to be valid over a wide range of turbulence present in the system. So, what we have we do understand the role eddy play eddies play in the transport process. It gives us the very good pictorial description of the intermixing between different layers and additional transport in turbulent flow as compared to laminar flow, but a qualitatively it gives a good clear picture, but it is so, difficult to express them quantitatively.

So, we have to adopt other approaches, which will discuss in the next class other approaches of heat transfer, other approaches have expressing turbulent flow heat transfer and turbulent flow momentum transfer, which rely heavily on empiricism.

So, even though it relies on imprecision there has been sufficient theoretical background people have tried to put sufficient theoretical support for the relations in turbulent flow, but at some point of time you need to have experimental results to quantify the transport the additional transport of heat and momentum as well as mass, in the case of turbulent flows. So, what we are we are going to do then is we are going to look at the how would be profile look like when you would see what is the, how does the temperature profile? and the velocity profile vary in here.

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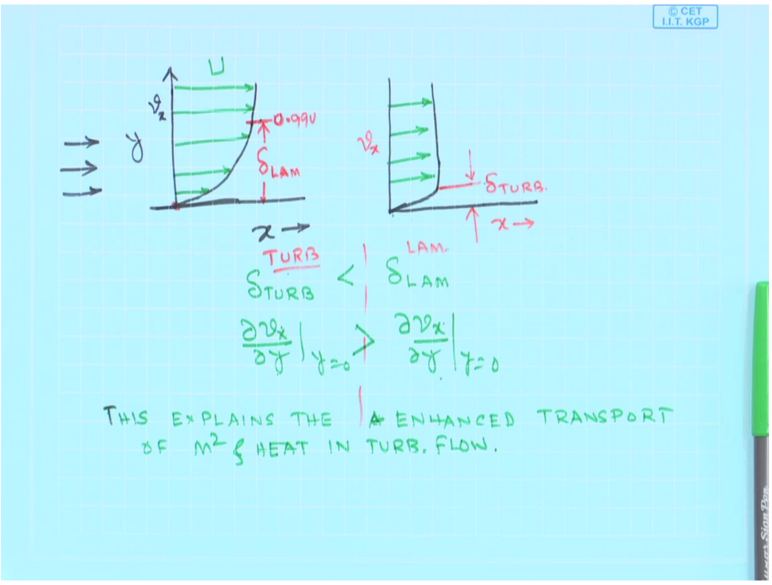

So, let us say you have this is a y direction the flow is taking place like this and this is your actual direction x, and you are trying to see what is the velocity profile?

So, the velocity profile is going to be something like this. So, the velocity profile; obviously, is 0 the velocity is 0 over here and it keeps on increasing as you move in this direction, and ultimately the velocity here is going to be equal to U. So, this is the velocity which is velocity in the x direction which is being plotted.

Similarly, if you do plot the same thing for the case of turbulent flow, you would expect a profile of velocity like this it took almost up to by with this point. So, this is the hydrodynamic boundary layer thickness and over this thickness the velocity varies from 0 to roughly about U or to be specific to about 0.99 U.

But, in this so, this is the case for laminar delta for the case of laminar, but in this case this is also velocity in the x direction and I am plotting it with y. In this case also in this case over a very small distance of delta this is called delta turbulent, the velocity changes from the 0 at the surface on the surface of the solid due to no slip condition to the velocity of the free stream. So, one can clearly see that delta turbulent is small as compared to delta laminar.

So, what can you say about the velocity if you velocity gradient. So, if you find out del v x del y at y equal to 0 and del v x del y again at y equal to 0. So, this is for the case of turbulent and this is get for the case of laminar. What you would see is that? This del v x del y at y equal to 0 for the case of turbulent flow will definitely be more than del v x del y at y equal to 0. Since the y over which the velocity changes from 0 to U 0 to U this, what is much smaller as compared to this? So, your del v x del y at y equal 0 for the case of turbulent flow is going to be more than del v x del y at y equal 0 for the case of laminar flow.

So, this explains this conceptually explains, the additional transport or the enhance transport of momentum and heat in turbulent flow. So, I have given you an idea I have given you a heuristically, I have shown you that why would you expect more heat transfer more momentum transfer in turbulent flow as compared to laminar flow. The con pictorially you can think of the formation of eddies; which edds in the transport additional transport of heat and momentum.

You can also express this something similar to Newton’s law or something similar to something similar to Fourier’s law, where the term eddy viscosity or eddy viscosity or the eddy transport eddy thermal conductivity would give you the additional transport. So, this is very good to understand the effect of turbulence, but as I have said before there are many statistical models, which try to explain, which try to quantify the eddy in turbulent flow the contribution of eddy in turbulent flow.

So, even though it is qualitatively gives a very good idea very good picture of the whole situation, you we still do not know how to have a model? Based on statistical fluctuations, based on a study of the fluctuation, based on a statistical study of the fluctuation in a in a flow field both in terms of velocity fluctuation and temperature fluctuation. And to come to a compact expression of the heat transfer or of momentum transfer.

So, we have to resort to semi empirical analysis of these processes and couple the results that, we have from our hydrodynamic treatment of turbulent flow and couple it with that of the thermal treatment, the treatment the or the quantification of the heat transfer in laminar in turbulent flow in turbulent heat transfer.

So, in tomorrow in the next class, we will look at turbulent heat transfer for from a flat plate in contact with the flowing fluid, where the temperature of the flowing fluid is different from that of the solid plate. So, when we have turbulent flow what would be the expression for Nussuelt number for such cases and, interestingly as I have said before the Reynolds number plot flow over flat plate the length scale that is defined is the actual position. So, the Reynolds number is defined as x rho v by mu and the x keeps on increasing as you move along the plate.

So, therefore, it is likely it is it is possible, that you are going to have laminar flow to begin with and at certain position on the plate the x becomes. So, large the Reynolds number become so, large that it changes from laminar to turbulent. Even though the transition from laminar to turbulent is going to take place over a length not at a length, you realistically cannot have a situation where at this point anywhere on this side is laminar and anywhere on the other side is turbulent, it never happens it always takes place over a range, but we put for convenience we put a value of 5 into 10 to the power 5 a Reynolds number a value of 5 into 10 to the 5 to quantify presence or absence of turbulence.

In our previous classes we have seen what is the Nusselt number relation for the first part, from 0 to x c over which the flow is laminar? In the next class we would see using semi empirical relations already available to us from momentum transfer what is going to be the expression for Nusselt number, when the flow switches from laminar to turbulent. Such that together it would give me a complete idea quantitative idea complete quantitative idea of heat transfer over a flat plate from a flat plate, when there is you have laminar flow and you have turbulent flow. Or in certain cases we can trip the flow from the very beginning at x equal to 0 such that the entire flow becomes turbulent.

So, these kind of different situations, we will derive and discuss in next in the subsequent class and there will be lots of lots of problems, which we will solve over here to give you an idea not only the concepts behind this equations, but how to use this equations for practical calculations of heat transfer.