Heat Transfer
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Lecture – 10
Transient Conduction

In this class we are going to start a new topic. So, far we have restricted our analysis to steady state situations; that means, when the temperature can be a function of location, so, for example, x y and z it was not a function of time. So, those all those steady state equations, relations, concepts that we have developed so far needs, modification when we try to apply it for situations where the where the steady state has not been reached and therefore, the temperature can in addition to the space coordinates can also be functions of time.

Now the transient process is prevalent in nature wherever you see wherever you change one of the boundary conditions allow across let see a solid object whenever you change the boundary conditions the solid material has to go through a transient state before a new steady state is reached. So, we quench a steel ball or when you expose a rode at a different temperature essentially what you are doing is, you are changing the boundary condition of the ball or that of the rod.

In both cases you change T infinity temperature of the surrounding medium which would impart a change in temperature inside the solid object and it would take some time before it reaches a new steady state value, when it reaches a steady state value from that point onwards the registered would be function of space coordinate so, once again. Incorporation of this time variation of temperature mix the process mix the equation are more complicated to solve. So, the heat diffusion equation that we have used so, far the right hand side of the heat diffusion equation contains the term 1 by alpha, alpha being the thermal diffusivity times del temperature by del time.

So, this right hand side is no longer 0. So, in our governing equation not only the relevant terms of the left hand side or the heat generation term they are to be kept on the left hand side, the right hand side also now contains temperature as a function of time which makes the process which makes the solution more difficult to obtain, more complicated and you require additional initial conditions on temperature. So, you at least need you need one condition on temperature for the one condition of temperature with respect to time to start solving that process, in many transit processes we have to resort to numerical solution of the partial differential equation.

But in some special case you could see that the situation can be resolved by making certain assumptions. So, these assumptions fortunately are valid in many situations and once you make this assumption the entire transient heat transfer problem can be resolved in a compact analytical form of solution is possible. So, we would like to see what would be that simplifying assumption that one can make in order to make the process in order to make the governing equation much more simpler for which analytical solution is possible.

So, therefore, we understand 2 things that transient conduction is something which is which is there everywhere we encounter transient conduction in many everyday processes industrial processes situations and the this arises mostly by the change of one of the boundary conditions and secondly, the variation of temperature with time mix the problem difficult to solve. So, first we are going to look at the simplifying assumptions and the symptoms before we do the simplifying assumption I am going to show conceptually what can happen when a steel ball or a copper ball of high thermal conductivity is quenched in cold water. So, a copper ball of 200 degree centigrade is suddenly put in a container which contains water at room temperature.

So, the temperature of the copper ball will decrease with time as the ball copper ball is going to lose it is energy lose it is heat and thereby reducing it is temperature. So, if I think conceptually what is going to happen inside the copper ball, it is temperature is decreasing with time, but since the thermal conductivity of copper is very high it is probably safe to assume that the variation in temperature inside the copper ball is not going to be significant, that is the assumption based on which all our subsequent analysis would be based would depend.

So, a copper ball of this big size is put in water in this was initially at a temperature of 200 and so, when it comes in contact with water at 30 degree the sides going to cool very fast. So, that side temperature the temperature that the periphery at the outer surface of the copper will decrease rapidly with time, but what is going to happen to the temperature let us at the centre of the copper ball. Can we assume that the entire copper ball during it is transient process remains specially isothermal that was there is no change in it is temperature with respect to x y z or since it is a spherical system r theta and phi it is a function only of temperature.

If I can assume that then the entire governing equation can be reduced to a simple ordinary differential equation and this kind of an assumption where the temperature of the solid is assumed to be in variant with position is known as the lumped capacitance model of transient conduction and that is what we are going to look at first, but let me first draw the diagram of what happens when you quench a steel ball in a liquid at a different temperature.

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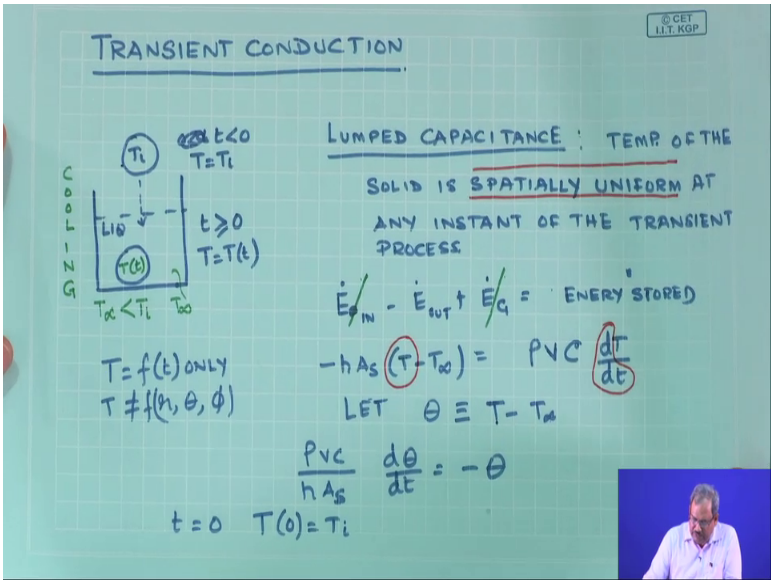

So, let see that we have this as the container which contains some liquid and you have an object whose temperature initially is T i the initial temperature and this is the situation at when t is less than 0. So, your temperature everywhere inside the solid object is going to be equal to T i and then you drop it in the liquid. So, it comes in here; however, in this case the temperature of the solid object is going to be a function of time ok. So, for any t greater than or equal to 0 the temperature is going to be a function of time.

In the temperature of this liquid is let say it is T infinity and temperature T infinity is less than T i. So, this is essentially a cooling of the solid object T i is greater than T infinity you put it in water and you see that the temperature changes with time. So, what is the lumped capacitance model, the lumped capacitance model or lumped capacitance assumption it tells you that the of the solid is spatially uniform at any instant of the transient process.

So, this one is important that the temperature is uniform, now what do you think when this is possible when you are going to have an object a solid object whose temperature is going to remain constant with respect to xyz or r theta phi, but it is going to decrease since it is in contact with a fluid whose temperature T infinity is less than the temperature of the object.

Now, common sense tells us that this is only possible theoretically only possible when the solid object has very high thermal conductivity or to be 100 percent accurate the thermal conductivity of the solid must be infinite. So, if the thermal conductivity of the solid is infinite then there is no resistance of conduction of heat inside the solid because we understand the let us say for a sorry for a plane valve the resistance the thermal resistance to conduction is L by k so, this k is the thermal conductivity.

So, when the thermal conductivity is very large; then L by k can be approximated to be equal to 0 and therefore, the resistance to conduction inside the solid is 0. So, if you do not have any resistance to conduction or very little resistance to conduction inside the solid then at any instant of time the temperature of the solid object would remain uniform, at the next instant the temperature is going to be less as compared to the previous instant; however, it will still be uniform temperature throughout the solid object.

So, this is what lumped capacitance assumption or lumped capacitance model for transient conduction is all about that you assume the temperature of the object will not change with space, but it will change with time. There are certain assumption certain advantages of major advantage of this lumped capacitance assumption is that in your heat diffusion equation it is heat in minus heat out plus or minus heat generated is equal to the energy rate of energy storage. So, E dot IN minus E dot OUT plus E dot generated is equal to E dot stored.

So, all these dots refer to the time rate of change of energy coming into the system which is 0 for the case that we are dealing with where we have just dropped a solid object in a liquid. So, there is no energy and the solid object temperature is higher than that of the liquid should no heat enters the solid. So, E dot in is 0 you will have E dot out definitely because the solid object is losing heat some energy is going out of the solid object and let us also assume that no energy is generated inside the solid object. So, the third term on the left hand side of the equation E dot IN minus E dot OUT plus E dot generated the only remaining term would be minus E dot OUT, the time rate of energy lost by the the solid object.

And on the right hand side it is rate of energy stored so, as a result of the solid object losing some energy due to maybe conviction in this case the total energy content of the solid will keep on decreasing. So, the non 0 term of this equation would be minus E dot OUT is equal to the time rate of energy stored in the system. So, that is the advantage that is the form of the equation, now we are going to write this and see how the lumped capacitance assumption makes the entire process simpler so, let us proceed with this.

So, we understand that as I mentioned E dot IN minus E dot OUT plus Generated is equal to energy stored again a dot over here. So, the guess this is 0, this is 0. So, E dot out would simply be minus h A S T minus T infinity energy stored is this is time rate of energy time rate of change of energy stored inside the system is rho V C this is a specific heat dT by d time mark what I have done over here and simply see T, I have used a normal ordinary differential.

So, what I am seeing is that T is a function of time only and T is not a function of for this case r theta and phi. So, if this is the case then if I write this equation at any instant this equation would be true, to this temperature is going to be a function of time, but it is not going to be function of r theta and phi. So, that is why I get a simple equation to deal with for a much more complicated process of transient conduction.

So, the beauty of lumped capacitance is essentially giving us a compact ordinary differential equation for a very complex process and we would later on see what is the quail quantitative criteria that must be satisfied before we can invoke lumped capacitance assumption, but before that let us see what this leads to let a theta is defined as T minus T infinity where the theta is essentially a temperature difference defined as the temperature of the solid object which is a function of time minus T infinity.

So, this makes it rho V C by h A S, this is a surface area times d theta by dt is equal to minus theta and upon this solid state straight forward equation which upon integration in the using the initial condition that at t is equal to 0, T at time 0 is equal to T i as it was the initial temperature of the solid. So, this is the initial condition and if I integrate this equation with the help of this initial condition what I am going to get is that.

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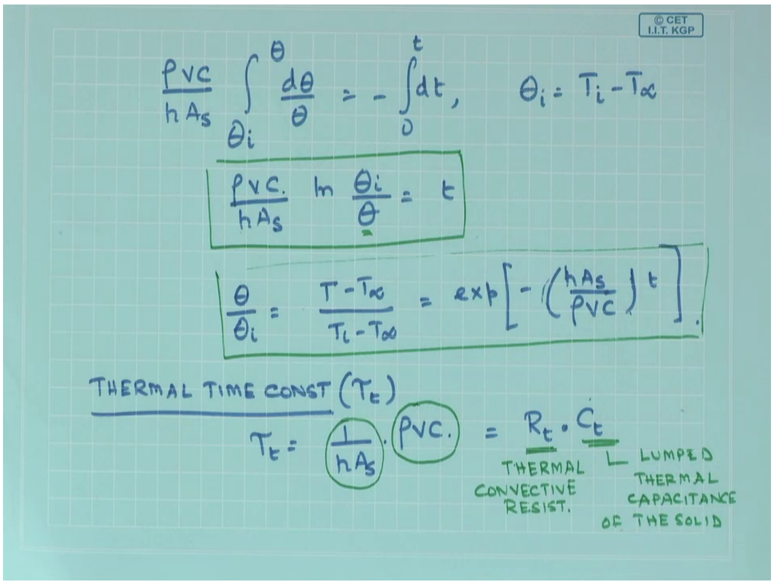

Theta i 2 theta, so, theta i is simply the initial temperature minus the temperature of the fluid surrounding it. So, this can clear this can be integrated. So, this would give me the variation of theta the temperature as a function of time also the thermo physical properties of interest would be rho the density, C the heat capacity, the geometric parameters are going to be volume of the solid, surface area of the solid and the operational parameter operational parameter would be the heat convective heat transfer coefficient which among other things depend on the conditions imposed by an external agency on the solid.

So, let see if you have a starter present in the fluid where you are going to put the solid then the speed of the starter would dictate what is the value of this h is going to be. So, externally imposed conditions in many cases dictate what would be the value of the heat and convective heat transfer coefficient. So, in that expression there are some thermo physical properties, some geometrical properties and some operational parameters together with those parameters or the clubbing of those parameters in the would give us the variation in temperature of a solid object as a function of time we could see that it is the temperature is going to change in a logarithmic function.

So, it is a very compact relation which you have obtained which is significant industrial uses because you can quickly estimate what is going to be the temperature of an object when it is quenched with something else. So, how is the temperature of a solid object is going to decrease how much time would it required for the solid to come to a some desirable temperature such that it is ready for further processing.

So, in many industry the formation of a specific material goes through discrete steps in some of the steps may required high temperature, some of the steps required curing the solid objects with a given temperature difference over a certain amount of time. So, and then repeating the process with different variables. So, with this variation in the property with this variation in the time may give rise to different difference in properties.

So, the desirable properties of a solid can be fine tuned by exposing it to a temperature environment and letting it heat or cool in a specific fashion. So, it is very important to know how long would it take for the solid to attain certain temperature and an easy way to estimate that is to use the lumped capacitance model which would exactly tell you that after how much time you were going to reach this temperature or in order to obtain your if you leave for x amount of time what is going to be it is temperature.

So, these are important calculations which are relevant in many industrial applications. So, let us prove it a little further and see what additional information we can obtain from this. So, if I expand it so, theta by theta i which is nothing, but T minus infinity by T i minus T infinity is going to be exponential. So, this is what the final form of the temperature expression for transient conduction assuming lumped capacitance should look like.

Now, will you see h A S you can recall that the heat transport is give by convection is given by h A S times delta t. So, this h A S is nothing, but an inverse of a resistance. So, if I try to express this wherever we have exponential some constant times t. So, inverse of this entire group is nothing, but a thermal constant of the system. So, if I write it in that form thermal time constant let us call it as tau T this tau T is equal to 1 by h A S times rho V C or the just the inverse of this and this is the resistance to come resistance to the convection pros convective heat transfer and this rho V C is the thermal capacitance of the system.

So, they are expressed as R t times C t where R t is the thermal convective resistance in C t is the lumped thermal capacitance of the solid. So, this R t is nothing, but the thermal convective resistance of the solid and C t is the lumped thermal capacitance of the solid. Now this can once I obtain the temperature the next thing that is of importance to me is to find out how much heat is lost by the solid is a given amount of time.

So, the expression that I have obtained gives me the temperature variation with time what are more interested all I am also interested to know is how much of heat is lost or gained by the solid in this transient process so, which I am going to find out which I am going to find out next.

So, what I do then is in order to obtain the amount of heat lost or the total energy total energy transfer.

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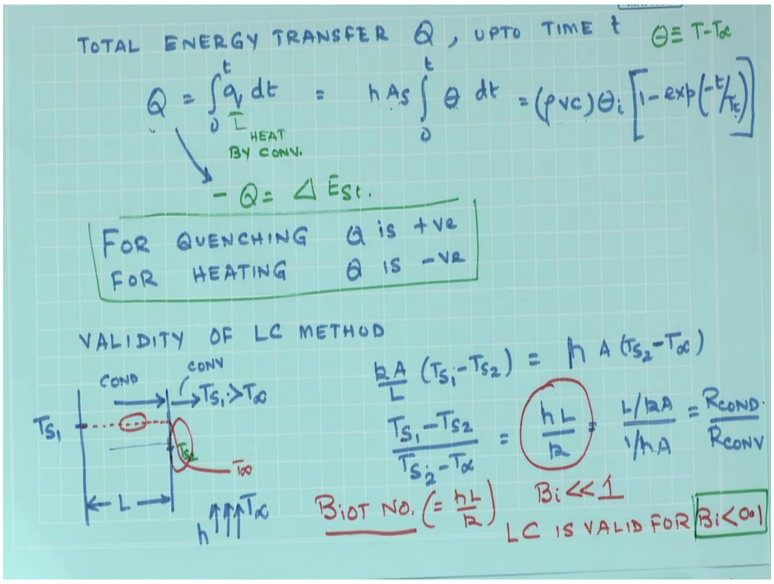

If I call that as Q up to time sometime up to sometime t this Q the total energy is 0 to t the amount of heat lost the amount of heat loss or gained by convection multiplied by dt. So, what is this Q the amount of energy lost or gained by the physical process of convection so, this is heat by convection. So, this is the heat that is lost or gained by the solid object due to convection. So, this would be nothing, but h A S times delta t delta t is essentially theta times dt.

So, if you look at the previous slide this theta is nothing, but T i minus T infinity and we have an expression for complete expression for theta here which is theta i multiplied by exponential of this form which when I put in this equation it simply becomes rho V C times theta i 1 minus exponential minus t by tau T. I would bring in the previous slide one more time so, that there is no confusion in here, heat lost by convection must be equal to h A delta t this theta is previously this theta was defined theta was defined as T minus T infinity.

So, this h A T minus T infinity gives you the amount of heat lost at any given instant when you integrated overtime, this gives you the total amount of heat lost during the period from 0 to t. So, this theta is to be substituted using the expression of it as obtained from the lumped capacitance model. So, what is that, this theta is this theta is theta I multiplied by exponential the entire thing the integration of that. So, once you put the expression of theta from this in here and integrate what you are going to get is this form of the expression. So, the total heat lost over a period t can be related to the thermo physical properties, the thermal time constant of the process, the thermal capacitance, the initial temperature difference and finally, with time.

So, this Q must this Q the total energy transfer must be equal to delta E st such that minus Q is equal to the change in the energy stored in the system. So, this Q can be substituted in here such that you can obtain the total energy stored in the system. Now for quenching; that means, when you are cooling the solid object this q is positive in the for heating; that means, when you are dropping the solid object in liquid whose temperature is more than that of the solid then Q is negative so, these 2 which you have to keep that in mind.

So, for quenching; obviously, the total energy content of the solid will decrease and when you heating the solid; that means, when you are dropping the solid in a liquid whose temperature is more than that of the that of the solid then the solid is going to gain energy.

So, this way the convention to follow then is that for quenching it is going to be positive the amount of heat transfer in for heating it is going to be negative since it moves in the other direction. So, one of the prerequisites of the lumped capacitance model or the use of the lumped capacitance model is that the temperature of the object is going to be specially uniform.

So, let us try to see physically what would have to happen in order for the solid to behave in that specific way or another words let us try to put a quantitative criteria criterion which would enable us to use the lumped capacitance model in many practical situations and in order to do that I am going to look at a solid object let us a solid wall which is initially at a constant temperature and suddenly the solid wall is exposed to a environment where the temperature is less than that.

So, these solid objects let us say it is 100 degrees and suddenly it brought into this room where the temperature is 25. So, from both the walls the solid is going to lose heat and it is temperature is going to decrease. Now how would that profile look like in realistically when you consider not only the thermal conductivity of the material, but also the geometric parameter.

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And the geometric parameter of interest is obviously you can see is the thickness of the solid. So, instead of this being the solid if we are considering this to be the solid only the cover of the mobile is the solid then it is pretty thin as compared to the overall one.

So, the chances of the solid remaining at a constant temperature at any given instant of time would be more for this cover as compared to the whole item itself. So, as we reduce the thickness of the solid it is likely that the constancy of temperature with position will most probably be valid. So, as this thing becomes thicker and thicker the centre temperature is going to be substantially different from the temperature of the solid. So, it is not only the thermal conductivity that plays a critical role in determining whether lumped capacitance is valid it is also the geometrical geometric dimensions of the object that play a role as well.

So, let us see pictorially how would it look like and come up with a number. So, what we are looking at is validity of lumped capacitance LC method yeah let us say we have a wall whose this is whose temperature at this end is T S 1 and here it is T infinity and T S 1 is definitely greater than T infinity and you also have a flow of the fluid which maintains a convective heat transfer coefficient of h. So, and this is let us say T S 2. So, the amount of heat which gets transferred through this is going to go out of this if I consider this one.

So, this is by conduction and this heat will goes out is by convection. So, at steady state the amount of heat which comes to this point must be equal to the amount of heat which goes out of it. So, the amount of heat which comes by conduction to this surface is going to be T S 1 minus T S 2 this is simply using Fourier's law where it is ka dtdx. So, k A T S 1 minus T this is T S 1 minus T S 2 by L, which must be equal to on the side of the interface as h A T S 2 minus T infinity so, at steady state this expression is valid.

So, minus k A dtdx which would essentially mean k A T S 1 minus T S 2 by L, where L is the thickness of the solid must be equal to the convective heat loss from this side of the interface. So, T S 1 minus T S 2 by T S 2 minus T infinity is equal to h L by k or this can be also written as L by k A by 1 by h A. So, what is L by k A from our previous study we know L by k A is simply the R conduction, the conduction resistance and one what is 1 by h A, this is the R convection.

So, whether or not T S 1 is going to be equal to or close to T S 2 would depend on the ratio of these 2. So, T S 1 would be equal to T S 2 if this part the conductive resistance is very small is 0 or the convective resistance is very large. So, whether or not T S 1 would be equal to T S 2 which is the fundamental assumption of lumped capacitance it would depend on the resistance of these 2 and the resistance of these 2 is simply h L by k and if you look at h L by k it is a dimensionless number and it is known as Biot number it is known as Biot number which is equal to h L by k.

The biot number therefore denotes that whether or not which one is important, conductive resistance or convective resistance and from this expression we see that the convective resistance has to be large as compared to conductive resistance in order for T S 1 to become equal or close to T S 2. So, if convection is large as compared to R conduction then the value of biot number must be quite small as compared to one for lumped capacitance to be valid.

So, the simple expressions simple thought experiment tells you that what is going to be the requirement of the conduction and convective resistances in everything is expressed in it compact number compact dimensionless number h L by k, what is h it is a convective resistance, L the thickness over which the temperature changes from T S 1 to T S 2. So, it is the geometric dimension of the solid across which the maximum temperature drop is taking place and what is k, k is the thermal conductivity of it.

So, this biot number has to be less than significantly smaller than 1 to ensure that T S 1 is equal to or close to T S 2 when can that happen either h has to be small or L has to be small, if you look at the definition of biot number here in order for biot number to be less than 1 either h has to be small, L has to be small or k has to be large. So, under this condition the temperature profile inside the solid will moved likely the very small or no variation in the temperature and then it is going to go to T infinity. So, this is what it is going to look like. So, most of the resistance is going to be the convective resistance and since the temperature here does not vary the conduction resistance is almost going to be very very small.

So, the biot the prerequisite for using the lumped capacitance model is to ensure is to calculate what is the value of biot number first and see if it is less than 1 ok, the common criteria for assuming lumped capacitance is biot number has to be less than 0.1 for using the lumped capacitance model. So, one can use biot number one LC is valid for biot number less than 0.1 this is the criteria generally used to used to ascertain whether or not lumped capacitance is valid in a system and if this is the case then it looks something the profile would look something like this and we can use the first equation that I have written over here that E dot OUT is h A S T minus infinity is the rate of energy stored which is rho V C ddT of temperature.

So, now since temperature is not a function of r theta and phi at any instant this equation is true which can then be integrated to obtain the total amount of heat which is lost or gained by the subject and not only that what is the temperature of the object as a function of all these parameters and time. So, it is the thermal time constant and which is this is inverse of the thermal time constant. So, a very convenient method for describing natural convection for describing the transient conduction is assuming it is to be space wise isothermal, but varying with time.

The moment I make space wise isothermal assumption the equation is an ordinary differential equation, which can be integrated with the help of an initial condition and the space wise isothermal condition can be used quantitatively when a newly defined dimensionless group known as Biot number is found to be less than 0.1 that check must be done to ensure the validity of lumped capacitance ones that is done it is a simple equation to solve which would give you important information about how an object would cool which would which time which has significant industrial applications in various sectors. So, we are going to go slightly different to the next class and solve a tutorial problem on transient heat transfer.