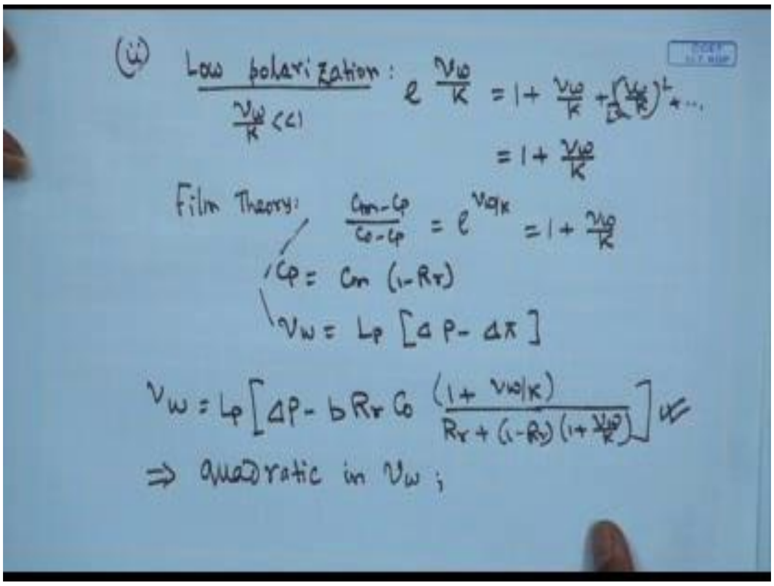
Introduction to Process Modeling in Membrane Separation Process

Prof. Sirshendu De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 09
Osmotic Pressure Controlling Filtration (Contd.)

Welcome to the session. We will be now we are looking into the some simplified versions of osmotic pressure control filtration. We have looked into the first simplified case there is the no concentration polarization and we have looked into the system how the system can behave and prediction can be obtained.

(Refer Slide Time: 00:40)



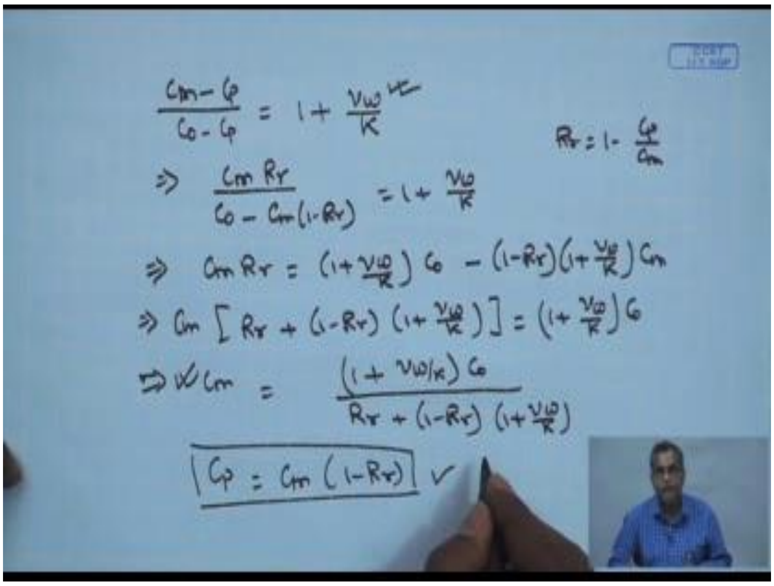
The second simplified case will be talking about is low polarization and low polarization can be realized by exponential Vw by K is going to be realized when Vw by K is much lesser than 1 that means, again in this case turbulence is quite high. So, Reynolds number will be quite high very high and therefore, the mass transfer co efficient will be high compared to the permeate flux. So, Vw by K will be much less than 1 and that will allow us to have an exponential expansion of e to the power Vw by K as 1 plus Vw by K plus Vw by kernel square of that 1 over factorial 2 here plus other higher order terms. We neglect the since Vw by K itself is much less than 1 Vw by k square on the higher terms will be neglected and this can be approximated as 1 plus Vw by K.

Now what we will be doing next? We will be writing the film theory the film theory. If you remembered this is Cm minus Cp divided by C not minus Cp e to the power Vw by K and this will be replaced by 1 plus Vw by K.

So, these a simplified fraction of the film theory and then will be writing Cp as Cm into 1 minus real retention and then will be writing the osmotic pressure model as Vw is equal to Lp deLp minus deLpi and we have seen earlier how deLpi can be replaced in terms of real retention and membrane surface concentration Cm. Now combining these 3 equation will be getting a simplified version as Vw is equal to Lp deLp minus b Rr C not 1 plus Vw by K divided by Rr plus 1 minus Rr into 1 plus Vw by K. Now in this case what will be ultimately getting? So, ultimately you will be getting a quadratic in Vw. So, all the other quantities are known to us Lp is membrane permeability deLp is the trans membrane pressure drop b is the osmotic co efficient known to us Rr is real retention known to us C not is the feed concentration K is the mass transfer co efficient that will be calculate and we have seen how to calculate the mass transfer co efficient.

So, we can calculate the Vw and from the Vw one can estimate. So, one can directly get the expression of Cm because Cm minus Cp divided by C not minus Cp can be expressed. So, Vw can be calculated independently. Let us see now how Cm will be calculated; Cp will be calculated.

(Refer Slide Time: 04:10)



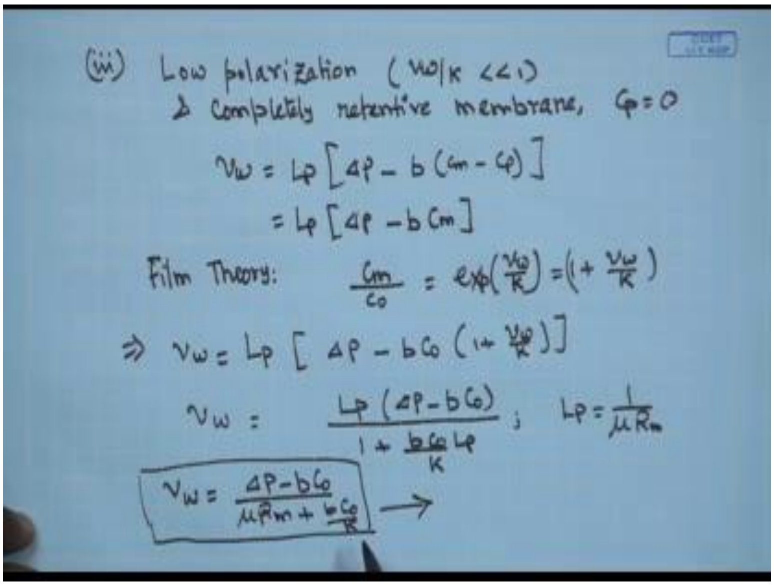
So, if you look into the osmotic expression film theory Cm minus Cp divided by C not minus Cp is equal to one plus Vw by K. We have already got an explicit expression in quadratic form of Vw. So, therefore, these Cm minus Cp can be replaced in terms of Cm. So, this will be nothing, but Cm times Rr because 1 of the definition of real retention 1 minus Cp by Cm. So, Cm minus Cp is nothing, but Cm time Rr and Cp will be the denominator Cp can be replaced Cm into 1 minus Rr is equal to 1 plus Vw by K.

So, just rearrange Cm. So, this become Cm Rr is equal to 1 plus Vw by K times C not minus 1 minus Rr 1 plus Vw by K times Cm. So, Cm will be Rr plus 1 minus Rr 1 plus Vw by K and here it will be 1 plus Vw by K times C not or one can estimate the value of Cm as 1 plus Vw by K time C not divided by the denominator Rr r plus 1 minus Rr into 1 plus Vw by K. So, Vw you have estimated already as a quadratic in the only few minutes back and K is known to as C not is known to us Rr is known to us and one can estimate the value of Cm. Once the Cm is estimated then permeate concentration can be estimated as well through their definition of real retention.

So, we can have an independent system prediction in terms of permeate concentration and permeate flux for a low polarization case. Where in the polarization is not low whether 1 it is significant we have already seen that we have to solve 2 equations and 2 unknown 3 equations in 3 unknown system by using Newton Raphson after combining all the equations we will be getting will be landing of with a single non-linear algebraic equation which will be solved iteratively by using the Newton Raphson method.

So, we will be doing another simplification, that the things will be physically quite significant and apparent to us.

(Refer Slide Time: 06:44)

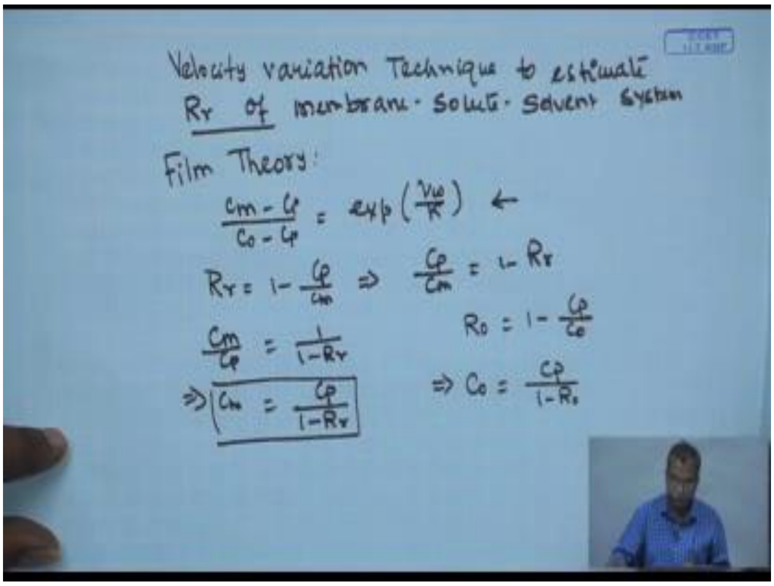


The third simplified case will be the low polarization and completely retentive membrane. Low polarization that is Vw by K is much much less than 1 and completely retentive membrane; that means, Cp is equal to 0. So, in this case permeate flux will be Lp deLp minus deLpi, deLpi will be b Cm minus Cp and Cp will be equal to 0. So, you will be having Lp deLp minus b Cm and from the film theory what you will be getting? For film theory you will be getting Cm by C not because all Cps are 0 it is a completely retentive membrane exponential Vw by K. So, it will be 1 plus Vw by K.

So, you will be getting permeate flux is equal to Lp del P minus b C not 1 plus Vw by K now if you bring Vw to the other side and write everything in terms of that. So, you will be getting Lp del P minus b C not divided by 1 plus b C not into Lp divided by mass transfer coefficient K and now if I replace Lp the membrane permeability in terms of membrane hydraulic resistance Rm then I will be getting a concrete expression of permeate flux as delta P minus b C not is equal to mu Rm plus b C not divided by K.

So, these expressions clearly show that, this is the effective driving force. This is the applied trans membrane pressure drop minus osmotic pressure divided by there will be 2 resistances in series one is membrane hydraulic resistance other is the resistance due to the mass transfer in the mass transfer boundary layer. So, two resistance in series that will be appearing in the denominator and in the numerator you will be having the effective trans membrane pressure drop which will be nothing, but the actual trans membrane pressure drop minus the osmotic pressure that is developed in to the system. So, this clearly shows that mass transfer resistance and membrane resistance they will be acting in series in an actual system. So, before going into the other variance of solution diffusion model I would like to now in at this point of time I would like to include or introduce velocity variation technique or one more technique.

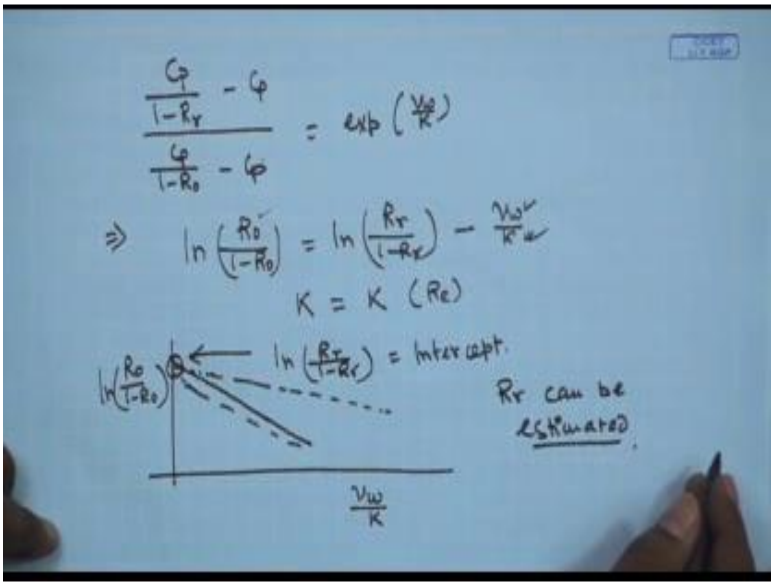
(Refer Slide Time: 10:11)



If you remember in an earlier class we have said that there are two ways or two experimental methods to estimate the real retention of the system. Now will be really looking into how to we have seen 1 method already how to conduct a separate set of experiments. So, that the real retention of the system can be estimated by under low polarization condition, that is low operating pressure low feed concentration high turbulence. Now in this will be looking into 1 more method to estimate the velocity variation technique which will be utilizing the film theory model to estimate the real retention of the membrane; technique to estimate Rr of membrane solute solvent system.

So, we take request to the film theory model and it is equation. So, Cm minus Cp divided by C not minus Cp is equal to exponential Vw by K. Next what we will be doing will be writing the definition of real retention and observe retention. So, real retention is 1 minus Cp by Cm. So, Cm can be written as. Cp by Cm will be 1 minus Rr Cm by Cp will be 1 over 1 minus Rr. So, therefore, Cm is nothing, but Cp divided by 1 minus Rr. Similarly from the definition of observe retention R not is 1 minus Cp by C not and we can estimate the value of C not in terms of Cp. So, C not will be nothing, but Cp divided by 1 minus R not. So, now, we are going to replace the expression of the Cm in favor of Cp and Rr and C not in favors of Cp and R not in this equation and see what we get.

(Refer Slide Time: 12:55)



So, if you really do that you will be getting Cp 1 minus Cp divided 1 minus Rr minus Cp is equal to Cp divided by 1 minus R not minus Cp exponential Vw by K. So, this can be. So, Cp will be canceling out from both the numerator and denominator and you will be getting an expression of observe retention and real retention and permeate flux and mass transfer coefficient. If you simplify this equation I am just writing the final form ln R not divided by 1 minus R not is equal to ln Rr by 1 minus Rr minus Vw by k.

Now in this expression if we have the real retention as constant and let us see how if you change your operating conditions operating conditions means here we cannot change the operating conditions in terms of pressure, because the mass transfer coefficient is appearing here and as you have already seen that mass transfer coefficient is a strong function of Reynolds number and Reynolds number will be basically will be a strong function of the cross flow velocity.

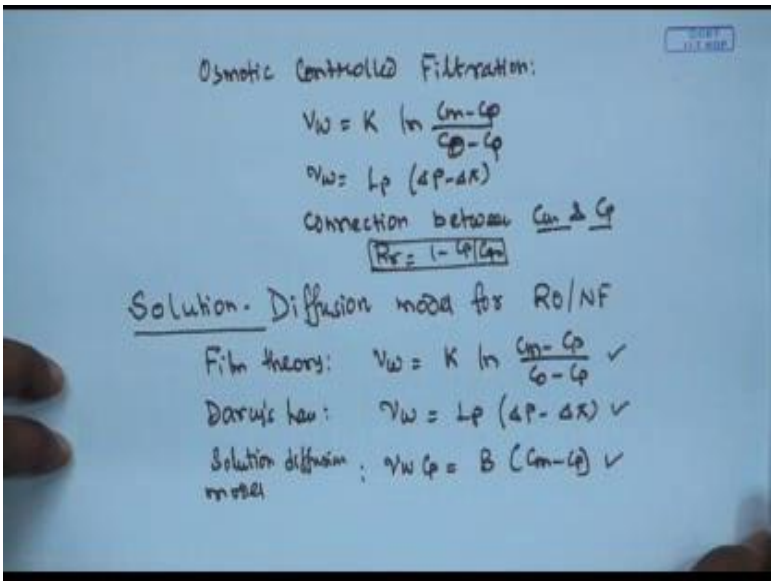
So, this method is known as velocity variation technique. What will be doing? Will be applied different cross flow velocity in the system that will alter the mass transfer coefficient; once the mass transfer coefficient will be altered the permeate flux will be altered and once the permeate flux will be altered as well as it will be affecting the permeate quality. So, observe retention because permeate concentration also be altered. So, observe retention will be altered. So, that is therefore, we change the cross flow velocity and get different type different values of permeate mass transfer coefficient and permeate flux and then we plot ln R not by 1 minus R not versus Vw by K.

So, now, if a plot ln R not divided by 1 minus R not versus Vw by K you will be getting a curve something like this. So, similarly actually these will be hitting the y axis the curve will be something like this and similarly it will be for the other values of mass transfer coefficient will be getting this. So, ultimately all these points will be going to the intercept R not by 1 minus R not. So, this is for a particular K this is that is for another particular K.

So, you will be getting from the intercept. What is the intercept? The intercept is ln Rr by 1 minus Rr. So, from the intercept one can estimate the value of real retention. So, Rr can be estimated. So, what is the drawback of this system? The major drawback of this system is that, the experiments should be very very accurate. If there is slight in accuracy in the experiments then in the log scale these will be even magnified and therefore, it will giving a very wrong estimation. So, there will be number of more number of experiment one has to conduct not only that not only the more number of experiments the experiments has to be very very accurate then only 1 can get an estimation of real retention appropriate estimation of real retention by this velocity variation method, OK.

Next, we will be looking into as you have discussed that the third equation let us look into once again the modeling equation for the osmotic pressure control filtration one dimensional model.

(Refer Slide Time: 16:56)

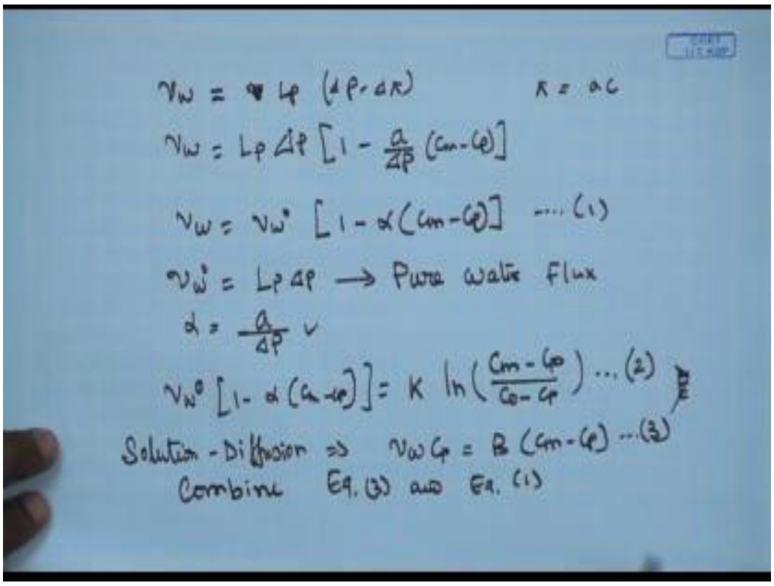


So, we have at the film theory Vw is equal to K ln Cm minus C not Cp minus C not we have the Darcy's law Vw is equal to Lp del P minus del pi and we have 2 equations 2 unknown Cm Cp Vw these are the 3 unknown and 2 equations though other equation is the connection between the Cm and Cp. Cm and Cp and we have already seen that 1 way of connecting Cm and Cp is the real retention that is nothing, but a partition co efficient between across the membrane of the solute partition co efficient across the membrane. Now we have already seen the all the formulations and the method for solving these third equation to get a system prediction of the system performance. Now we are what will be doing? Will be replacing if the expression of real retention that will be valid more valid for the ultra filtration system and various cut off ultra filtration system by the solution diffusion model for the for the reverse osmosis nano filtration system.

So, real retention is a concept which will be more valid for the ultra filtration system, but for the reverse osmosis and nano filtration will not beyond be able to use the expression of real retention as partition co efficient 1 minus Cp by Cm. You have to use the other variance of the connection between the Cp and Cm that is the solution diffusion model and it is various variations or modifications. So, first will be talking about the solution diffusion model for reverse osmosis and nano filtration system; for RO-NF system, now, let us look in to the 1 of the various governing equations. So, 1 equation will be the film theory is Vw is equal to k l n Cm minus Cp. So, is not Cp C not minus Cp Cm minus Cp divided by C not minus Cp the other 1 is the Darcy's law which is nothing, but the solvent flux through the membrane. Vw is equal to Lp del P minus del pi and the third 1 is the solute flux through the membrane, which is the solution diffusion model.

So, these will be giving you the solvent flux through the mass transfer boundary layer, solvent flux to the porous membrane and the solute flux to the porous membrane Vw Cp is equal to b Cm minus Cp. So, we will be having 3 equations and 3 unknowns Vw Cm and Cp and can get the system prediction. So, I will just give a small algebraic manipulation in order to solve this set of equations.

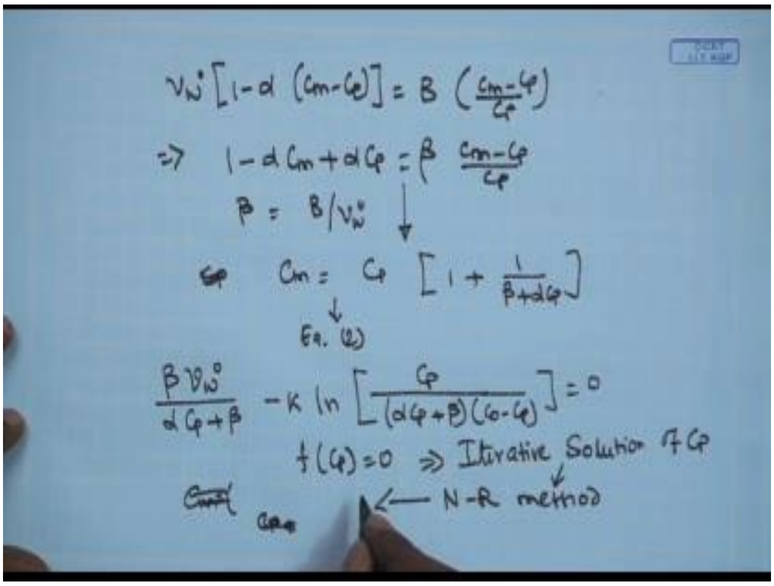
(Refer Slide Time: 20:34)



So, Vw can be the solvent flux to the membrane can be written as Vw is equal to Lp del P minus del pi. So, this will be, if u take Lp out del P out. So, it will be giving 1 minus del pi and del pi is let us say pi is equal to ac. So, del pi will be basically a by delta P Cm minus Cp. So, I write Vw is equal to Vw not 1 minus alpha Cm minus Cp. So, this is my equation number 1 and what is Vw not? Vw not is basically Lp times del P it is a pure water flux. Pure water flux and what is the parameter alpha? So, these will be Vw not will be known us because Lp is known to us and delta P is the trans membrane pressure drop. So, what is alpha? Alpha is the parameter a by delta P.

So, it is non dimensional. So, there will be equation number 1 then will be equating these with the film theory equation. So, Vw not 1 minus alpha Cm minus Cp will be equal to K ln Cm minus Cp divided by C not minus Cp. So, these will be the combination of the film theory and the osmotic pressure model Darcy's law. So, I write it a; this is a equation number 2, then we will be having the solution diffusion model for the solute flux through membrane. That will be Vw Cp is equal to b Cm minus Cp. So, now, we can combine these two equations and finally, will be getting into these. So, we will be combining the equation number 3 solution diffusion model and Darcy's law that is equation 1 you can combine these 2 equations and see what we will get? If you combine this two equation you will be getting Vw not 1 minus alpha Cm minus Cp is equal to b Cm minus Cp divided by Cp.

(Refer Slide Time: 23:17)



So, these equations can be written in this form 1 minus alpha times Cm plus alpha times Cp is equal to beta Cm minus Cp divided by Cp. Where alpha we have already defined and beta is b by Vw not. So, from these equations the membrane surface concentration is obtained in terms of permeate concentration. So, from these we can get Cm membrane surface concentration as function of permeate concentration Cp into 1 plus 1 plus beta plus alpha Cp.

Now, this expression of Cm can be put into the equation that we have already talked about. Equation number 2. We are going to put the value of Cm that we are obtained in the in these equation - equation number 2 if you really do that put into equation number 2 you will be getting 1 non-linear algebraic equation in the form of I will be giving the final expression beta Vw not alpha Cp plus beta minus K ln Cp divided by alpha Cp plus beta into C not minus Cp is equal to 0. Again these expression is nothing, but a non-linear algebraic equation in Cp and again 1 has to take care go by a Newton-Raphson method iteratively. So, one has to have an iterative solution. iterative solution of Cp and the method can be used as Newton Raphson method. once you get the value of Cp you will be getting the value of Cm Cp is equal will be getting because from this equation you will be getting the value of Cm because Cp you have already estimated. once you know the value of Cp and Cm then 1 can go through the film theory equation and can get the expression of permeate flux.

So, again in this method also, we can land up with 3 equations only difference is the definition of real retention which basically acts as a partition co-efficient between the membrane surface concentration solute concentration in the membrane surface and in the permeate will be replaced by the solution diffusion equation which is basically nothing, but the solute transport equation for the solute flux through the membrane. Then again these 3 equations can be solved and can be recast in the form of single equation which will be algebraic non-linear equation and that can be solved by using an iterative method. By that 1 can get the permeate concentration membrane surface concentration and permeate flux ultimately.

So, I will stopping in this class in the next class next session will be looking into the other variance of solution diffusion model and how to use these solution diffusion model, what there will be the there will be a modifications of we need in order to get the system prediction perfectly.

Thank you very much.