Heat Transfer
Prof. Sunando Dasgupta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 12
Transient Heat Conduction

So, far we have we have been studying Transient Conduction. In civil assumptions associated with it, we have seen that incorporating lumped capacitance model, that is if we can assume that the temperature is going to be space vice isothermal as the object is getting cold; that means, temperature inside the solid will remain in variant which position, it is a function of time only, in that case significant simplification of the entire problem is possible. And we are going to get a closed form solution, for the variation of temperature with time.

We have also seen that this assumption can only be used, when the Biot number which gives us some idea about the conviction the conduction resistance and convection resistance. It is the ratio of conviction resistance to conduction resistance, when the Biot number is small ideally when it is less than 0.1, then we can safely use the lumped capacitance model in transient conduction.

So, the significance of Biot number and that of another dimensions number which is called Fourier number is clear to all of us. So, what I would do in this class is I will I am going to solve another problem on transient conduction, where you would see that the lumped capacitance model is valid, but it is slightly different than the one that we have attempted so far. So, far we have what we have done is when I write the energy equation, it is rate of energy in minus rate of energy out plus any generation of energy rate of any generation of energy inside the control volume, must be equal to the rate of energy stored in the control volume.

The formula the methodology the modeling which we have adopted so, far assumes that there is no energy which comes in it is let us say it is a Quentin problem. So, this no energy in to the control volume energy goes out principally by convection by a convection convective process and, that is also no energy generated in the system. And therefore, the energy going out is simply going to be equal to the rate at which the energy is stored, inside or stored or it depleted inside the control volume. And the equations which we have obtained essentially assumes that only two terms, one term on the left hand side and the term on the right hand side, they will remain in the physical description of the situation.

But what happens in the case, where we have energy generation as well in the control volume. So, the whenever you are going to use any formula be careful to identify whether, the basis on which the physical system based on which the formula has been derived, whether or not its identical with the problem that you are solving. So, the first tutorial problem of today is when an electric current passes through an electric wire and the temperature obviously will rise as a result of the heat generation and, let us also assume that the wire is exposed to a conviction environment.

As the temperature of the wire rises it is going to lose heat by convection and, part of the heat which is which is generated in the wire is going to be used to raise the energy the energy stored, the capacity energy stored in the system. So, rate of energy out minus of rate of energy out, plus rate of energy generation must be equal to the rate of energy stored. So, this way the inclusion of the generation term mix the problem different than the than those which we have already discussed and solved in this.

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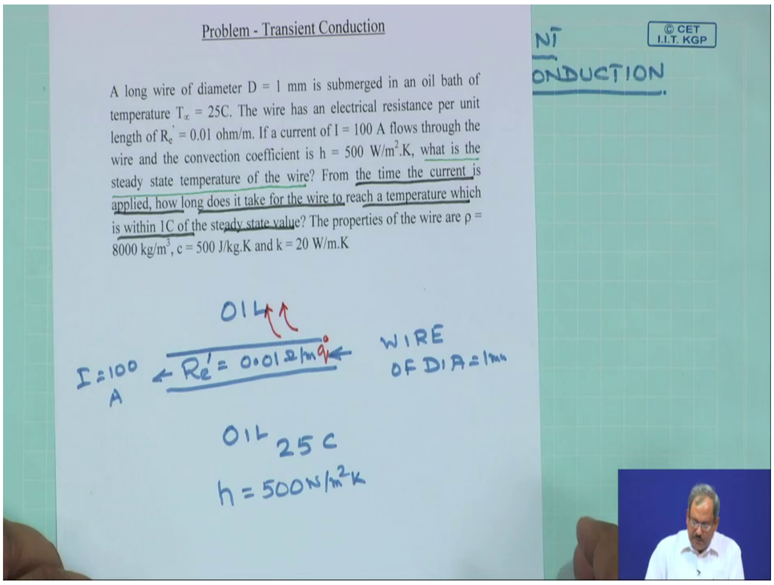

So, the first problem today and I hope you can read this which it says is that a long wire of diameter D equals 1 millimeters. So, you have a diameter wire of diameter 1 millimeter is submerged in an oil bath. So, it is it has oil everywhere and the oil bath is at a temperature of 25 degree centigrade.

And the world has electrical resistance per unit length. So, I call it as re prime to be 0.01 ohm per meter and the current which flows through this, that I is equal to 100 ampere. The oil bath creates a convective environment, where h can be the value of h is equal to 500 watt per meter square Kelvin. So, there are three parts, or few parts of the problem the first one is what is the what is the steady state temperature of the wire ok. The second part is from the time the current is applied, how long does it take for the wire to reach a temperature, which is within 1 degree centigrade of the steady state value.

So, there are the and there are some properties of the wire are given for example, rho c and k thermal conductivity all these are provided. So, but essentially the problem is that of heat transfer by convection to the oil heat generation, because of the flow of current unit. And since it is a transient process it would require some amount of heat, it would also d denote that some amount of heat is going to be stored in it.

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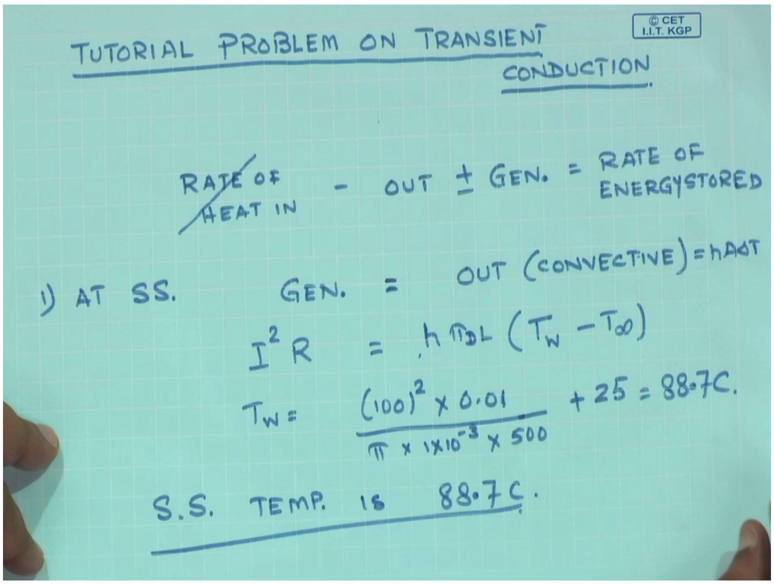

So, the equation if I write that rate of heat in minus out plus or minus generation, or depletion should be equal to rate of energy stored in the system.

So, the first part of the problem, we can write that at steady state, when the steady state is there, the right hand side of this equation is 0 because, at steady state it is the temperature is not going to be a function of time. Since the temperature it could still be a function it could still be a function of position, but at but at steady state there would be no change in the in the in the energy stored inside the system. There is no heat which comes into the system. So, out should be equal to generation. So, at steady state the amount of heat generated because of because of the resistance of the wire and the current which is flowing through it, must be equal to the energy which is going out.

And this out is by a convective process, we know the value of h and so, on. So, the energy generated must be equal to energy out. So, if I is a current which is flowing through it and R is the resistance of the entire wire, then it should be Newton's law of cooling it should give us just h a delta T and by h what we mean as the convective heat transfer coefficient times pi d L, where L is entered length and this is the T of the wire at steady state minus T infinity. So, at steady state this equality must hold, in order to obtain in order to obtain what is the steady state temperature. Now, we it is a simple equation which can be or where you can substitute the values and, get the final value of the temperature of the wire, which is going to be a constant which is which will not change with location.

And therefore, the value of T w can be obtained from this relation one point to note here is that this value of the resistance per unit length is provided in the problem. So, if I divide the left hand side this side by re by L, then I have all the all the way very all the numbers with me so, the a is hundred amperes square into R by L is 0.01 ohm per meter. And then pi sorry I missed a D over here pi times D is 1 into 10 to the power minus 3 into h is 500 plus T infinity which is given as 25.

So, this should be about 88.7 degree centigrade. So, the steady state temperature steady state temperature of the wire 88.7 degree centigrade. See the next part of the problem tells us that we need to find out, what is the what is the what time does it take for the wire to reach the temperature, which is within one degree centigrade of this steady state value. So, starting at 25 degree centigrade, how much time has to has to after how much time the temperature will be 1 degree centigrade of the steady state value; that means, what is the time required for the wire to reach a temperature of 87.7 degree centigrade.

One thing we have to keep in mind here is that, we are using the lumped capacitance assumption. Whenever we use lumped capacity assumption, or lumped model is imperative that we find out whatever we have done, whether or not it is going to be valid. So, the way to check the validity of lumped capacitance model is to find out what is the value of Biot number.

So, once the value of the Biot number is calculated and, if it is less than 0.1, then whatever we have done so far or whatever we are going to do by incorporating the lumped capacitance model would be correct. So, the next step or the one of the major steps of solving transient conduction problem, using lumped capacitance is to show that your assumption is valid by calculating; what is the value of the Biot number. So, that is what we should do next.

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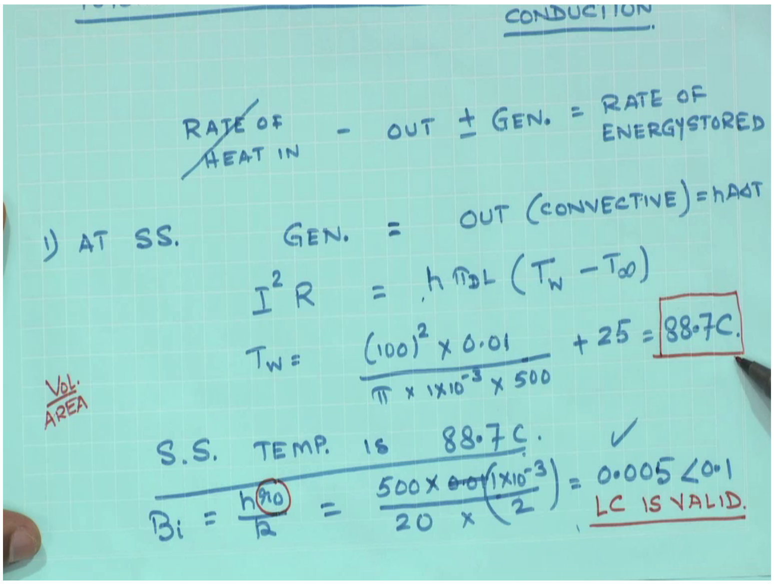

So, this value of the Biot number, I can calculate right now which would be h times r 0 by k and see I have used r 0 ideally this length scale is simply volume by area. So, if it is volume by area for a cylindrical 1 it is not going to be r 0, it is going to be most likely r 0 by 2, what as I have mentioned in order to be on the conservative side of calculations on the conservative side the length scale is always chosen as the dimension across which we are getting the maximum temperature drop. So, for the case of a cylindrical system, you would expect the maximum drop in temperature in the solid between the centre line and that of the surface.

So, to be really to be conservative about the number the magnitude of the Biot number we will always r 0 for the case of cylinders and, for the case of spheres as well whereas, for the case of a plane wall, we will use half the thickness of the wall, when it is heated when it is heated from both sides, uniformly from both sides heated or cold, uniformly from both sides. So, that is the length scale that we should use for calculating the value of the Biot number. So, we put the values in here what you would get is the value of h is 500 the value of r 0 can be can be substituted in here, in the value of so, this is by 2 and the value of k which should give you the value of Biot number to be really small definitely less than 0.1.

And therefore, the major this thing is LC lumped capacitance as a model is valid in here. So, once we know that lumped capacitance is valid, then we should be able to able to calculate, what is the time required for this thing to reach 1 within 1 degree of the steady state temperature. And we know that the steady state temperature is 88.7.

So, essentially we have to find out how long does it take for the wire to reach a temperature of 87.7 degree centigrade, which is one degree of the steady state temperature. In order to do that I am going to first write physically what is going to happen what is happening in this case.

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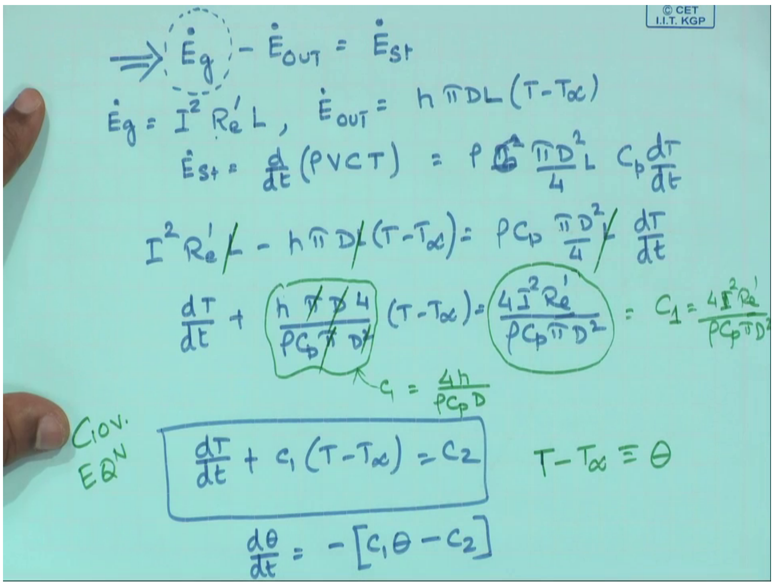

So, in this case the governing equation takes the form that E dot generation minus E dot out by convection, is equal to the rate of change of energy stored in the system.

So, this E dot g is the additional term, which we are getting for this specific problem. Now, what is E dot g if we express everything in terms in terms of energy only. So, this is R e prime which is a resistance per unit length, times the length of the length of the wire in E dot out would simply be h times a which is pi D times L T minus T infinity, this T is the time variation time varying temperature of the system, but since Biot number is less than 0.1, the T inside the wire will remain a constant ok. So, it is not going to vary in space, but it is going to vary in time.

And E dot store would simply be equal to d dt the time rate of change of energy contained within the control volume. So, energy should always be expressed in terms of reference, but since that reference is a constant I can simply write it in this specific form, which is row pi D square times L. So, that is that is the volume times C times dT by d time. So, this is this is this is the m cp that rate of change of time with the rate of change of temperature with time. So, this I am going to put in my physical equation the understanding here.

And the link can be cancelled from all sides. So, what we end up with is the variation of temperature with time. So, this pi will cancel D and D from here would cancel, if you look at this part this is nothing, but a constant let us call this as C 1. And this is also a constant 4 h by rho C p times D. So, this is another constant let us call it as C 1. So, my C 1 is 4 h by rho C p D and C 1 is simply 4 I square r E prime by rho C p by D square.

So, that is straight forward so, we whatever we got is a straight forward expression in terms of the variation of temperature with time which is so, this is the governing equation, which we need to solve and it requires 1 boundary condition. So, but before that I think we can also define that T minus T infinity to be equals theta, it is just a it I am defining it in this way. So, my governing equation would therefore, would become d theta dt to be equal to minus C 1 theta minus C 2. So, this would be my more compact form of the same governing equation ok.

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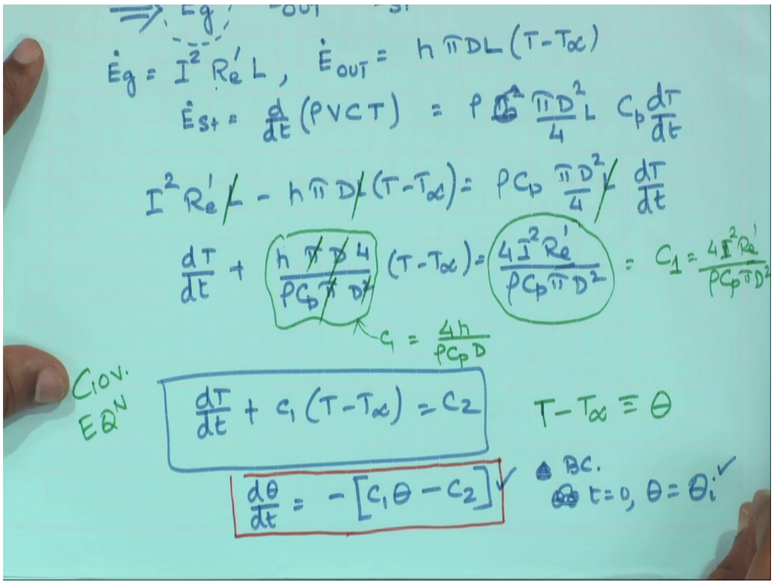

And at the boundary condition would be at theta equals sorry at time t equals 0, theta is theta i which is the initial temperature difference between the wire and the surrounding medium. So, this equation has to be solved with this initial condition to obtain the variation of theta with time. Now, the solution of this is pretty simple and I would only show you some of the steps.

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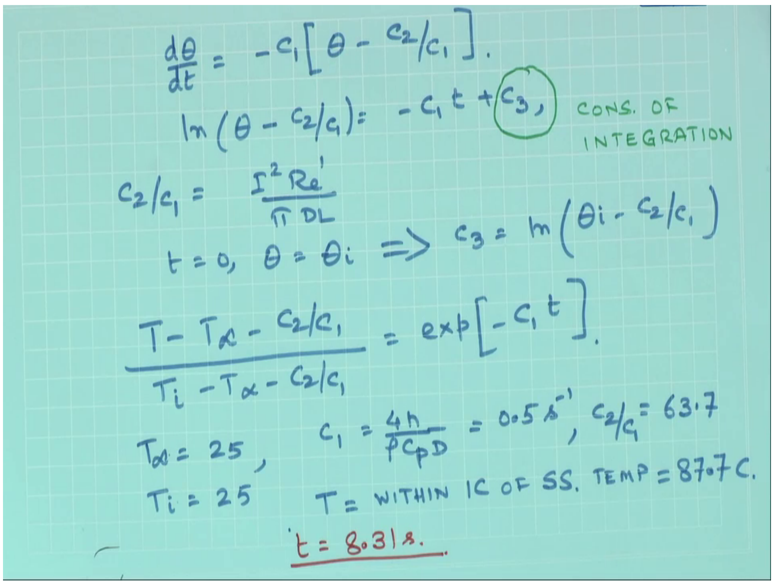

And, I think you should do it on your own and what you get this thing that is d theta dt would be if I C 1 outside theta minus C 2 by C 1 in the 1 integration unit l in theta minus C 2 by C 1 equals minus C 1 time plus C 3.

Where we have we can see that from our previous expression for C 2 and C 1 C 2 by C 1 would simply be I square R e prime by pi D L. And this C 3 is the constant of integration which can be evaluated, if we supply the appropriate boundary conditions. So, the appropriate boundary condition at t equals 0, theta is equal to theta i, which would give rise to C 3 to be l n of theta i minus C 2 by C 1.

So, after everything is said and done what you get is T minus T infinity minus C 2 by C 1, you should check it on your own to ensure write the numbers correct minus C 2 by C 1 would be equal to exponential minus C 1 t. So, if you look once carefully to this equation once again what you would what you can do it should not is that we are getting similar form to that, what we have obtained, when we did not have any energy generation.

But the presence of the energy generation term because of the joule heating of the of the wire joule heating present in the wire creates, additional or introduces additional terms into the governing equation, so which must be taken into account. So, the entire purpose of any modeling is for you to ensure, that you did not miss out on any of the physical processes which are taking place in the system.

So, once you identify the physical processes, then it would it would be easy to substitute the corresponding form corresponding mathematical form of the physical process in your equation, and when you do the algebraic sum of the contribution of all physical processes, which are contributing to the total amount of energy which is stored in the system, then what you have is your governing equation that is all that that is so, just identify the processes put the terms in the conservation equation. And, the sum must be equal to the right hand side which is the time rate of change of energy stored inside the system, it is that simple.

And once you do that then it is going to be a case of integration of an ordinary first order ordinary differential equation, and you require a boundary condition in this case an initial condition at time t equals 0.

So, what you see is if the time at t equal to 0 that is initially you know what is the temperature of the solid to start with, then that provides you with the boundary condition as is the case in this specific problem. So, what remains here is to put the values of T i the values of the thermophysical properties for example, rho C p k rho and C p rho C p and k.

And you use the value of convective heat transfer coefficient h, which is a parameter which depends on many things including the operating conditions and, what you what you are going to get is the unknown temperature. And we need to calculate the unknown time I am sorry, you need to calculate the time required for the wire to reach 1 degree centigrade of the steady state temperature.

So, your T the temperature that you are that you are shooting for is one degree less than the steady state temperature. And if that is known since the steady state temperature is known the only unknown in that equation is the time ok. So, that time you can calculate easily. So, when put the values in here what you would get is that your T infinity is 25 the other values that you get the values of C 1, you can calculate for h by row C p D, which should turn out to be 0.5 second inverse and C 2 by C 1. Once you put the values should be 63.7 and your T i initial temperature.

When there is no current flowing through the wire, the initial temperature must also be equal to T infinity. So, T i is T infinity and T is within 1 centigrade of steady state temperature. So, this must be equal to 87.7 degree centigrade so, when you put all these values in there what you should get as the time required is 8.31 second. So, this gives you some idea about how to look at a problem, which is which is different than what is given what you have done what you have seen in your text book ok. So, you should be you should be prepared to modify the equation, given in your text depending on what you have what you have in the present scenario. So, this is all about lumped capacities that I wanted to I wanted to teach in this class.

What there is still a vast number of situations, in which the lumped capacitance would not be valid. So, how do you treat such a system the fundamental equation the fundamental conduction equation the differential equation will be valid, but how you are going to solve them is the question. So, the problems shifts from that of physical understanding and heat transfer to the problem of solving partial differential equations, where you may have been in the simplest case, with temperature is a function of time and it is also a function of one of the dimensions so, let us say x. So, your governing equation would be k times del to T del x square is equal to alpha is equal to rho C p times D temperature by del time.

So, this kind of equation can be solved in a number of ways the p d one of the ways of solving the pd is the method of combination of variables the method of separation of variables. So, these different types of methods are available in your text, which I would not cover in this in this specific course, but in most of the cases when you solve a two dimensional heat transfer problem, you are going to end up with a series solution. And that series solution can be truncated for some special cases and, those truncated solution of the series solution would can be expressed in graphical form. So, that is one way of analytical finding out what is the how does the temperature vary as a function of x y and time.

That is another method, which is the numerical method and you have probably heard about the finite difference method, if not you are going to read it in your numerical the in your course on numerical computation, where each equation can the differential equation can be approximated as a difference equation. So, you can define definite nodes in the control volume and write the difference equation across that node, and what you would end up with is us is a series of equation series of algebraic equations, which can then be solved to obtain what is the temperature of each of these nodes.

So, the finite difference is a powerful tool of numerically solving differential equations, the differential the heat diffusion equation in order to obtain the temperature at every node, which essentially is a numerical computation problem. So, I will not teach that in this in this course, but it would it should remain, you should you should be you should be familiar with how to convert a governing, how to convert a differential equation to a difference equation. And, then solve the resulting algebraic equation to obtain the temperature at every node.

Now, in your textbook if you are interested, you can see how the difference equation can be written there are difference different ways by which, you can write the difference equations, what you are going to what would be the form of the equation, if one node is surrounded by other nodes in the free space or what if the node is essentially sitting on the surface. Based on that your difference equation would be would be slightly different, but there is a specific methodology which one can adopt which is there in the in the textbook and, if you are interested you can take a look at that by which you can convert the difference equation the differential equation into difference equations.

Now, what would be the spacing between the nodes or so, to say what is going to be the size of the grid and, other numerical considerations in order to in order to increase the accuracy of your method, in order to ensure that your solution remains stable. All these are part of a numerical solution of differential equations course, which either you have done or you are going to do.

So, if you are interested in take a look at the finite difference method of solution of heat diffusion equations. So, the major part which we have covered is the lumped capacitance model, which gives you which gives you a closed form solution, but there is there is a vast literature available for analytical solution, as well as for numerical solution.

In some special cases these numerical solutions, or the reduced form of the analytical solutions can be expressed in graphical form, which is very convenient to use from an engineering stand point. So, in that next class we are going to see what is going to happen, if we can we can truncate the series solution arising from the arising from the analytical solution of the heat diffusion equation and express them graphically. And use them to solve for temperature variation with time and, temperature variation with space for systems in which the Biot number is not less than 0.1 such that the lumped capacitance model is not valid.

So, we will see the graphical solution of transient conduction problems, where lumped capacitance assumption cannot be made in the next class.