Heat Transfer
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Lecture - 11
Lumped Capacitance (Contd.) and Tutorial Problem

So, we have seen how Lumped Capacitance model can be used to analyse convict the transient heat transfer process. And therein we have also seen the importance of Biot number to decide whether the temperature inside the solid can remain space wise isothermal. And if it is so, then there it is a compact expression, which can be obtained starting with the fundamental energy equation e dot in minus e dot out plus e dot generated is equal to e dot store.

So, we get a compact expression, which time and at time constant. And the time constant would contain h which is the heat transfer convective heat transfer coefficient, a which is the surface area the thermal conductivity the capacity and so on.

So, we understood that Biot number has to be less than 1, in order for the lumped capacitance model to be valid. And we have put a number which has been obtained x, which has been observed experimentally to correctly reflect the transient process that is if Biot number is less than 0.1, then the lumped capacitance is valid. So, quantitatively one has to make sure that the Biot number for the specific system under consideration is less than 0.1, before he or she can proceed with the equation derived with lumped capacitance model. Now what happens when Biot number is greater than 0.1?

In that case the conduction resistance can be significant or as compared to the conviction resistance and this simplification is not valid and temperature inside the object is going to be a function of its location not only of the time and therefore, it is more complicated and we would quickly see towards in the next class, how to solve problems in which case the Biot number is greater than 0.1. But they did not that one more thing which we must ascertain before we move on is the Biot number contains h which is heat transfer convective heat transfer coefficient, there are several relations or correlations available by which one would be able to calculate what is the heat transfer coefficient depending on which fluid is being used, what is the temperature range of operation, what is the velocity with which the fluid is moving over the solid or the fluid is made to move over the solid to obtain the value of the heat convective heat transfer coefficient.

The value of k is a thermo physical property, which is fixed the moment you fix the solid which is to be which is to be cooled, which is to be subjected to transient conduction h and k are of known to us now what about n c? So, we call nc as characteristic link, if you look at the definition if you look about how we have arrived at this l, l is nothing but v by A S where v is the volume of the system and a s is the surface area which is which is which is available.

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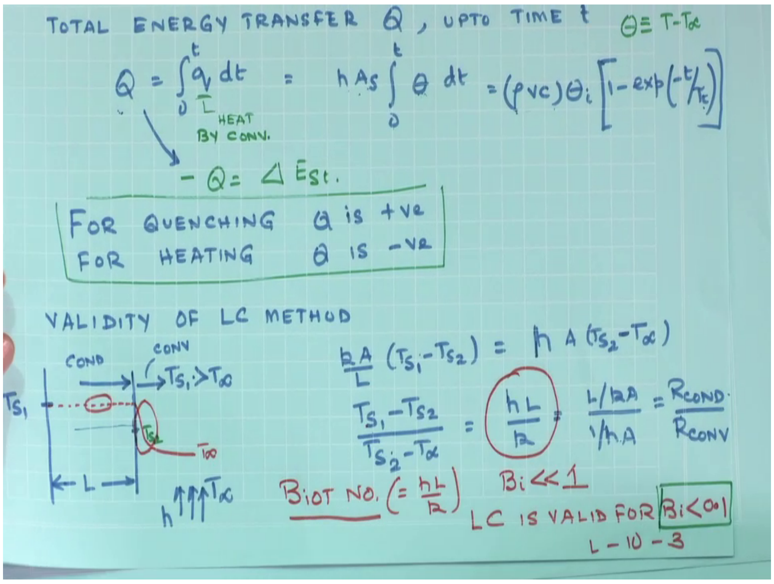

So, the l that appeared if you look at the expression just in the in the previous slide which I think is this one if you look at the expression over here, this l has come from this its v by A S.

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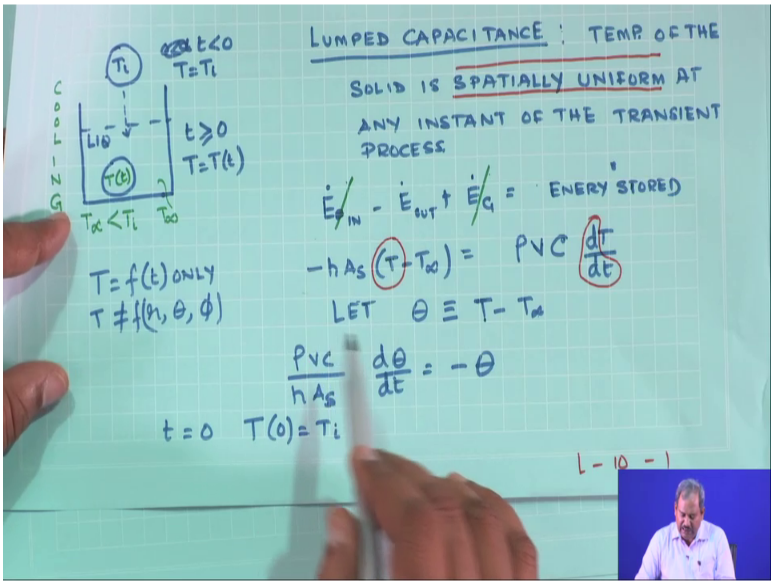

So, v by A S is the l. So, how do we how do we find out the expression how do you find out the expression for Lc?

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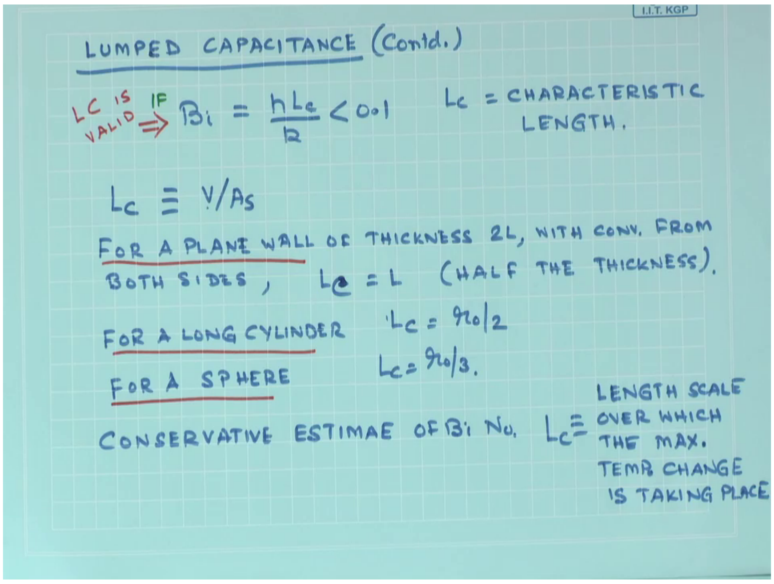

Lc is defined as v by A S in when v is the volume of the solid; A S is the surface area. So, for a plane wall of thickness 2 L with convection from both sides L c is equal to L. So, it is half the thickness. If you think of for a long cylinder if you use this formula L c equal to v by A S you would see that L c is going to be equal to r naught by 2 where r naught is radius of the cylinder and for a sphere again.

If you use this formula, your L c is simply going to make holds to r 0 by 3. So, the expression therefore, gives you what length scale to be used in order to evaluate the Biot number. It is either half the thickness where both the valves are experience in conviction, for a long cylinder it is a r 0 by 2 and for a sphere it is going to be equal to r 0 by 3.

However, for the conservative estimate of Biot number L c is defined as the length scale over which the maximum temperature change is taking place. So, what is L c once again it is the length scale over which the maximum temperature change is taking place. So, when you think of a cylinder, in case a long cylinder which is exposed to a conviction environment, the maximum temperature is going to be at the center line and the minimum temperature is going to be on the surface of the cylinder; that means, over a distance equal to r.

So, even though the theory suggests that L c equals v by A S, it suggests that the L c would be equals r 0 by 2, in order to be remain conservative on the estimate of Biot on the estimation of Biot number, L c is taken to be equal to r naught the distance over which you would expect the maximum change in temperature. Similarly for a sphere; the maximum change in temperature is going to be over the radius of the sphere.

So, if you consider the centre point and if you considered the surface, that is the length scale over which maximum change in temperature is expected. So, instead of taking L c to be equal to be r 0 by 3 for a sphere, the L c is taken to be equal to r. So, what we are doing here is that, we are making the Biot number calculation and its applicability for the lumped capacitance approximation more stringent.

So, if Biot number with L c equals r 0 by 3 is less than 0.1, it may happen that Biot number with L c equals r is not less than 0.1, but to ensure the validity and accuracy of lumped capacitance model, its always the thickness the length scale over which maximum temperature drop is taking place is taken to be the length scale to be used for the calculation of Biot number this is something which you have to keep in mind. So, that is how all these Biot numbers are calculated.

Now we are going to do a little bit of more jugglery with the expression of cube or with expression of temperature and to see what is the role of temperature can we express the role of temperature front moving inside a solid, in a more coherent form. So, we know how the temperature is changing, but let us say a solid is suddenly put in a liquid with a lower temperature.

So, how fast the how fast the solid is losing heat or how fast the temperature front is progressing inside the solid, we would probably be able to see a slightly more insight into the process is if you rearrange the terms a little bit more. So, let us do that. So, what I have then is my starting point, which is the heat which is getting into the system with L c equals.

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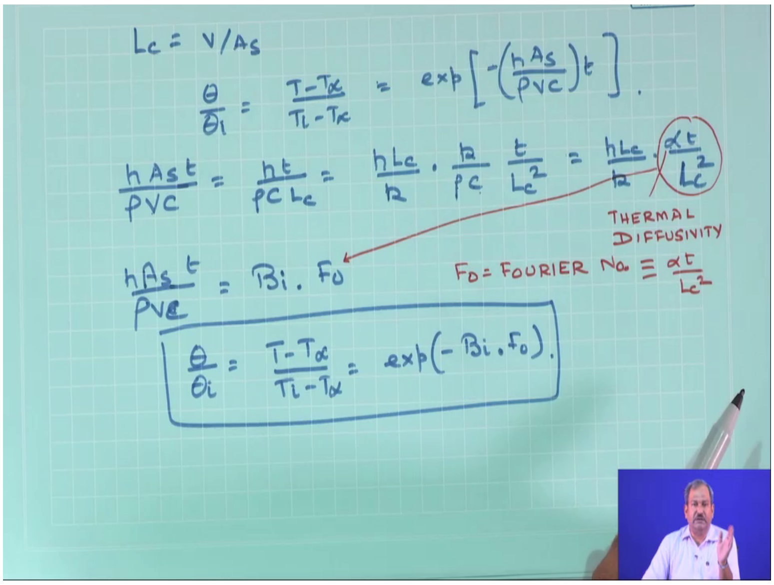

V by A s the expression is theta by theta I, T minus T infinity by T i minus T infinity which is exponential minus h A S by rho C times the with time. This h A S by rho V C is can be expressed ht by rho C L c where L c is v by a the characteristic length. So, this is h L c by k and bringing and L c in the numerator. So, it becomes h L c and I am bringing a k in here as well and putting this keyword here.

So, I have not done anything except that I in order to bring Biot number in here I put an L c in the numerator. So, I have to have another l c. So, therefore, L c square in the denominator I brought in k. So, I brought k over here and that is all there it is. So, this is further simplified as h Lc by k times alpha t by L c square. So, k by rho c is nothing, but the thermal diffusivity alpha. So, k by rho c is substituted by alpha is as I have told you before is the thermal diffusivity in this h therefore, h A S t by rho v c this whole thing is equal to Biot number times F o. This entire thing is dimensionless and this is what is expressed by F o where F o is the short form of Fourier number this Fourier number which is defined as alpha t by L c square hm.

So, the expression theta by theta i which is T minus T infinity by T i minus T infinity is exponential minus of Biot number times Fourier number. This is another way of writing the same equation. So, this Fourier number alpha t by L c square it gives it compares the penetration depth of the temperature front inside the solid. And therefore, together with Biot and Fourier number one would be able to obtain the variation of temperature inside a solid undergoing transient conduction and it somewhat decouples to effects.

One is the Biot number which is the resistance to conduction by resistance to convection and the second is Fourier number, which tells us something about the time dependent part something about how a link skill can be compared with the penetration depth of the temperature front. So, this is the concept is not new as compared to what we have derived in the first class.

But it gives you some idea about how the if the entire temperature process the temperature change in transient process can be expressed in terms of physically explainable phenomena, which are to be which can be expressed in terms of thermophysical properties; the dimension of the system that is L c and conditions imposed by external agency on the outside of the solid, which would dictate what is going to be the value of the thermal the convection thermal convection coefficient.

So, the if convection coefficient h the geometric parameter L c and the thermophysical property for example, k rho c all together would define, what is going to be the temperature of a solid when it is experiencing transient conduction. So, that is all I wanted to cover in transient conduction with lumped capacitance, what I am going to do now is solve the problem. There will be many more problems in the tutorial on transient conduction, which you should solve yourself and if there is any query you ask us the question put us the questions and we will get back to you with clarification whenever needed.

In the I would require half a class more to give an idea of if the Biot number is greater than 1 greater than 0.1 how to handle that situation and there is a way of solving that using graphical that is a graphical method of solving such cases. So, I will introduce the graphical method for analysing transient conduction problems. And I will give you an idea how analytically at least one or 2 simple situations that can be handled. In for rest of the cases one has to resort to numerical computation numerical solution of the governing equation. But now let us see sample problem on the use of lumped capacitance model in the transient conduction. So, let us look at a problem now. The problem that we are looking at is that of a thermocouple.

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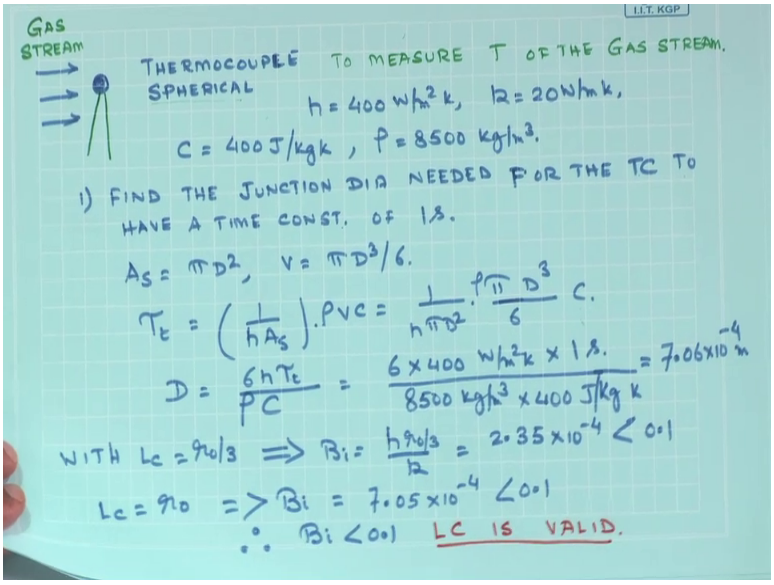

So, we have a thermocouple, which has these are the leads of the thermocouple. So, this thermocouple let us assume that it is spherical in shape. And this thermocouple is kept in a kept in a is gas stream to measure what is the temperature of this. So, a thermocouple junction which is which is approximated as this sphere is used for temperature measurement in a gas stream.

So, the purpose of this is to measure temperature of the gas stream ok. The properties which are given h is for the condition for the flow of the gas, thermophysical properties etcetera the value of h is 400 watt per metre square Kelvin, the thermal conductivity of the material of the thermocouple is 20 watt permetre Kelvin, the thermal capacity is 400 joule per kg Kelvin that of the thermocouple material. The rho the density is 8500 kg per metre cube the first part of the problem is find the thermocouple junction thermocouple junction diameter.

So, essentially the diameter of this needed for the thermo couple thermo couple, I write it as t c to have a time constant of one second ok. So, the problem here is that, I am not sure if the lumped capacitance model can be used in this case see is a junction diameter is not known to me I cannot calculate Biot number, which is h L c by k. So, L c is unknown.

I cannot calculate that I I cannot calculate Biot number, but let us assume that the Biot number is less than point one the lumped capacitance approximation is valid calculate the diameter of the diameter of the thermocouple junction and then re then calculate what is the Biot number to check whether the Biot number that you obtain by this method is lesson 0.1.

So, it this way we will be able to obtain what is the diameter as well as check the validity of lumped capacitance model so, as to ensure that the method that we have adopted is correct. So, if assume that lumped capacitance model is valid, then I know exactly what is the expression for the thermal constant the thermal constant. So, we put the thermal constant expression and equate that to the thermal constant required which is one second. So, equate that to one second to obtain what is the diameter. So, that is what I am going to do now.

So, the surface area is simply going to be pi D square the volume is going to be pi D cube by 6. So, tau t the thermal constant of the process as we have derived before h A S times rho v c equal to 1 by h pi D squared times pi D cube rho pi D cube by 6 times c. So, the diameter of the thermocouple junction would be 6 h the thermal constant by rho density and the specific heat. So, when you put the numbers are going to be 400 watt per metre square Kelvin into one second, because in the problem it is given that for the thermocouple to have a time constant of one second. So, that is why I put in one second here and rho is 8500 joule per kg per Kelvin.

So, this would give you the value to be equals to 7.06 into 10 to the power minus 4 meters. So, that is a diameter once I have the diameter, I must check what is Biot number. So, with L c to be equals r 0 by 3 what we get is, Biot number to be h r 0 by 3 times k which when you put in the values, you would see the number is going to be 2.35 into10 to the power minus 4, which is definitely less than 0.1.

So, if it is less than point one then whatever we have decided for the validity of lumped capacitance is going to be going to be honored going to be correct since the value of Biot number that we calculate comes out to be less than 0.1. Now I have used I could have used the more conservative calculation procedure to see what the Biot number would turn out to be. So; that means, instead of the characteristic length to be at 0 by 3, which follows directly from v by A S, I should probably use the length scale over which you get the maximum temperature. (Refer Time: 25:10) The maximum temperature change takes place between the centre and the periphery; that means, over the radius r which in this case over the radius r 0.

So, let us see what would the value of Biot number be if I use the conservative estimate of the characteristic length L c and see what is the value of Biot number. So, if I use this L c to be equals not r 0 by 3, but simply by r this should give to a Biot number Biot number value equal to 7.05 into 10 to the power minus 4. So, in both cases you can see that the value of Biot number is less than 0.1, and once that happens you know that L c is valid. So, once L c is valid. So, whatever you have done over here must is correct.

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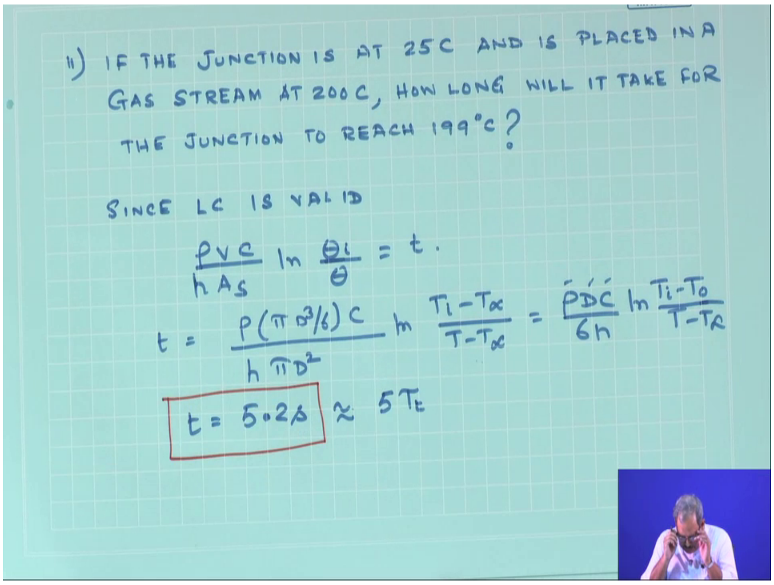

The second part of the problem tells us to find out if the junction is at 25 C and is placed in a gas stream how long will it take.

So, the problem is the junction is at a temperature some temperature room temperature 25 degree centigrade. In any measurement device that time required to sends the new value that whether response of the measurement techniques should be should be very fast. So, as at ones place the thermocouple in the gas stream, it should very quickly find out what is the temperature of the gas stream. So, the response of a measurement device is plays an important role in deciding which thermocouple to choose and therefore, its a parameter which detects the performance so which is the indicator of the performance of the system.

So, the thermocouple initially is at 25 suddenly you place it measure the gas temperature, which is 200 degree centigrade and need to find out how long would it take for the thermocouple to reach very close to the temperature of the gas stream very close to the original temperature of the gas stream because we should look at the expression it is an exponential expression. So, the temperature of the thermocouple is going to be equal to the temperature of the gas stream, it will reach the gas stream temperature asymptotically.

But we would like to have within a plus minus 1 degree accuracy, how long would it take for the thermocouple to record a temperature of 199 degree centigrade, which is within one degree of the actual temperature. And in the previous part we have already established that the Biot number approximation is Biot number the lumped capacitance approximation is valid and therefore, we now can proceed to find out what is the time required. So, that is let us see how this can be done. So, since L c is valid lump capacitance is valid, rho V c by h A S times l n theta i initial temperature difference by theta is equal to t. So, your temperature would be rho.

Once you put the values of this rho DC h T i etcetera, the time you are going to get is 5.2 seconds, which is approximately equal to if you see the expression of the thermal constant. So, its roughly about 5 times the thermal constant of the system. So, from the time you place the place the thermocouple junction into the gas stream, it would require roughly about 5 seconds for a to correctly measure the temperature of the with an accuracy of plus minus 1 degree centigrade the temperature of the gas treatment which is flowing.

So, this gives an estimate of the efficiency or the quality or the or the usefulness of the of the object thermocouple detector. However, the way this problem is solved it did not take into account certain things we must mention that. So, that our understanding is complete. So, we know that we have been an approximation. So, we would be aware of the value that you have obtained is subject to the following assumptions. So, what are the assumptions? For getting about lumped capacitance which you have shown to be valid, the other thing that we have neglected here is the radiative losses.

So, if you think of the thermocouple junction its temperature is rising and its going towards 200 degree centigrade, at that temperature it is this possible that some amount of heat would be lost from the thermocouple bid thermocouple junction apart from the convective losses, but we did not take into account the radiative losses from the thermocouple junction first assumption. The second assumption the thermocouple is junction is not is not floating in the air, there are some lids which are made of solid materials that places the thermocouple in the flowing stream of the gases. This thermocouple the lids of the thermocouple which as you can see from this figure these are the lids of the thermocouple, they are made of solids and they will also conduct some heat away from the thermocouple.

So, not only we have not taken into account the radiative losses from the thermocouple bid we also did not take into account the conductive heat flow through the lids of the thermocouple. So, the value that we have obtained as 5.2 seconds, it subjected to these 2 assumptions. But as the first estimate it is safe to see that the response time of the thermocouple to reach the final temperature is roughly going to be about 5 seconds. So, this shows you one of the clear examples of the utility of transient heat transfer and the way the entire transient process can be simplified by assuming its a lumped capacitance system.

So, we will solve one more problem of lumped capacitance in the next class and we will also see how for the case of Biot number greater than 0.1, how it can be handled and shown in the world one analytical method which gives rise to an interesting result for the case of transient conduction. When a solid is at uniform temperature it is a cuboid solid, and suddenly one of the phases the temperature of one of the phase is changed to a new value and kept constant at that value we just want to see how the temperature front progress is inside the solid.

So, those are the things we would cover in the next class.