Heat Transfer

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Lecture - 17
Equations of Change for Non-isothermal Systems

We are going to start with a, the Fundamental Derivation of Energy Equation in the, I am not going to show you all the steps, I will rather tell you the significance of the major steps and I will skip the rearrangement of terms etcetera where known new concepts are involved, but I would simply try to show you that based on the conservation of energy one would be able to write and energy equation which is a pity, that would take care of the transient affects, the conduction, the convection and the rate of work done by the system or on the system.

So, if we can derive this equation then based on the application at hand, we will be able to simplify the solution, a simplify the equation end get, this should give rise to the governing equation for the entire process. And we are also going to check the equation with something that is already known to us. For example, we know that what is the heat conduction equation, the heat conduction equation is simply k del to t del x square plus del to t del y square, plus del to t del z square plus q dot. Where q dot is the heat generated per unit volume by k, on the other hand side it should be rho c p times del del del t of temperature.

So, some of the, some of the limiting equations are already known to us. So, you would see that based on the expanded, the full form of energy equation, if you can impose certain conditions whether or not we recover the solution the form which is known to us. But before we do into, go into that that part I would like to introduce a concept which we started discussing about in the last class, which is called the shell balance. Now, this shell balance can be of momentum, it can be of energy it can also be offers species.

So, when we do shell momentum balance. So, what we do get is a governing equation that describes the velocity, the order rather the change in velocity as a function of x, y, z and time. When we write the shell momentum shell heat balance we should get what is known as the equation of energy. Similarly, if I write it for a species, which is let s say reacting with another species in a flowing fluid field, then we would get the species conservation equation which is going to be very important in mass transfer, but you will not going to that. Let us try to concentrate on how we can write a shell heat balance, the trick is to define a shell of let us say some size del x, del y, del z which is fixed in space.

So, a cuboid shape of size del x, del y, del z we have 6 faces and through these 6 faces, one is the x face which is perpendicular to the x direction, one is the y face which is perpendicular to the y direction and the other is going to be that z face which is perpendicular to the z direction.

Each of these faces will have areas associated with them and through these areas heat energy can come into the control volume. So, the control volume is defined by a box of size del x, del y and del z, through this faces energy can come into the control volume, into the defined shell. And as a result of which if the energy which comes in and energy which leaves the system if they are not balanced then the internal energy content of the box would increase.

Now, when we talk about energy will we are not going to talk only about the internal energy, we also have to consider the kinetic energy. So, a fluid may come through the x face with certain velocity and therefore, certain kinetic energy, at a temperature which is different form the temperature of the fluid contained in the box. So, therefore, the entering fluid will have some internal energy and some kinetic energy associated with it. So, it will enter the x face and will leave the x plus delta x face, similarly it would come through y, leave at y plus delta y, come at z and leave at z plus delta z.

So, I would once again show you the way this box is drawn, it is fixed in space and we would identify the terms which are associated with convection and conduction. And then will think of the work done by the system or on thus, to the surrounding or on the system by the surrounding and what kind of different work can be, can be done by the system against some forces, which could be a body force or a surface force, as we have discussed in the previous class.

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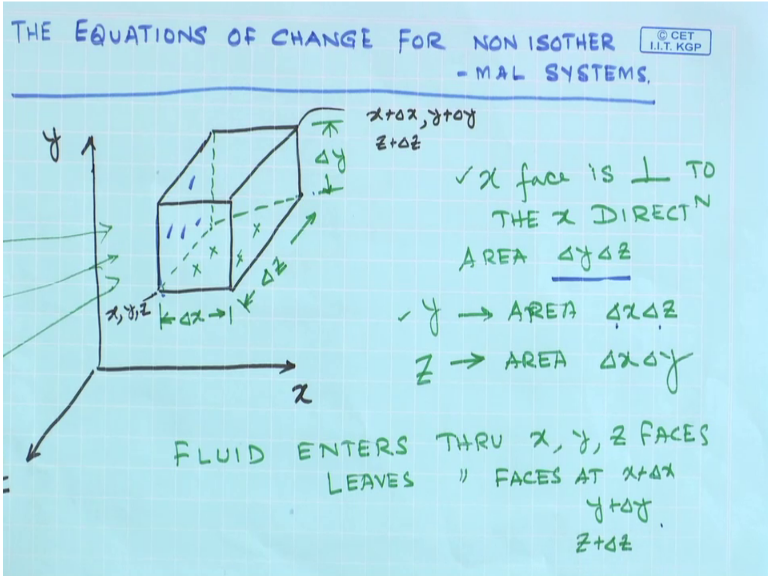

But let us once again go through this which is equation of change for a non isothermal system, here I have drawn this box which is del x, del z and del y.

So, these are the x, y and z direction and the x face which is this one is perpendicular to the x sorry, this is the x face, the x face is perpendicular to the x direction and therefore, its area is del y, del z. So, it is del y times del z would be the area of the x face, similarly the y face which is perpendicular to the y direction would should have area of del x, del z. And this z face which is the front face that you see in this figure, it would have an area of del x times del y.

So, fluid enters through x, and x, y and z faces and the leave through the faces at x plus del x, y plus del y and z plus del z. So, this is what we have covered in the last class and we are going to now write what would be the form of the energy equation for a system of size del x, del y, del z which is which is fixed in space and this is our coordinate system.

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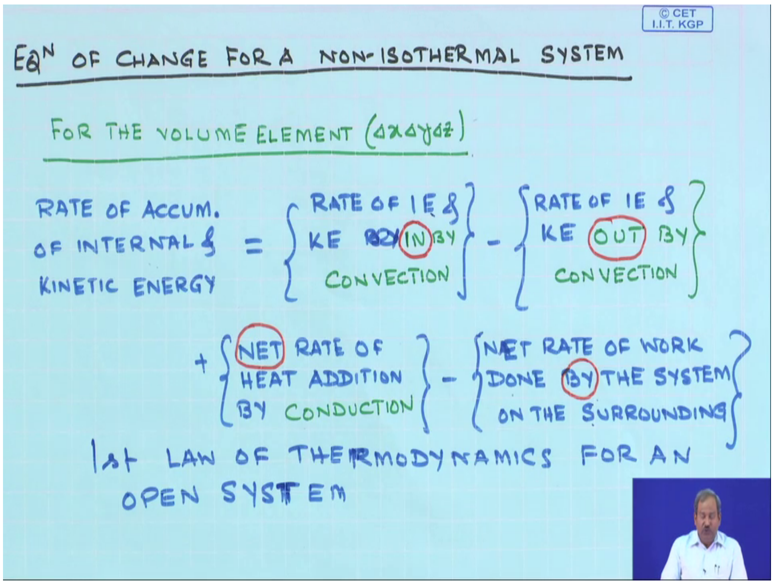

So, let us see how would the equation of change for a non isothermal system look like. If I write it in words, then for the volume element which I have defined as size del x del y del z, the rate of accumulation of internal and kinetic energy, in the volume element that I have defined. Which is del x del y del z the rate is going to be equal to the rate of internal energy I E and kinetic energy K E which comes into the volume element by convection.

So, when there is flow some amount of energy comes with the flow and when I talk about energy I speak about both the internal and the kinetic energy. So, some internal and kinetic energy can come into the control volume, into this volume element by convection and it is going to go out again by convection, from the x plus del x, y plus del y and z plus delta z faces. So, this is the in and the out by convection, if there is a temperature difference which exists in, in the x direction. Let us I have a temperature difference exists in the x direction. So, there then; obviously, I am going to have some flow of heat through conduction through this x face.

So, a difference in temperature either in x or in y or in z would give rise to even if the fluid is still, it would still give rise to conductive heat transfer. So, if there is a temperature difference, let us going to be a conductive heat transfer and that is what I have written over here is the net and when I write the word net, what I mean is that it is the summation of energy in and out. In minus out that I have written separately over here, I have combined them together to write it in the form of net rate of heat addition to the volume element by conduction.

So, these two terms are for convection and the come, together if I combined them and I I identify the mechanism by which its heat is going to come into the system by conduction. So, this takes care of all the heat that comes to the system by conduction or by convection. However, there is one missing term that I should take into account at this point is the rate of work done by the system on the surrounding. And since it is by the system, as a result of doing work by the system on the surrounding the internal, and internal and kinetic energy the total energy of the system should reduce, should decrease and that is why we have this minus sign. Had this been a case of work being done on the system, then this sign should be positive.

So, based on whether the system does work, in which case this the energy content to decrease, energy content of the volume element to decrease or work is being done on the system in which case the at the total energy of the system would increase. So, what I have written over here is nothing, but the first law of thermodynamics, for an x and since I am allowing fluid to enter and leave, this must be for an open system.

So, the statement which I have written over here is nothing, but the first law of thermodynamics for an open system. Now, from this generalized energy equation one should be able to, one should be able to deduct or to subtract the commonly available equation for kinetic energy of a system. And therefore, what you would left out with is the energy equation where we are only construing in internal energy, as a part of the heat transfer course we are mostly interested in what happens to the internal energy, which is manifested by a change in temperature.

So, in order to, in order to subtract, in order to take care of the kinetic energy change for this, from the more general energy equation that I have written over here we can subtract the kinetic energy equation, in order to obtain purely the energy equation where we are considering only the internal energy. So, that is what we will we will do next, but first of all we have to identify what would be the terms for convection, for conduction and the rate of work done by the by the system or on the system.

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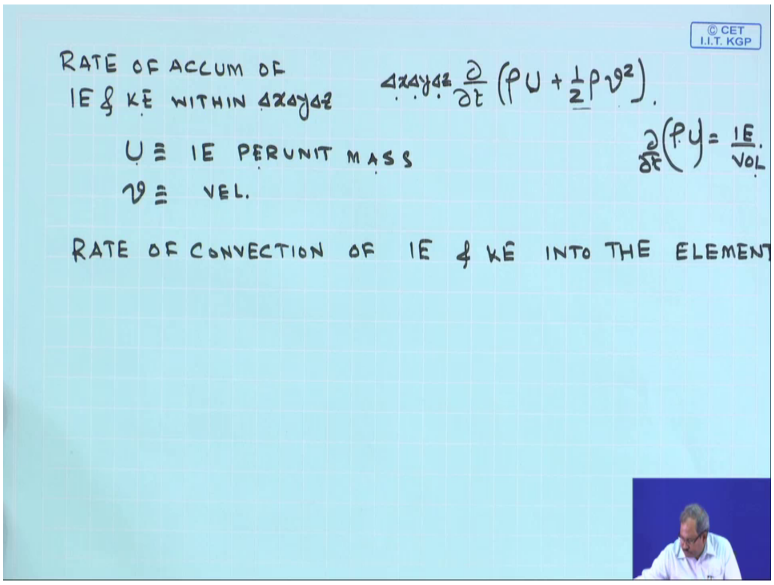

So, will go term by term and see how do they appear in the, how do they appear in this equation. So, first of all let us see what is going to be the rate of accumulation of internal energy and kinetic energy within the system, which is defined as del x del y del z. So, this must be del t, time rate of change of rho times u plus half rho v square where, this U is nothing, but the internal energy per unit mass and v is the velocity. So, this U is internal energy per unit mass, so here I have multiplied it with rho. So, this becomes internal energy this, two terms together they become internal energy per unit volume.

So, if it is internal energy, if rho times U have has units of, so rho U is I internal energy per unit volume, since my U is internal energy per unit mass, rho times U is internal energy per unit volume. So, del t of this is the rate of change of internal energy per unit volume, but you would like to, we know what is the volume of it. So, if I would like to find out what is the total rate of accumulation of internal and kinetic energy within x, y, z this must be multiplied by del x del y del z, which makes it the rate of change of internal energy for a system, whose dimensions or del x del y and del z. As long as my U is defined as internal energy per unit mass, rho is the density.

So, therefore, del x del y del z times del t of rho U would simply give us the amount, the rate of accumulation of internal energy within del x del y del z and similar the same logic will also applied to half rho v square. The next one is, I am going to write what is would be the convection of internal energy and kinetic energy into the element, the element what where the element has size del x del y del z. So, what is that going to be, let us pick this figure.

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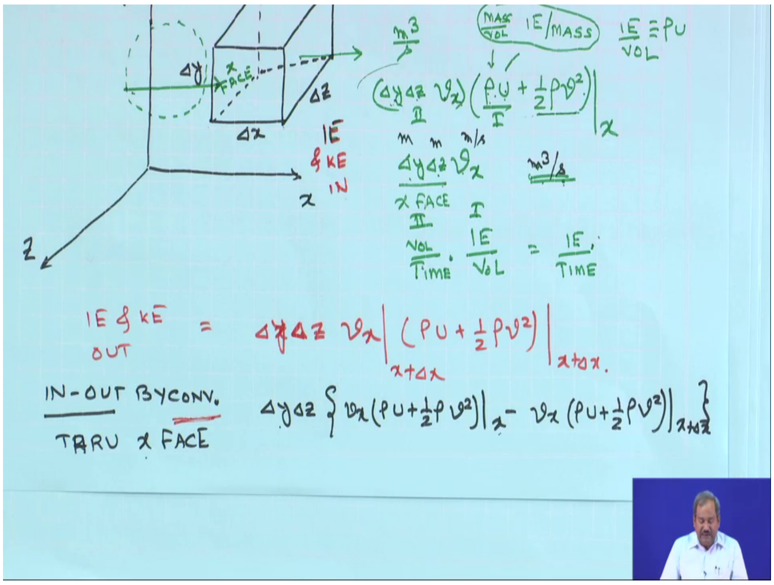

In this figure, some amount of let us, this is my x face, which has area of del y and del z, some amount of fluid is going to enter through this x face ok.

And let us say it is, this velocity with which it comes into this comes into this control is del x del y del z is v x, rho U plus half rho v square, this is the internal and kinetic internal and kinetic energy. So, what is the fluid mass which enters through this x face per unit time? So, the area of this face is del y del z so, if this is del y del z what is the mass that comes in through the x face by unit time, the other this must be equal to v x. If you look at the units of these so this is meter and meter per second. So, the unit turns out to be meter cube per second, which is fine this is the volumetric flow rate of fluid through the x face.

So, this is x face, through the x face. So this the volumetric flow rate volumetric flow rate per unit time, volume per unit time. So, through the x face the amount of, amount of volume of fluid which comes in must be equal to del y del z times v x, this is internal energy per unit mass, this is mass rho is mass per unit volume. So, what I have done is the product of these two, when I take the product of these two this is internal energy per unit volume. So, rho times U is internal energy per unit volume and this term del y del z times v x, this is units of meter cube per second.

So, what then, what I have then here, this term is internal energy per unit volume. So, if I call it term one this is internal energy per unit volume and this one if I call it term two, the term two is nothing, but volume per unit time. So, the product of del y del z v x and rho U is simply internal energy per unit time. So, this gives the rate of change of internal energy due to flow of the fluid through the x face, the same applies for the kinetic energy part of it.

So, when you multiply half flow v square, which term two which is del y del z times v x it is going to give you the kinetic energy per unit time. So, together this whole term gives you the end, this is evaluated at x at this point term this face. So, it comes in through the x face and goes out through the face at x plus del x. So, what is, so this is it I E, I E and K E in, what would be I E and K E out, this must be equal to del x del sorry, del y del z as before this is v x, but v x is not at x, but evaluated whatever be the velocity at x plus delta x.

So, the entire thing is going to be at x plus delta x times rho U plus half rho v square, all evaluated at x plus delta x. So, the amount of energy, internal and kinetic energy in due to convection. Why due to convection? Because I have a non zero v x, due to convection would be this term, which I have explain term by term and the internal and kinetic energy out which is at x plus delta x, must be everything remaining same except the quantities are now evaluated not at x like this, but is evaluated at x plus delta x. So, when I considered the net, so if I go in minus out by convection. So, if I take this, so I am simply going to have del y del z times v x, rho U plus half rho v square evaluated x minus v x rho U plus half rho v square, evaluated at x plus delta x.

So, this is what the expression for in minus out by convection through the x face would look like. So, I guess this is clear to all of you that how I have arrived at this expression for the convective heat flow through the x face. Now, I can write the same thing for the y face, the only thing which will be different here is the y face has an area equal to del x del z and since it is, we are talking about y then it the v x must be replaced with y, v y and over here v x is to be replaced again by v y. Instead of evaluating it at x, it is going to be evaluated at y and y plus delta y, similarly for z face this area is going to be del x del y, v x is to be replaced by v z and everything else will remain same.

So, in total to take into account the convective flow of internal and kinetic energy into the volume element, del x del y del z, I will have 6 terms. Two terms each for x y and z face would give me total of 6 terms that would signify what is the total amount of heat which comes into the system by convection. Similarly, now I am going to write what is so the rate of convection of internal in kinetic energy into the element, there will be total 6 terms.

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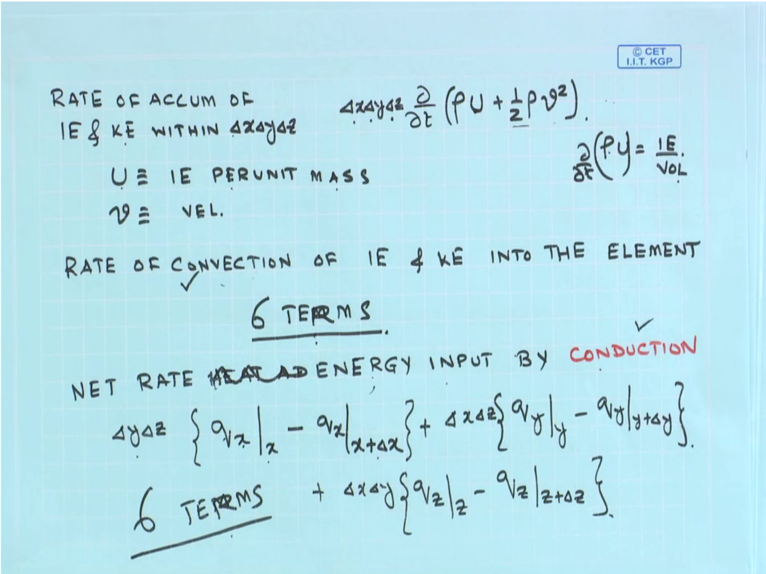

As I have explained before, and then we have to think about net rate of heat addition by conduction, I would not say heat here, energy because I also have the kinetic energy to take care of, net rate of energy input by conduction. And here I am going to express it in terms of the heat flux, the heat flux in the x direction, the component of heat flux in the x direction.

So, this is, this is the heat in per unit area per unit time. So, I must multiply it with the appropriate area since its x face this must be equal to delta y delta z. So, this x denotes, it tells me what is the heat that comes to the control volume through the x face and the one that goes out through the x plus delta x face would be this much. So, these two terms together one at x and one at x plus del x multiplied by delta y delta z, together they tell us about the net rate of heat addition by conduction through the x face.

Similarly, I am going to have the y face which will have del x del z in the area and the heat flux is are going to be q y, at y minus q y at y plus delta y and for the z face it is going to be del x del y times q z, evaluated at z minus q z at z plus delta z. So, this 6 terms again, 6 terms for each of the faces would tell me about the net energy input by conduction and so I have taken care of the convection and I have taken care of conduction. So, what is left in my, in my remaining one is I have already taken care of in by convection, out by convection and net by conduction.

So, what is left is work done by the system on the surrounding. So, what is work done by the system on the surrounding? We have to, if we can considered that then my equation is complete.

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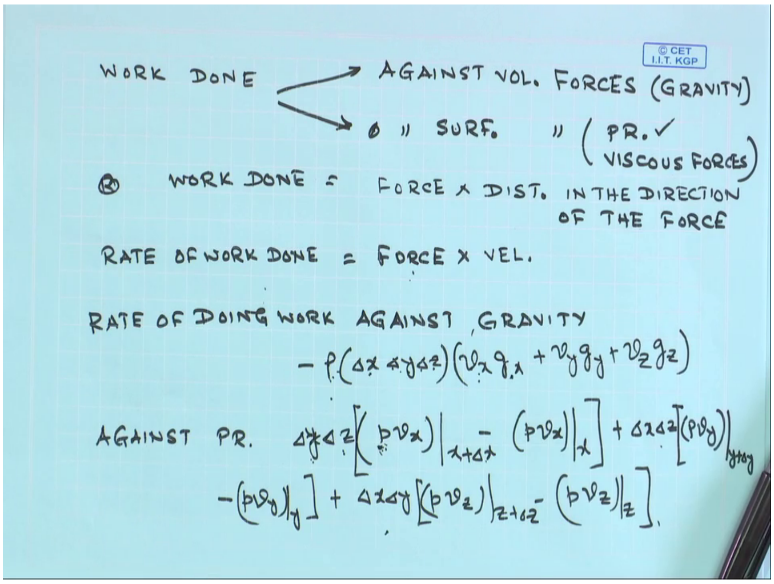

Now, work done as we all know it simply, it can be against volumetric forces, volumetric forces that which are acting on the entire volume of the volume element. The, the common example would be gravity and the second one is against surface forces, surface forces which could be against pressure, which could be against viscous forces. So, these two are again the common examples of surface force which is, which you can see pressure force and the viscous force.

Now, let us see the, I am not interested in work done, I am interested so work done is, we all understand gets force times distance in the direction of the force. So, what would be rate of work done that is the time rate of work done, it would be force times distance by time.

So, distance by time; obviously, would give you the velocity. So, this is what you are going to get for the rate of work done. So, rate of work done would simply be expressed as, this is time rate of work done would simply be expressed as force times velocity. So, let us quickly right the expression for the force against gravity forces, so rate of doing work against gravity would simply be equal to minus since, it is against work is done against gravity.

So, v x times g x velocity in this is acceleration due to gravity plus v y g y plus v z g z. So, when you, when you see this equation you would be able to see that it is the force in the x direction which is v x, del x del y del z times rho and this is this is multiplied by g x. So, this totally gives you the rho del x del y del z times g x is the force, because this is mass, this is mass per unit volume this is volume and this is the acceleration. So, this gives you the force and force multiplied by the velocity in the appropriate direction. So, v x v y and v z would give you the rate of work, rate of doing work against gravity.

So, again what is going to be the form for against pressure, it should be the area on which let u say the x face del z times p v x, evaluated at x plus delta x minus p v x at x. So, this is one term, plus del x del z, p v y at y plus del y minus p v y at y, this is going to be the second term plus del x del y, p v z times z plus del z minus p v z. Look at this terms one more time and see what the mean, pressure is force per unit area. Whatever be the pressure at x plus del x is multiplied by the appropriate area which is del y del z, to give to give us the force in the x direction acting on the control volume, acting on the volume element at x plus del x and we understand that the rate of work done is force times velocity.

So, that is why I am multiplying del y del z times p with v x in order to obtain what is the pressure, what is the work done against pressure forces at x plus del x and similarly this is the rate of work done against the forces at x and I am going to write that for the x face and this is for the z face. So, these 6 terms together would give us the work done against the pressure forces by the, by the by the volume element del x del y del z. What is remaining here is the work done against viscous forces, now work done against viscous forces this I am going to neglect for the time being because work done against viscous forces is something similar to solid friction.

So, what happens when you work against friction forces, you are pulling object, pulling an object over a rough surface. So, you have to overcome the viscous forces, overcome the forces frictional forces exerted by the rough surface on the system that your pulling overhead. So, as a result of which there is going to be heat generation in a net and the energy change, the any work that you do that you will get the work that you explain in order to make that block move over a rough surface, it is going to be converted into heat and it will change the energy of the of the system. Similarly, when fluid flows specially at high speed through a small dot, there is going to be tremendous velocity gradient which is present.

So, let us say I have a jet which is the, which is very thin and the fluid is coming at a very high velocity. So, the velocity is large and if the velocity is large and the gap is small, then the velocity gradient would be very large and we understand that the viscous force is related to velocity gradient in, the shear stress is mu times velocity gradient. So, if the velocity gradient is large end or the viscosity is large, in that case you will have a strong force that you need to overcome in order to make the fluid flow through that thing can do it at a very high velocity.

If that happens then you do substantial work against the viscous forces and whenever you do that kind of, that kind of work against viscous forces the temperature will increase. And that increase in temperature which is obtained at the expense of work done by the system must be taken into account for any energy equation, for any form of energy equation.

However this is only relevant in some special situations, we do not get the heat generation due to viscosity in many of the practical problems. As you would see it requires high velocity gradient, very high velocity gradient and very high viscosity. So, where are the, what are the places in which they become relevant, when a rocket reentered earth’s atmosphere its velocity is very large, the atmosphere is still, but the rocket is coming down with a very high velocity. So, near the boundary layer, formed close to the rocket the velocity changes from that of the rocket which is very large, to velocity equal to 0 which is the velocity of the atmosphere.

So, this thinnest of the boundary layer and the very high speed of the rocket reentry, at reentry would ensure that the frictional heat generated is tremendous. And that is why you would see that the rocket comes as red, almost like a red hot and there has to be special protective arrangements to ensure that the safety of the astronauts inside the rocket is not compromised.

So, that is an extreme example, in some cases viscous polymer is extruded by making it flow through a very thin gap, if that is the case then the viscosity is high, the velocity is large as you would like to have higher throughput of the polymer when you are making a rare note of it or the sheet out of it.

So, the velocity combined with the high viscosity of the polymer and the very high speed with which it comes out of the die, ensures that you cannot neglect viscous dissipation. However, in this in the example that we are going to do in heat transfer we will neglect viscous heat dissipation, wherever the viscous heat dissipation is relevant. I will tell you that how to incorporate additional terms into the energy equation which would give the, which would take into account viscous heat generation.

So, if you look at your textbook and look at the full form of the energy equation, you would see there are a bunch of terms which have mu, which are multiplied with mu which has mu in front of them. So, the easiest way to identify which term of your energy equation in your text relates to be, relate to viscous heat generation, look for terms containing mu. If in your problem the viscous heat generation is negligible drop the entire set of terms containing mu and what you would have is the energy equation that we, we are going to use for most of the realistic applications.

So, since those terms you complicated, I am dropping them for the time being, but making you aware that in some special situations you need to add them to ensure that your energy equation is complete. So, whatever we have done in today’s class, what will you do next is, we will add all these terms. The rate of doing work against gravity, the rate of doing works against pressure, the convective terms which are added, the net convective terms the net energy input by convection and equate that to the rate of accumulation of internal and kinetic energy within the volume element del x del y del z.

So, when we write that what do you end and we divide both sides by del x del y del z and take in the limit when del x del y and del z approach 0. What we would get is we can convert the difference equation, which is a statement of the physics of the problem into a differential equation that contains among other things the velocity due to convection and the temperature.

So, we would have to see how we can solve that partial differential equation in order to obtain the temperature distribution, so that is what we are going to do in the next class. But first of all you would see whether by neglecting convection are, we getting back the equation of conduction, with which we are more familiar with at this point of time. So, the beginning of the next class would be to finally, get what is the energy equation and try to reduce it, try to simplify it for some applications in which the temperature could be just a function of one variable, it is a steady state and it is a function of x or of y or of z.

So, we start with energy equation, see where it confirms to conduction equations that, we are familiar with and then go into convection. So, that is our plan for the next few classes.