# Heat Transfer

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## Lecture - 18
## Equations of Change for Non-isothermal Systems (Contd.)

We will continue with our study of the Derivation of the Equation for Non Isothermal Systems. This we have started in the last class and we have preceded by defining a control volume of dimensions del x del y del z. And we have identified through each of the six faces that is at face at x and at x plus del x, y and y plus del y, z and del plus del z, some amount of energy comes into this defined volume by means of convection and by means of conduction. So, whenever there is convective flow into the volume element that we have defined, that flow carries with it some amount of internal and kinetic energy and its going to come at x and it is going to leave at x plus del x.

So, we have identified three pairs of terms, one showing the amount of energy both kinetic and th internal, that comes in at some face in living at x plus del x, living at y plus del y and living at z plus del z. So, these 6 terms together would give us the total amount of energy that is being added to the volume element due to convection. Similarly, we have also identified the conductive energy which flows into the defined volume element because of a difference in temperature. So, that is no velocity is involved since conduction does not require any movement of the medium.

So, we have expressed the heat flux at each of these locations, such as q x, q y, q z those three are the in terms that is the flux and the one that goes out at x plus del x is simply denoted by q x plus d x, q y plus d y and q z plus d z. So, the net amount of energy, thermal energy which is being added to the defined control volume, the volume element that you have defined should be, if you just considered the x face is the amount of energy that comes in per unit, per unit time would be q x which is the flux, multiplied by the area.

And the area of the x face is simply del y times del z. So, the amount of energy, thermal energy that is be internal energy that is being added to the volume element would simply be q x evaluated at x, multiplied by del y del z. And the going out term it will be minus of q, x plus d x that is q x evaluated at location x plus d x, multiplied by the area which remains the same, that is it would be equal to del y del z. So, these two terms would give us the conductive energy which is being added to the volume element through the face at x and x plus del x.

Similarly, I should be able to write what a, what is the energy be added through y and y plus del y and z and z plus del z. So, the algebraic sum of these 6 term should give us the total amount of internal energy which is being added by conduction to the volume element. So, I have 6 terms for convective flow of energy and 6 terms for conductive flow of energy, that together would give me the total amount of energy being added to the system.

In this system we have to identify that it is an open system such that the fluid is allowed to enter and leave to the volume element. So, that gives us the total amount of energy being added to the system and if we think about the first of thermodynamics we would also have to take into account the work done by the system or work done on the system. Work done by the system is going to result in a reduction of its energy.

So, it would come with minus sign in front of it and work being done on the system, since it increases the energy of the system. So, it is going to come as plus with a plus sign and the work done can be against different forces. The forces can be categorized into two distinct groups; one is a volumetric force, the force which is acting on every point inside the volume element.

So, an a common example of that would be gravity and it could also be done against surface forces and one of the surface forces which we can easily identify is the pressure. So, when we talk about the work done against volumetric forces or against surface forces, the work done would be simply force times to distance and the force, the rate of work done would therefore, be force times distance by time and since distance by time is velocity. So, the rate of work done against or by external forces be volumetric or be its surface forces, would simply with the force multiplied by the velocity in that direction.

So, when you talk about the force being exerted due to pressure on the x face, would simply be p at x pressure evaluated at x multiplied by the area, the area of the x face being del y del z. So, this is the force p at x multiplied by del y del z, this is the force and it has to be multiplied by the component of velocity in the x direction which is v x evaluated at x.

So, the entire work done against pressure forces would simply be p x evaluated at x multiplied by del y del z, multiplied by the component of velocity which is v x, also evaluated at x. And the work done on the other side at x plus delta x would still be the same, except the pressure is no evaluated at p at x plus del x and the velocity is evaluated at x plus del x.

In a similar fashion I should be able to find out what are the other 4 terms for example, the y term would be p y multiplied by the area of the y face which is del x del z multiplied by the component of velocity in the y direction which is v y. And the one that that is on the y plus del y face would simply be p y at y plus del y, multiplied by del x del z multiplied by v y, evaluated at y plus del y.

So, these 6 terms together give us; will give us the force, the work done against the surface forces. Similarly I have also; I have shown is in last class the expression for the work done against volumetric forces, I have omitted, purposefully omitted the viscous dissipation term. That is the force, that is the work to be done against viscous forces, as I have said the viscous forces, that viscous dissipation or the work done against viscous forces, they become prominent as compared to the other terms under some specialized conditions.

The specialized conditions generally refer to, if the viscosity is large or if the velocity gradient is large. So, the velocity has to be large and the length scale over which the velocity changes is small, such that del v x del y, if that is a velocity gradient we are talking about this has to be large and mu has to be large, the viscosity has to be large.

So, if those two conditions are satisfied then the viscous work done against viscous forces, which generally is manifested by a change in temperature; since it is a dissipation function, it is going to give rise to a change in, give rise to energy where is and therefore, the temperature will also change.

And this term which takes into account the work done against viscous forces would only be important in two or three cases as I have mentioned it could be for reentry of a rocket or for very high viscous fluid when it is flowing through a small conduit, a thin conduit. Such that, the velocity may not be high, but the distance over which the velocity changes is very small, which would be which would be relevant in an upcoming area of fluid mechanics heat transfer and application which is known as micro fluidics.

Where the structure of the of the system in which this change in velocity is occurring is very small, they are of the order of 10's of microns or maybe 100's of microns. So, even though the velocity is small since, the area since the length scale over which the velocity change is of the order of microns, the velocity gradient itself is large.

So, in some very high speed extrusion of polymers reentry of rockets or in some micro fluidic systems, this viscous dissipation can lead to significant changes in energy and those terms must be included in the energy equation. But, we would, I would not derive those terms, but I will show you the from the textbook what would the energy equation look like and you can easily identify the terms which correspond to viscous dissipation that we are not considering at this point of time.

So, the result of all these heat input all these energy coming in by convection and conduction and the energy change as a result of work done by the system against volumetric forces or again surface forces. When you combine all of them together and if the system is at unsteady condition, if the system is not at steady state, then the algebraic sum of all these terms should result in and change of the net energy content of the defined volume element. So, the change in energy content both internal energy and kinetic energy of a volume element in an open system is a result of several factors, the conductive flow of energy the convective flow of energy and all terms that refer to work done by or on the system.

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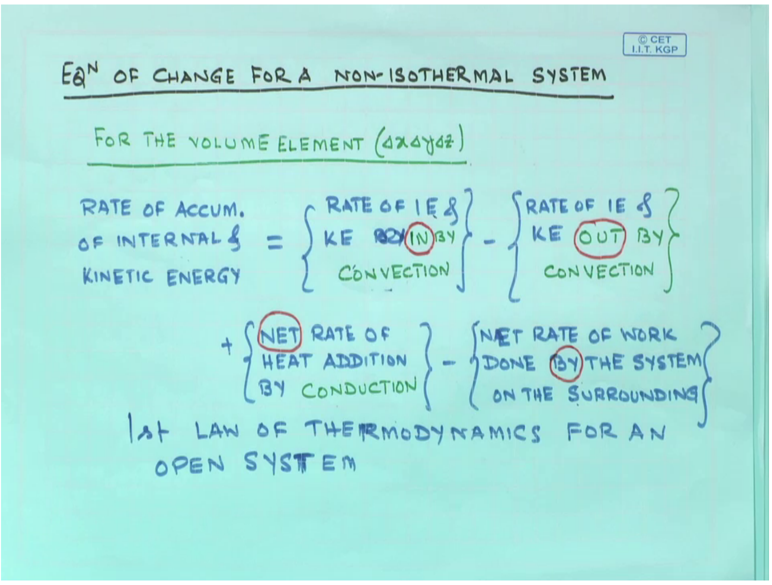

So, this much we have covered in the last class in when you when you see this equation, the one that I have shown over here is what I have described. The rate of accumulation of internal and kinetic energy is the rate, is equal to the rate of internal and kinetic energy in by convection, out by convection and the net rate of heat addition by conduction and the net rate of work done by the system on the surrounding. And since it is by the system that is why it comes with its preceded by a negative sign so which is nothing, but a first law of thermodynamics for an open system.

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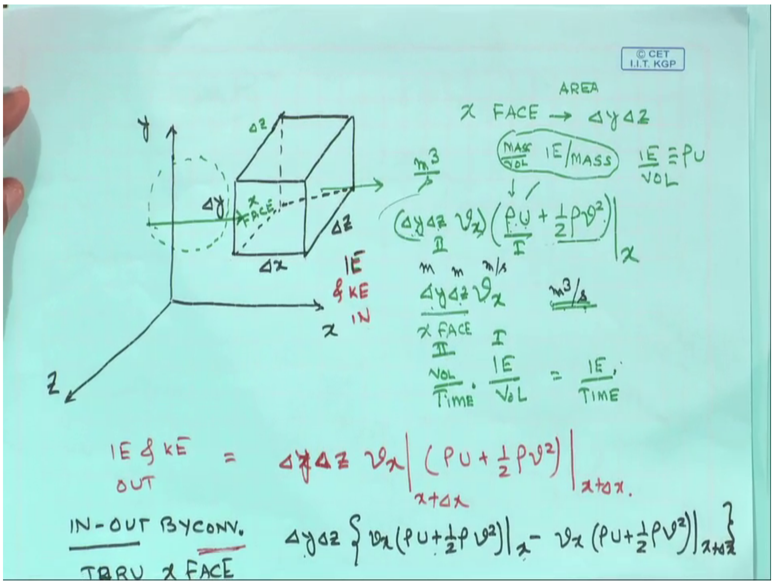


So, after that what I have, what I have done is I have defined the volume element in here, as this is the volume element and I have identified what are the terms for convection.

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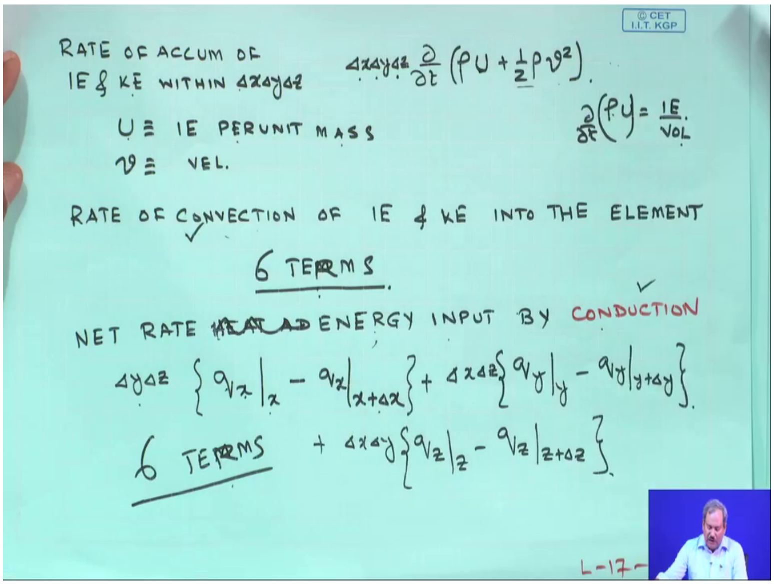


I have also identified what is the energy input by conduction, so these are the 6 terms that I have refer to.

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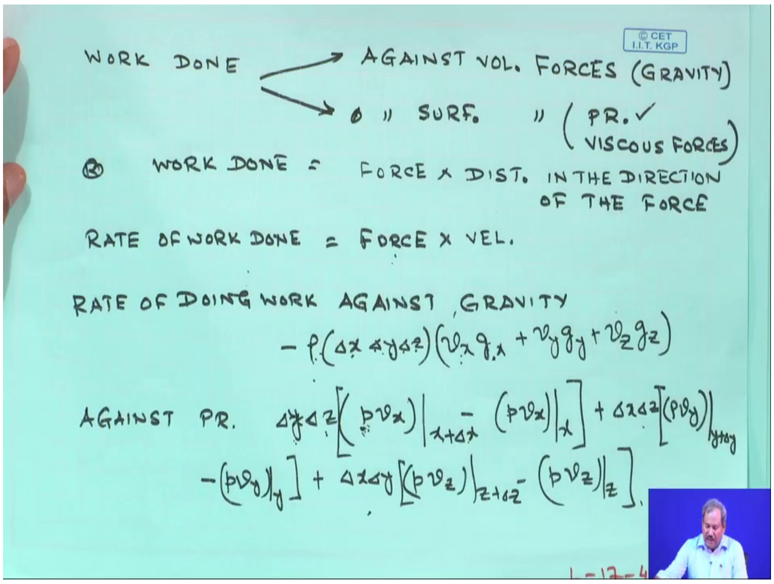


And then I have also identified the work done against volumetric forces for example, against gravity and against pressure, which would be these terms.

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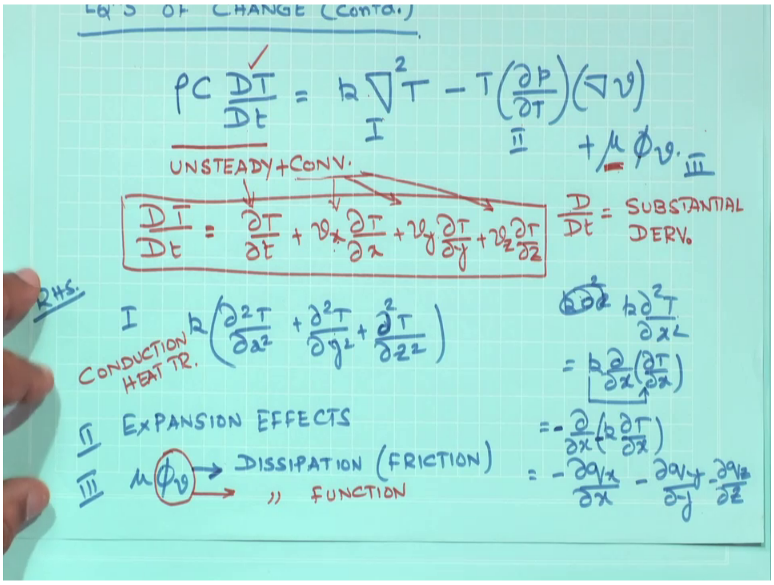


So, when I combine all these terms together and divide the, divide both sides by del x del y del z and then, take the limit when all of them approach to 0. What I get is a reduced form of the, reduced form of the equation and I am going to write the final form of the equation as rho C D temperature by D time is equal to k del square T, divergence of T minus T of del P del T times velocity, this is velocity plus mu phi v.

I have omitted large number of steps in between ok, which would be available in your textbooks so I am not going through all the steps in the class. What I would like to do is a like to clarify the genesis of each of these terms as I have described as a result of conduction, convection work done and unsteady state effects.

So, what you get out of that is a energy equation, now from that generalized energy equation I subtract the mechanical energy equation and what I get is known as the as the heat transfer equation, the energy equation where only internal energies are taken into account..

So, if you look at the terms over here, the first term this is the unsteady term, this D DT has a special meaning this D Dt, the capital D Dt this capital D of Dt its known as substantial derivative. This is a special mathematical function which simply tells you this is going to be equals del temperature by del time plus v x del temperature del by del x plus v y del temperature by del y plus v z del temperature by del z.

So, this is the expanded form of the D Dt, this one substantial derivative which is defined in this way. So, here if you see this term is definitely the unsteady term, what about these terms v x, v y, v z and with the temperature, temperature gradient in there. So, whenever you have velocity associated with any term in energy equation, that must refer to that should refer to convective flow of heat, because only in convection you have a, you have the velocity associated.

So, conduction takes place only when the medium has a velocity of its own. So, if you concentrate on, if you look at the first term on the left hand side, the first part of that del del T of temperature that is the unsteady term. The other three terms all contain the components of velocity, which are v x, v y and v z in a rectilinear coordinate system.

So, since they have velocity in that in its form so these three terms refers to the convective heat transfer process. So, therefore, this whole term is unsteady, plus convection where this term is due to the unsteady behavior of the system and the convection is manifested, represented by these three terms. Now, what is the first term over here, I will call it a second and this is the third term the first term on the right hand side.

If a constant the first term on the right hand side, it is going to be k del 2 T del x square plus del 2 T del z y square plus del 2 t by del z square and you can clearly see what is the significance of these term they have the k the, thermal conductivity and they talk about temperature gradient. So, k del 2 t del x square is nothing, but the different, the gradient in the heat flux, as you can write k del 2, k del 2 T del x square as k times del del x of del T del x and you can bring this k in here. So, this becomes del del x of k, del T del x and if I put a minus sign over here and a minus sign to balance the minus that have introduced here it is going to be minus del q x by del x.

Since by Fourier's law minus k del T del x is equal to q x. So, this term gives me the del x of the q x, the scalar component of the heat flux vector in the x direction. Similarly, del 2 T del y square would simply be minus del q y by del y and this term would be minus del q z by del z ok. So, this three terms on the right hand side, which I denoted by one is nothing, but it represent conduction heat transfer or conduction energy transfer.

So, the left hand side is unsteady and convective heat transfer, the first term on the right hand side is conduction heat transfer as I have shown here, the second term, the term 2 is due to expansion effects. And it is if this is expansion effects it essentially tells you that what would be what would be the effect of this expansion effect, what will be the work done against special forces and so on. And the third term which is mu times phi v, this is known as the work done against the surface forces which is which is the frictional effects, this is known as the dissipation.

And this dissipation is due to friction, this phi v is known as the dissipation function and the form of this phi v is quite complicated and I did not derived it. So, therefore, the equation, the energy equation, the complete form of the energy equation I did not derive all the terms, but what I have done is, I have shown you the significance of each of these terms in the overall scheme of things.

So, for most of the practical cases this third term can be neglected, the phi v the dissipation function or the energy change of a system due to viscous dissipation is neglected and can you clearly identify, easily identify the term by noting that you have a mu in front of this.

So, all the terms which of the viscosity in front of them in the energy equation they referred to the dissipation function and therefore, for most of the practical situations this third term can be neglected. The second term may or may not be relevant, so the second term will have will have different, different connotation in terms of its significance in the energy equation. And let us see what it would, what when it would be relevant and when it is not relevant and we can get back to an equation that we are more familiar way.

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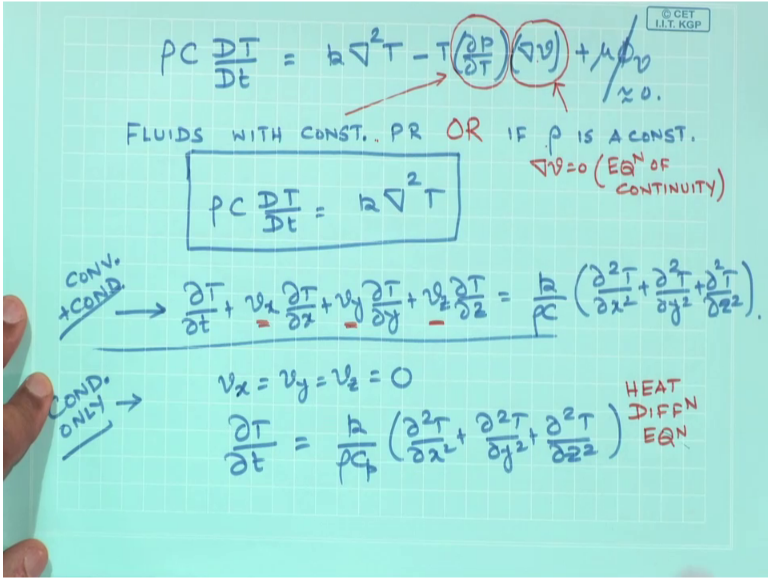


So, I am writing this equation once again, it is rho C, C is the heat capacity the substantial derivative of temperature which is equal to k times del square T which is the conductive heat transfer. T times del p by del T times divergence of the velocity vector plus mu phi v that I am setting equal to 0 for our case.

Now, if it is a, for fluids with constant pressure, which is a reasonable assumption in many situations or if the rho, the density is a constant. Then a fluid is with constant pressure then this part is going to be 0 and if rho is a constant, then this part is going to be 0.

So, if rho is a constant then del v would be equal to 0 which is nothing, but one form of equation of continuity that you must have studied in fluid mechanics. So, if rho is constant, this part would be 0, if fluid is are constant pressure then this term would be 0 and this is anyway to be equal to 0. If you use any of these conditions, then the equation would be, this is the form of the equation energy equation which is mostly used for convection as well as conduction.

Now, let us see what is, what is going to happen if it is just if you have, if its convection plus conduction both are present. I expand this term which would be del temperature by del time plus v x, del t by del x plus v y del t by del y plus v z del t by del z which is nothing, but the expanded form of the substantial derivative as I have described before. Would be k by rho C C, I have brought the c rho c on this side times del 2 t del x square plus del 2 t del y square and for the z. if this is the case and if it is conduction only case. So, if this is a conduction and convection, both are present then this would be the form for a rectilinear coordinate system.

If it is a conduction only case, now what is going to be the conduction only case, what is this what is this what are the simplifications that I can make this v x, v y and v z are going to be 0 since in conduction you do not have any velocity of the medium. So, since in conduction you do not have any velocity of the medium, velocity of the fluid itself then v x, v y and v z all are set equal to 0. And therefore, this equation becomes del T by del 1 is equal to k by rho C p rho C p or C times del 2 T del x square plus del 2 T, del y square plus del 2 T del z square.

So, I think you can recognize this equation electoral equation now; this is the equation which is also known as the heat diffusion equation that we have studied before, heat diffusion equation that we have studied before.

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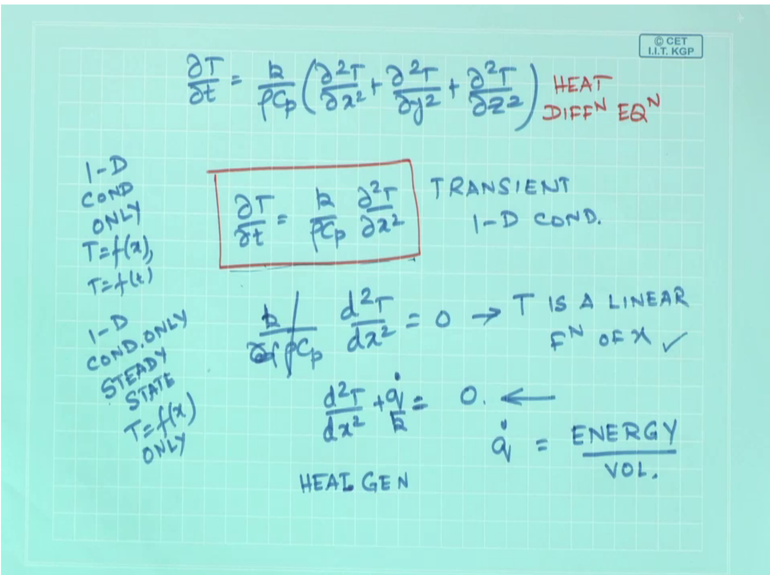


So, I will write it once again, the heat diffusion equation. One of the one of the things one should always do is when ever develop a new equation you should always try to see if it reverts to the known form for limiting cases.

So, the equation that we have derived is a general equation, is energy equation where we have taken into account the unsteady state, unsteady effect we have taken the effect, we are considered the effect of convection, conduction work done and viscous dissipation that we are not considering.

So, to this equation, now I am going to impose the condition that there is no expansion effects or the density is constant, so I take care of the second term on the right hand side. So, I am left with one term on the left hand side and one term on the right hand side, the term on the left hand side is the unsteady state effect and convection together, the term on the right hand side is only the conduction term.

What I have done after that is I will set convection to be equal to 0 by setting the velocity is to be equal to 0, the velocity components to be equal to 0. And therefore, what I have is the pure the heat transfer due to pure conduction.

So, heat transfer by pure conduction we have we have seen before, we have derived before in conduction which is the heat diffusion equation. So, in the limiting case of no convection and no work done what we get is the heat diffusion equation. So, if you see this equation once again, if this is for a one dimensional conduction only situation. Let us say the temperature is a function of x only, it is not a function of y or z, then this term this equation would be k by rho C p times del 2 T by del x square.

So, this would be the equation for this and if I simplified it further it is one dimensional conduction only situation, but at steady state. So, if it is a steady state case, then this temperature is a function only of x, it is not a function of time. So, what you have then is k by rho C, rho C p times d 2 T d x square to be equal to 0. It is temperature is a function of x only, it is not a function x and temperature is also a function of time in this case, here temperature is a function of x only.

So, this can be cancelled and what I have then is d 2 T d x square is equal to 0. So, this is this must look very familiar to you now since, you have studied conduction. If you have heat generation due to some electrical sources heat generation in the system, then you are going to have plus q dot by k to be equal to 0 so this form of the equation you have seen before. So, which is the one dimensional conduction only steady state situation with heat generation and if you set heat generation to be equal to 0, this is what you are going to get and where you should the result the solution of this would be T is a linear function of x.

So, one we can you can clearly see that starting with the more most general and slightly complicated expression, you can see this is the expression for where you have conduction, you have convection and conduction both present. Which is this form, if you set the convection to be equal to 0 what you get is the heat diffusion equation. If you simplify the heat diffusion equation, by assuming it is a one dimensional conduction only case you get the equation which is the equation for transient one dimensional conductions.

So, this is the case of transient, one dimensional conduction case and when you drop the transient make it a steady state is simply get this d 2 T d x square is equal to 0, no need to use a partial differential since temperature is a function of x only. So, t is a linear function of x and if it is a system with heat generation, you simply at the heat generation term where q dot is the energy generated in the system per unit volume.

So, with this, that concludes the development of the diffusion equation in our for most of the cases. And if you look at your textbook Incropera and Devita or any textbook will have this energy equation in, I am I have discuss the Cartesian coordinate equation, you would see that in cylindrical coordinates and you would see that in spherical coordinates as well.

So, the trick is to first identify whether you have cylindrical system or a Cartesian system or a spherical system. Then look at the equation, look at the full form of the energy equation and then depending on the situation at hand cancel the terms which are not relevant.

So, if it is a conduction only case cancel all terms which contains the velocity ok, velocity in its, velocity if it is a steady state case cancel the term which has the temperature derivative of temperature sorry, time derivative of temperature. If it is a situation in which viscous dissipation is not relevant drop all terms that contain the viscosity mu. So, I will just show you an example which is which is the full form of the energy equation in Cartesian and in cylindrical and so systems. So, this is the equation that you would see and it is available in your textbook.

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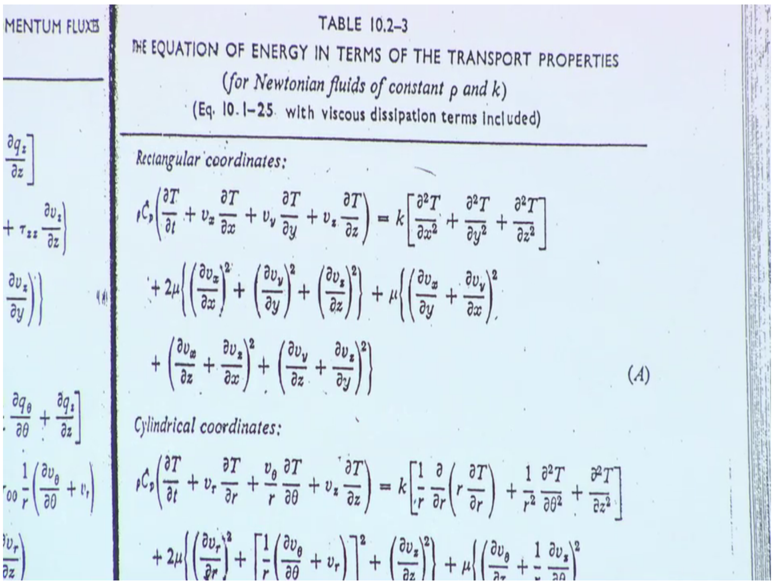

TABLE 10.2-3

THE EQUATION OF ENERGY IN TERMS OF THE TRANSPORT PROPERTIES

(*for Newtonian fluids of constant* $\rho$ *and* $k$)

(Eq. 10.1-25 with viscous dissipation terms included)

*Rectangular coordinates:*

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + 2\mu\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right\} \quad (A)$$

*Cylindrical coordinates:*

$$\rho \hat{C}_p\left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] + 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2 + \left[\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right)\right]^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)^2 \ldots\right.$$

So, if you look at carefully over here, this is rho C p I do not know if you can see it properly, I will read it for you. So, that it becomes clear to you, rho c p del temperature by del time plus v x times del T del x v y times del T del y v z times del T del z. So, you have a transient term and 3 terms which contain velocity, so they must refer to convection. As I have shown you the entire left hand side is a combination of transient effects and convective effects, when you come to the right hand side you have k del 2 T del x square del 2 T del y square and del 2 t del z square.

This terms contains k the thermal conductivity and I have shown you that these three terms referred to conductive heat, the heat transfer due to conduction only and finally, there is a complex set of terms which are nothing, but the gradient square del v x del x square del v y del y square and so on. But the important point to note here is that all these terms they contain mu explicitly.

So, since they contained mu without even knowing the full derivation, you know that these terms that contain mu referred to viscous dissipation, the dissipation of energy due to viscosity of the solid and as I said it becomes relevant only for high speed, high viscosity flow or flow in small micro fluidic systems. So, for most of the practical purposes you do not need to consider this term, these terms at all, you are going to considered only on the left hand side and on the right hand side the right hand side for convection the left hand side for transient and for convection.

So, that is the governing equation which we are going to use for all our treatment of convection subsequently, similar to the rectangular coordinates we have the equation in cylindrical coordinates as well as in spherical coordinates.

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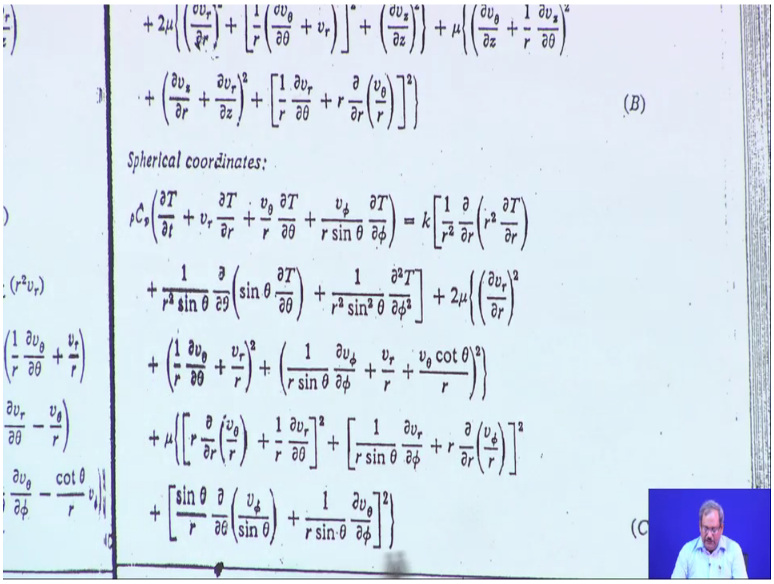


The significance remains the same, the left hand side contains a transient term end terms which contains v r, v theta, v z so they must refer to convection. The right hand side, there would be three terms, these three terms which contain k. So, they refer to the conductive transport of energy, all the other terms that you see contain mu.

So, this entire group of terms they refer to viscous dissipation in cylindrical systems, that can be neglected in most of the, for most of the applications. And when you come to spherical coordinates again the same thing transient, convection, convection, convection, conduction, conduction, conduction and the rest of the terms contain mu. So, this entire set of terms is nothing, but the viscous dissipation which can be neglected.

So, what we have seen in this class is the derivation, the simplified derivation of the energy equation where all effects are considered. The energy equation that you have derived starts from a very fundamental law, which is the first law of thermodynamics for an open system, we have taken care of convection, conduction the work effects and everything else.

So, this would be the starting point for our studies in convection. So, this is slightly complicated concept, for I think once you go through the textbook and read and read look at the equations carefully. I am sure the concepts would be clear to you and if there are any questions I will be more than glad to answer, answer them and once you master these equations at this the significance of the equations then the rest can move a in a much smoother fashion.