Heat Transfer
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Lecture - 13
Transient Conduction - Heisler Chart

The last part of Transient Conduction is to use the graphical form of the series solution. Some of the in some of the cases the series solution can be truncated and the results for many situations are expressed in a graphic in graphical form. So, that gives a powerful tool for practicing engineers to find out what would be the temperature as a function of position. Remember this in this case the Biot number is not less than 0.1. And therefore, the temperature is a function of the special coordinates. Unlike the case where you have assume that the temperature inside the solid remains constant with respect to space it only varies with time.

But this is a case in which the temperature inside the solid can be a function both of space and of time. So, let us look at how it is done and you should look at your textbook to see how the different charts and graphs are provided. I am only going to give you a brief introduction of those charts, in how they are used to calculate through an example which will follow this lecture through an example of how it can be used to obtain the temperature. So, this type of series solutions are commonly known as Heisler charts.

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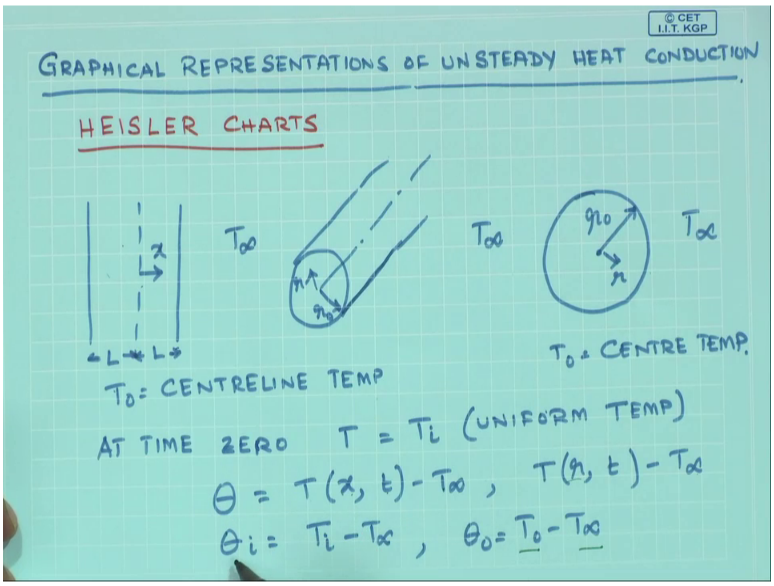

So, this let us take a look with look at how we define this. So, we are looking at graphical representations of unsteady heat conduction. The result is the results are provided in graphical form which is known as Heisler charts.

In the graphical representations are valid for three of the most common cases of transient heat transfer that is a plane wall of thickness twice cell. It is it is also there for radial systems where r 0 is the easy is the radius of the wire and it is also there for a spherical system where r 0 is the radius this sphere.

In all cases T 0 is the centerline temperature in these two cases T 0 is the centerline temperature that means, the temperature along the axis. Whereas, for the case of a spherical system T 0 simply refers to the centre temperature and all of these the plane wall, the cylinder and the sphere. They are exposed to convection environment which is let us call the lets say that the conviction environment the temperature is T infinity.

So, as time progress is based on the difference between the initial and the temperature to which it is exposed to the temperature inside the solid will vary with time ok. So, we will say that at time that is the initial temperature at time 0 T of the solid would simply be equal to a constant temperature T i which is uniform throughout. So, this is uniform temperature. Initially the temperature of each of these solid is at a constant value equal to T i and the time varying temperature is given as T which is a which is a function of both x and time minus T infinity. So, this is this is some sort of a difference temperature this is for a plane wall whereas, for the case of a cylinder it is going to be T which is a which could be a function of the radius, the radial position and time.

So, this is this is what we have for theta under this theta I which is the initial temperature difference is T i minus T infinity. And finally, what we have as theta 0 to be equals T 0 minus T infinity. So, one more time here we see that the temperature is a function of location as well as time.

So, theta the difference temperature denotes T as a function of x and t minus T infinity for the case of plane wall whereas, for the case of cylinders or spheres this theta is simply T r t minus T infinity. So, for the case of a cylinder this is simply the central line temperature, central line temperature as compared to the T infinity.

And for the case of sphere it is the centre temperature with centre temperatures or centre where the centre temperature with respect to the T infinity. So, theta i is T i minus T infinity, the solid of it could be hotter or cooler. So, that is what it is read over here and theta naught which is the central line temperature minus T infinity. So, I guess the three nomenclature of theta i and theta 0 are clear, theta is the temperature as compared to T infinity theta i is the initial temperature as compared to T infinity and theta 0 is the centerline temperature in comparison to T infinity.

So, the results of this the results of this are expressed in graphical form and I will show you one or two of representative graphs how they look like.

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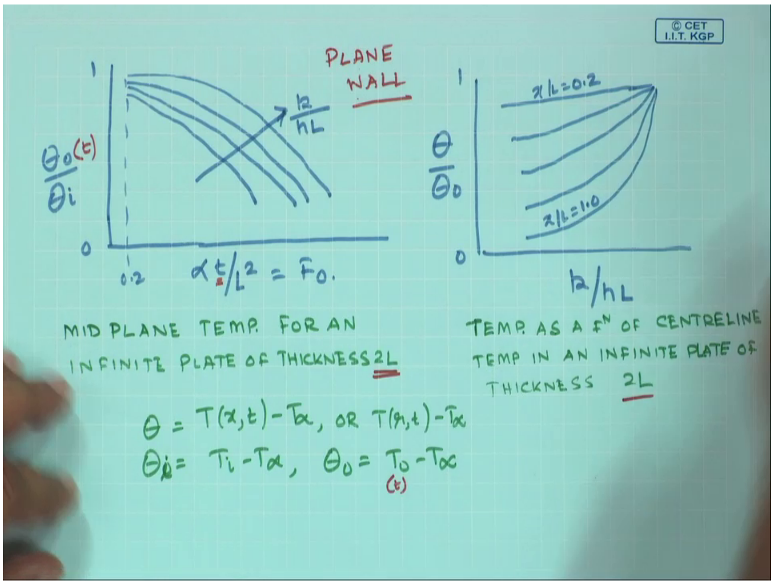

So, let us think about the lets take the plane walls you unit text, you should you would see that this theta 0 by theta i. This is a central line temperature and this is the initial temperature. They are plotted as a function of Fourier number between 0 to 1. So, at t equal to 0 that is at this point the theta 0 must be equal to theta i.

So, all points in the solid will be at a constant temperature at time t equal to 0. As the time progresses you are going to get a family of curves like this. So, this value is roughly about 0.2 up to 0.2 it has been done and this is increasing value of the inverse of Biot number.

So, this is one figure which is provided in the text. So, this essentially gives you the theta 0 which is the mid plane temperature for an infinite plate of thickness 2 L. So, what I mean by infinite thickness is that the plate is really the plate is really wide. However, its thickness is finite and that thickness is equal to 12. So, this curve provides you with what is the centerline temperature. How does the central line temperature vary with time for different values of Biot number.

So, having this graph for the plane wall would let you find out what is the centerline temperature as a function of time ok. But you do you do not you need definitely you need the centerline temperature, but you also need the temperature at every point in between. So, what is to be done if you required not just the centerline temperature, but the temperature at every point at every point in between. So, for that there is a second curve. So, which you can also see in your text what I am going to draw it over here so, that we know how what this curves look like and how to use those curves for calculating the variation in temperature.

So, what it look like is the this curve is plotted as a function of k by h L and theta by theta 0; again from 0 to 1 in roughly they would look something like this. So, this is x by L equals to 1 and this is x by L equals 0.2 and some point and in between. So, this gives temperature as a function of centre centerline temperature or centerline temperature in this case, in and infinite plate of thickness 2 L.

So, again if you look careful and just to be just to be just to remind you, I write the expression I write what are theta as T x t minus T infinity or T r t minus T infinity, theta naught is T i minus T infinity and sorry theta i and theta naught to be equals T 0 minus T infinity. So, let us see you would like to find out and with these are for plane walls.

All the two curves that you see are for plane wall in the there the in your text you would see the similar curves are there for the case of spheres as well as cylinders. So, let us say you would like to find out what is going to be the temperature of a plane wall which is experiencing conviction, convective heat loss or gain from two sides the thickness of the plate is twice L and it is really wide in other directions. So, how do you find the temperature knowing that lumped capacitance model cannot be used. So, use Heisler chart.

The first Heisler chart where the x axis is going to be to be Fourier number so, if you look over here the x axis is Fourier. The x axis is Fourier number which contains the time and the y axis is the centerline temperature so, for different values of Biot number or rather inverse of Biot number.

So, what you need to do is first find out what is the Biot number for your system, it is greater than 0.1, but you still you exactly know what is the value of Biot number and find out what is the value of Fourier number after what is the value of Fourier number corresponding to the time that that you that you at which you would required the temperature.

So, let us say T equals 10 minutes you know the value of alpha, you know the characteristic length. So, you know the value of Fourier number you go all the way up to the Biot number which relates to your material and then come to this side find out what is the value of theta 0. The theta 0 is nothing but T 0 you understand that this T 0 is also a function of time whereas, T infinity remains constant this T i, the initial temperature is not a function of time.

So, therefore, this has a fixed value. So, in this theta 0 is a function of time. So, through the use of this curve at a given t for a given value of Biot number or inverse of Biot number you find out how the central line temperature is changing with time. Then you come to curve two, in curve two again you see that inverse of Biot number is there in the x axis and the family of curves that you see are at different locations in the solid.

If my L to be equal to 1 refers to this point whereas, x by L to be 0 refers to the centre line. Thus, if you want to find out what is the value of temperature at let us say at a point exactly midpoint between this these two, these two planes right over here; that means, x by L equal to 0.5. So, from the value of inverse of Biot number you go all the way up to x by L equals to 0.5 come to this side and find out what is theta by theta 0. What is theta? Is the temperature. What is theta 0? You have already obtained from curve one.

So, in order to obtain the centerline temperature you require just one curve, one chart. In order to evaluate the temperature at points in between the central line and the surface you required this cover as well, where from the value of the inverse of Biot number you go to the curve corresponding to the location, corresponding to the dimensionless location and read what is the value of theta by theta 0.

Since, theta 0 is known theta can be found out. So, this is the way by which one can obtain for simple geometries, plane wall, cylinders or spheres what would be the value of temperature as a function of position and as a function of time. All these are valid for Biot number less than, sorry Biot number greater than 0.1 where the lumped capacitance model is not valid.

Now, there is a second part to it which in some cases you are not only interested in to know what is the temperature, but you will also like to know how much of heat the control volume. Let us say the sphere has gained while it is being heated; how much heat is gained by a cylinder while it is being heated or if it is being cooled how much heat it has lost.

Since, the temperature is not space wise isothermal you would exactly like to know what is the energy content of the body at a given instant of time and that brings us to the third figure of a third chart of Heisler chart which will give us the value of the energy content of the object as a function of time. So, the first two charts are to find out what is the central line temperature and what is the temperature at any point in the control volume. The third chart of Heisler, the third figure of Heisler chart refers to how much energy, what is the energy content of the body as a function of time.

So, I will I will draw that and which will complete our study of transient conduction.

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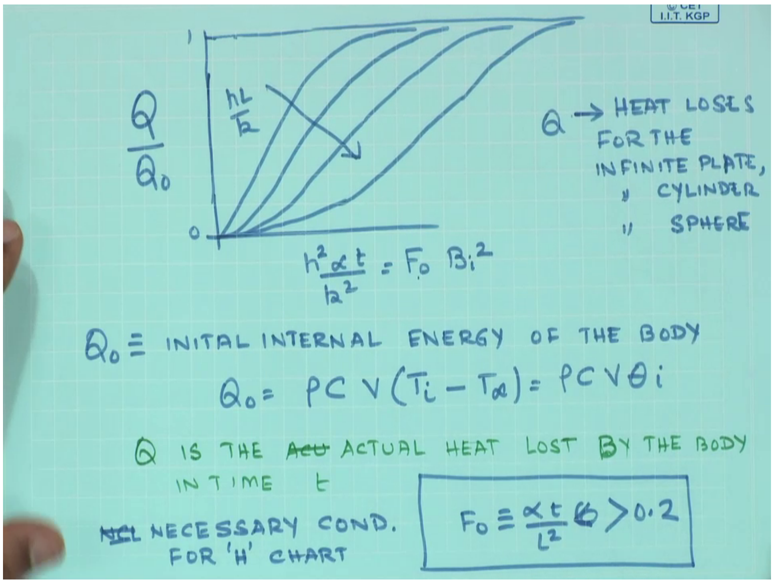

So, the curves looks something like this, this is Q by Q naught from 0 to 1 looks something like this. So, this is h square alpha t by k square which is nothing but Fourier number times Biot number square and these are h L by k.

So, this Q is the heat let us say it is a quenching problem. So, heat losses this Q refers to heat loses for the infinite plane infinite plate or infinite cylinder and infinite sphere. And Q 0 is the initial internal energy of the body, we know that the initially the entire object is at a temperature of T i which is same everywhere, which is a constant. So, Q 0 can be replaced rho C V T i which is the initial temperature minus T infinity nothing but rho C V theta i ok. So, in this figure Q is the actual heat lost or gained depending on whether it is heating problem or a cooling problem by the body in time t.

So, if you know what is the time, if you know all the thermophysical properties. Essentially, if you know the Fourier number and the Biot number then using these graphs using these curves corresponding to a different values of Biot number you should be able to calculate what is Q. Since, your Q naught is known to you.

So, since you know the initial internal energy content of the object you would be able to find out what is the energy that the body has lost. If it is a quenching problem or the body has gained, if it is it is a problem where the object is going to be heated; that means, it is exposed to conviction environment where the temperature is more than that of the object itself.

So, through this chart you then would be able to calculate the total amount of energy gained or lost in the system. So, this is the third phase Heisler chart which can be used for many of the engineering calculations provided you are dealing with the plane wall or you are dealing with an infinite cylinder or you are dealing with a sphere ok.

But there is one word of caution here this truncation of the infinite series solution which has been used to obtain this to generate this curves has a limitation. It is only valid when the necessary condition for Heisler chart to be valid is that Fourier number which is defined as alpha t by L square must be greater than 0.2, where this L is the characteristic dimension.

So, this condition has to be kept in mind while using the Heisler chart for calculating the calculating the energy, calculating the central line temperature or calculating the temperature at any point in between the centre and the surface of the object.

So, this is another useful and compact way of finding out what is going to be the temperature for situations where lumped capacitance model cannot be used. I will quickly give you a problem for you to practice. So, that is going to be another tutorial problem, but I will not solve it completely in the class you try to your own and if there is any question I will discuss it or TA's will discuss with you for the solution.

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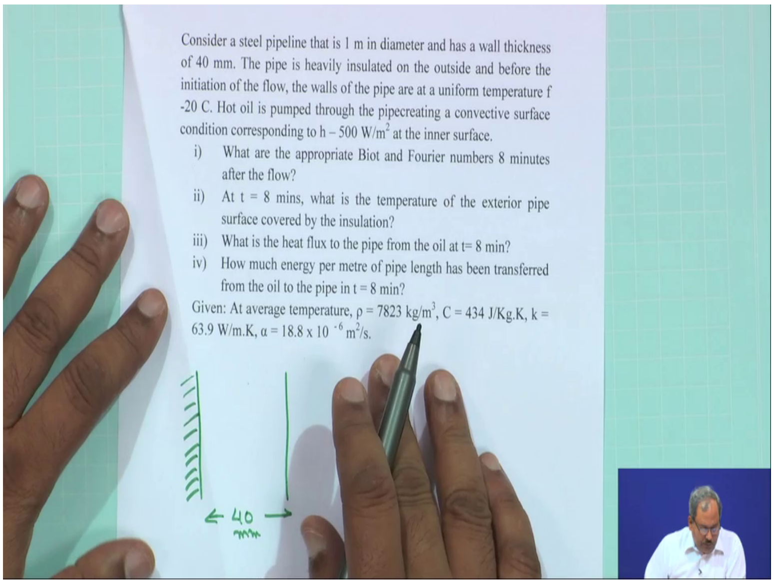

So, the problem that we are going to deal with is this; a problem on transient conduction again.

So, you consider a steel pipe line which is 1 meter in diameter. So, you have pipeline whose has a wall thickness of 40 millimeter. So, this length is 40 millimeters and it is heavily insulated on the outside. So, this side is insulated then this pipe is before the initiation of the flow, the walls of the pipe are at a uniform temperature of 20 degree centigrade.

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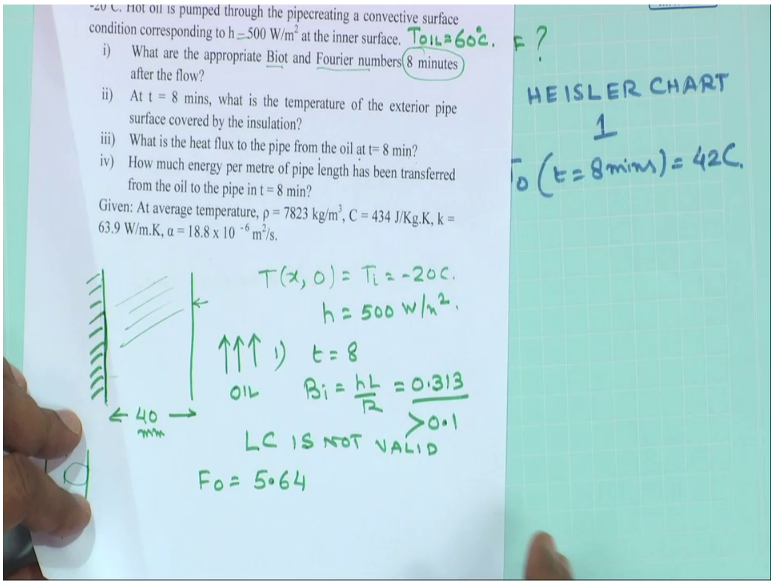

So, what we have initial is that T at any x but 0 time is equal to T i could minus 20 degree centigrade and then we have hot oil which is pumped through this pipe with a convective heat transfer coefficient corresponding to h equals h equals 500. So, h is equal to 500 watt per meter square at the inner at the inner surface.

Now, you have to find out what is going to be the appropriate Biot and the Fourier numbers 8 minutes after you start the flow of the hot oil. So, initially everything was at a temperature of minus 20 and then suddenly you start passing current sorry passing an hot oil through this and you would like to find out how the temperature will temperature will change at different points.

So, what we can do then is first of all calculate the Biot number after t equals t equals 8 minutes. So, when you go for t equals 8 minute the Biot number is h L by k and by putting the values in there you would see that the Biot number is equal to 0.313 so, it is definitely greater than 0.1. So, LC is not valid.

So, that is one observation and you can also obtain the value of Fourier number and you should check yourself that is this is going to come to 5.64. So, this is the first part of the problem. The second part of the problem it says is after t equals 8 minutes, what is the temperature of the exterior pipe surface covered by the insulation; that means, what is the temperature of this surface after 8 minutes.

Now, the interesting thing what you need to realize or need to see is that this is exactly like half of a pipe of thickness 2 L. Now, if you remember what happens at x equal to 0 that means, at the centre plane of the of the centre plane of the system plan planer system, it acts like an adiabatic wall. The temperature is maximum at that point and therefore, x equals 0 from x 0 to L is identical at from x 0 to minus L.

So, having the pipe insulated at x equal to 0 ensures that I can use this; I can use the solution of the plane wall for this case as well. So, the in the outside of the pipe wall this is nothing but the centerline temperature of Heisler chart. So, we are only looking at half of this, we are only looking at this part which is over here.

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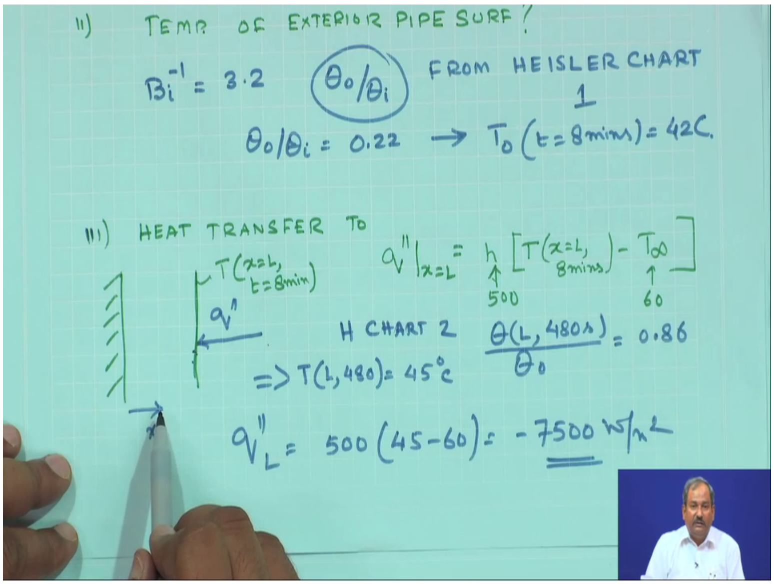

So, since this is an adiabatic plane and this is an insulated plane this part and this part are identical and therefore, Heisler chart is going to be valid. So, that is part two is part two of this is what is going to be the temperature of the exterior pipe surface. So, if you if you need to find that out then what you do need to do is you are going to use Heisler chart and for a Biot number inverse of Biot number to be equal to 3.2 you find out what is theta 0 by theta i from Heisler chart.

So, what do you have this here is essentially you are using the first curve. So, for t equals 8 minutes that means, you are calculating the value of Fourier number, you know the Biot number. You go all the way up to the value of Biot number corresponding Biot number for this plane wall, come to the side and find out what is the value of theta 0 where theta 0 is nothing but the central line temperature minus T infinity.

So, with this from Heisler chart number 1 you should be able to calculate this theta 0 by theta i to be equal to 0.22 which would give you the centerline temperature at t equals 8 minutes to be equal to 42 degree centigrade. The third part of this problem asks you what is the heat transfer; heat transfer to the inner surface the by inner surface I mean this surface. What is the heat flux to the pipe from the oil at 8 minutes?

So, how much of heat gets transferred from the oil to the pipe. So, that is that is essentially the problem. The temperature of the hot oil which I think is not mentioned over here, the temperature of the oil is at 60 degree. So, hot oil T of the oil is 60 degree which is missing here 60 degree centigrade.

So, when it says what is the heat flux to the pipe from the oil at 8 t equals 8 minutes then I am essentially trying to find out; if I can find out what is the temperature at x equals L and t equals 8 minutes. If I can find this, then the heat flux would simply be equal to the heat flux would simply at x equals L will simply be equal to h times temperature at x equal to L at 8 minutes minus T infinity where T infinity is that of the oil. So, this is 60, this is provided as 500.

Now, you have to find this. How do you find this? In order to find the temperature at x equal to L what you need to do is use a second chart and in the in the in the second in the second chart you know what is the inverse of Biot number. You go all the way x by L to be equal to 1 x by L to be equal to 1 and read what is the value of the x axis.

So, here theta by theta naught, theta naught you have already calculated from here. So, you can calculate theta which is corresponding to x equal to L. So, this value would give you the temperature of the inner wall of the pipe after at a time equals to 8 minutes.

So, when you use Heisler chart 2 number 2; you should be able to find, you should be able to see that your theta at L corresponding to a time of 8 minutes which is 480 seconds by theta naught to be equal to be equal to 0.86, which would give as T L 480 to be 45 degree centigrade. And therefore, q double prime L would be 500 times 45 minus 60 equals minus 7500 watt per meter square. So that means this much of heat flux q double prime is going to come from the oil to the inner surface at 8 minutes after 8 minutes.

So, since with increase in temperature with an increase in time the temperature of the wall will also increase. Therefore, the amount of heat which would get transferred from the hot oil to the cold wall will decrease with time. It is going to be a function of time and in order to obtain the value of the heat flux at any specific instant of time, you need Heisler chart 2.

But in order to use Heisler chart 2 you should first calculate from Heisler chart 1 what is the centerline temperature; because the temperature at any point in the solid will be given as a function is going to be non-dimensionalized by Q naught which is the centerline temperature.

So, when you do that you are going to see that this value of q double prime would come out to be 7500 and the third part of the or the at the at the last part is how much of energy is has been transferred from the oil to the pipe in 8 minutes.

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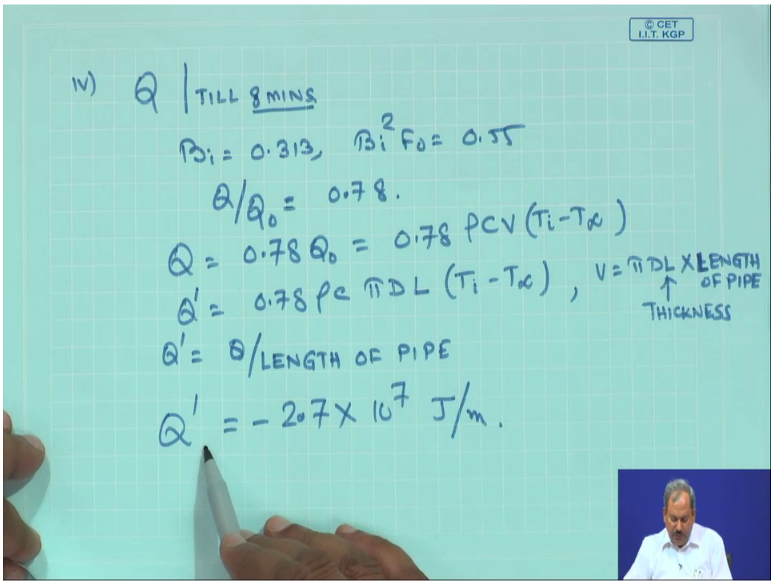

So, the last part is you have to find out what is the value of Q till 8 minutes. What is the value of heat that has that has been transferred from the hot oil to the pipe over the time period of 8 minutes. So, obviously here you are going to use the 4th chart, the 3rd chart where this is the amount of heat which has been transferred. This is the amount of heat which was which was initially contained in the solid wall and here you have the Fourier number which contains time and Biot number square.

So, at 8 at time equal to 8 minutes or 480 seconds you calculate this one, you go all the way up to the Biot number to the curve corresponding to the Biot number in this specific case, come over here and find the value Q by Q naught. When you do that for this specific problem, you would see that for a Biot number of the present system to be equal to 0.313 and Biot number square times Fourier number equal to 0.55 your Q by Q naught from the curve you will read as 0.78.

So, your Q is 0.78 Q naught and this should be Q naught should be equal to rho C V T i minus T infinity. So, this Q then is 0.78 rho C and V is V is pi D times at times L so, this T i minus T infinity.

So, this is that will be volume is pi DL where L is the thickness of the pipe times length of the pipe. So, when you do this is Q prime and Q prime is simply Q by length of the pipe and when you put the numbers you would see Q prime to be equal to minus 2.7 into 10 to the power 7 joule per meter. The minus sign in the total amount of heat that is transferred from the oil to the pipe or the minus sign what you would get that heat transfer the heat flux from the oil to the pipe simply refers to that the heat travels in a in the direction of minus x. So, the only significance of the minus sign is the direction of heat transfer.

So, now at the end what we have in transient conduction is that we know there are two principle methods for solving transient conduction problems. One is the lumped capacitance which is easy to use, which gives which is which gives us the compact form, but it has certain limitations. It can only be used if the value of Biot number which is conduction resistance by convection resistance, if it is less than 0.1 and we have solved a problem in fact, a number of problems to see how it can be done.

The second one is when the lumped capacitance model cannot be used, what is the alternative. The alternatives are either go for an analytic solutions, go for a numerical solution or in a in some cases this series solution arising out of the analytical solution, those they have been they have been presented in the form of curves for geometric geometries like pipe, the plane wall, the cylinder or the sphere.

And these charts can be successfully and effectively used to obtain not only the centerline temperature, but temperature at any point in between. Apart from that the Heisler chart can also be used to find out what is the total amount of energy transferred from the object to the outside ambient or vice versa and as a function of the initial energy content of the object.

So, the 3rd Heisler chart presents in the x axis what you get is Q the total amount of energy transfer divided by Q naught, where Q naught is a energy contain and the x axis contains both time and the length scale thermo physical property all combinations by having Fourier number times Biot number square.

So, through the use of all three curves one would be able to obtain what is the temperature and what is the energy that that gets transferred over a certain period of time for a specific solid, for a specific convective heat transfer coefficient and so on. So, in the last class on conduction heat transfer we will look into something interesting some interesting applications of conduction heat transfer to reduce the temperature or to increase the heat dissipation from some specific systems. We are we are talking about fins. What fins are, what extended surfaces are, what are their applications and when we can use, when we can prescribe the use of fins to increase the heat transfer from an hot object.

So, that would be the last topic in conduction heat transfer. And then we will move to convective heat transfer from the next to next class.