Introduction to Process Modeling in Membrane Separation Process

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Lecture – 07
Osmotic Pressure Controlling Filtration

Well good morning everyone. So, in this session we will see how the Osmotic Pressure Modeling will for measuring filtration can be done 14 hour to get appropriate system performance. In the in the last class what do we have seen is that, we have looked into the transport phenomena based modeling for the reverse osmosis for flow through the porous medium, and we have found out that how the solute flux, what will be the expression of solute flux and solvent flux through the porous medium, that is membrane now as we have discussed in the last class that that is not the end of the story one has to hook up the mass transfer from boundary layer that is there, that will of formed in the flow channel and so, once that will be hooked up with the transport phenomena in the porous medium, then in the component theory will be providing an estimate of system performance in terms of permeate flux and permeate concentration.

Now, today will be looking into the modeling of concentration polarization that that there is a transport that is occurring external to the membrane surface in the flow channel, within the flow channel. So, now there will be building up the model from a very simplistic approach, to a very complicated one and which will be quite realistic. So, therefore, first will be assuming that there will be a constant a film of solute that will forming over the membrane surface which will be having a constant thickness. So, this constant thickness film is known as the, known as film theory and it is one dimensional approach and this film theory will be giving an estimate and of permeate flux and permeate concentration when its properly hooked up with the Darcy's law and the solute solution diffusion model or definition of real retention that will be the transport that is occurring the within the mass transfer, with within the membrane surface, with in the membrane matrix.

So, will be first looking into a one dimensional model of model of mass transfer boundary layer within the flow channel then will be looking into the short coming of one dimensional model and the film theory which will be identical film theory, and then will be looking into how these short coming can be overcome and one can do a you know detailed modeling with a two dimensional modeling and how the osmotic pressure counter filtration will then really be modeled. At the outset I will do the one dimensional modeling of concentration polarization.

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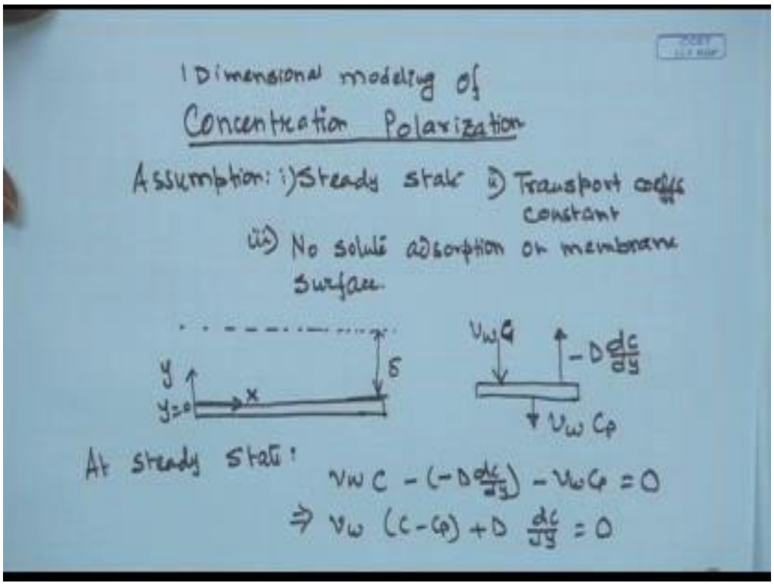

Now, the assumptions are basically it is a steady state transport process; second assumption is the transport coefficients are constant, and third assumption is no solute absorption on membrane surface.

Now let us have a schematic and fix up a coordinate system suppose this is the membrane surface and we are assuming a const a film of solute of constant thickness is form over the membrane surface. And this thickness constant thickness is given as delta and we fix up a coordinate system why from the bottom of the channel. So, these y is equal to 0 and this direction is x direction or axial direction.

So, let us write down the various solute fluxes those are the appearing to the membrane surface, once one will be the convective flux VwC that will be the convective plus towards the membrane surface because the solvent is flowing towards the membrane because of the applied pressure in the channel. Because of that some solute will be also drugged along with it. So, total solvent total solute flux towards the membrane by convection will be VwC. Then, there will a backward diffusion from the membranes surface to the bulk why because solute will be deposited over the membrane surface; therefore, as we have discussed in the last class the membrane are solute concentration on the membrane surface will be hired compare to the bulk concentration and therefore, that will cause a backward diffusion from the membrane surface towards bulk. So, it will be minus d, d c d y and the solute will be going away from the membrane surface in order to whenever there will some solute that will be coming to the permeate. So, it will be Vw Cp.

So, at the steady state summation of all 3 fluxes will be equal to 0. So, VwC minus of minus d, d c d y minus Vw Cp is equal to 0. So, therefore, VwC is towards the membrane surface Vw Cp is away from the membrane surface; therefore, there is a minus sign here minus d, d c d y is the diffusive flux away from the membrane surface. Therefore, there is you know minus sign here. So, we will be getting as VwC minus Cpplus d, d c d y will be equal to 0. So, this one dimensional governing equation is formed to in order to quantify the mass transfer boundary layer within the membrane within the when, the solute concentration balance solute balance within the mass transfer boundary layer. Now let us try to solve this equation and see what we get.

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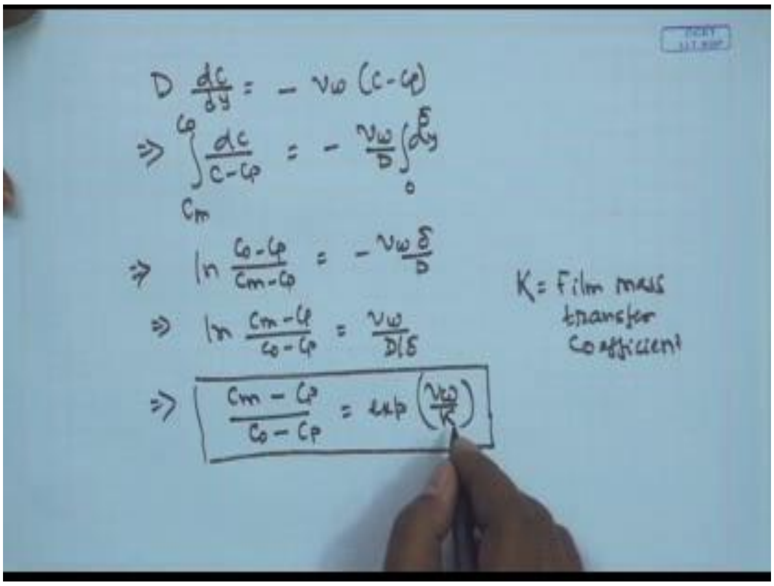

So, it will give you d, d c d y is equal to minus VwC minus Cp. Therefore, d c d by C minus Cp is equal to minus Vw by d d y then these has to be integrated over the mass transfer boundary layer thickness 0 to delta and from membrane surface concentration to bulk concentration. So, you will be getting L n c naught minus Cp divided Cm minus Cp is equal to minus Vw delta by d and if take the minus sign incorporate. So, Therefore, this will be giving you equal to Vw by d by delta and what is d by delta. Delta at d by delta is nothing, but the film mass transfer coefficient.

Therefore, 1 will be quantifying the concentration polarization Cm minus Cp divided by c naught minus Cp is equal to exponential Vw by k this is a one dimensional model of to quantify the concentration polarization or mass transfer boundary layer in the channel, if it will consider a one dimensional film that is formed over the membrane surface. Now let us look into the how the mass transfer coefficient will be calculated or estimated then, will be will be looking into the how this will be hooked up with the Darcy’s law in order to get a system performance and various i know simplified version of this. So, this mass transfer coefficient can be obtained from the non dimensional Sherwood number relationship.

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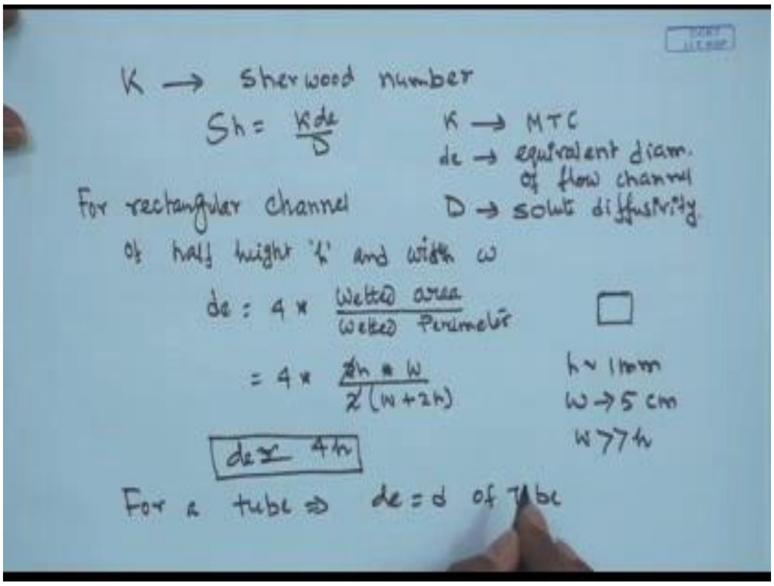

K is obtained or estimated from the Sherwood number relationship. Sherwood is defined as Kd equivalent by d; where K is the mass transfer coefficient, d equivalent is the equivalent diameter of flow channel and capital d is the solute diffusivity. Therefore, for rectangular channel, channel of height h and width w 1 can estimate the equivalent diameter as four into weighted area divided by weighted perimeter.

So, let us find out what it comes out to be 4 into weighted area is basically, if this is the cross session of the channel weighted area will be basically the 2 h multiplied by w. So, 2 h into w divided by weighted perimeter is two times w plus 2 h. So, this turns out to be this 2. 2 will be canceling out and generally h is the in the order of millimeter and w will be in the order of centimeter. Let us say 5 centimeter, 6 centimeter like that this will be order of magnitude; 3 order or 2 order of magnitude less.

So, therefore, w is much, much greater than 8. So, one will be getting these; 2 h will be very, very small and w w will be canceling out. So, one will be getting d equal to roughly four into half height. So, that is the equivalent diameter of the there is equivalent diameter of that channel and for a tube or pipe d equivalent is equal to internal diameter of tube. So, there is how the equivalent diameter is defined in both the causes for the flow through the channel or a flow through a tube flow through it channel will be similar to flow through a spiral wound model and flow through a tube will be basically the flow flow through a hollow fibers or flow through the tubular module.

Next will be looking into the variation relations of Sherwood number expression for various flow geometries and flow domains, so that the mass transfer coefficient can be estimated.

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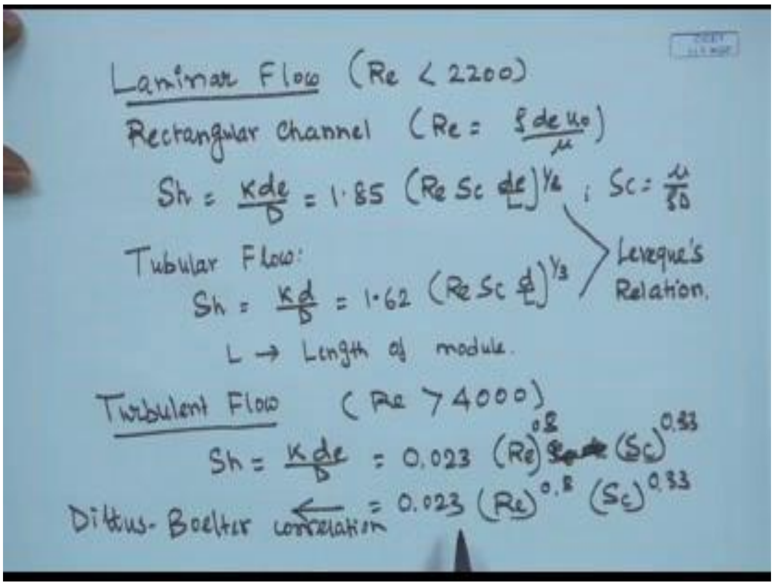

Now, let us look into the two domains first two flow domains laminar flow where Reynolds number is less than 2200 and for rectangular and we will considering two geometries rectangular channel Reynolds number is equal to rho d equivalent velocity in the flow channel divided by mu where is a definition of Reynolds number and mass transfer coefficient will be estimated from k d e by d is equal to 1.85 Reynolds schmidt d by l rest to the power one upon three where schmidt is equal to mu by rho d schmidt number. So, for tubular module or flow through a tube Sherwood number is defined as k d over d is equal to 1.6 to Reynolds schmidt d by l rest to the power one upon three where in both the cases l is the length of the module; that means, for rectangular channel l is the length of the channel for tubular for the tube l is the length of the tube.

So, mass transfer coefficient or will be calculated from this solute number and these relations are known as the levegues relation for turbulent flow, where the Reynolds number is greater than four thousand the schmidt number relationship is obtained from famous Dittus Boelter relationship. So, this becomes k d equivalent by d is equal to 0.023 Reynolds schmidt Reynolds to the power 0.8 schmidt to the power 0.3. There is no geometric factor like d by l. So, in case of rectangular channel d is the d equivalent in case of tubular flow d equivalent equal to diameter of the tube these becomes 0.023 Reynolds to the power 0.8 and schmidt to the power 0.33 this relationship is famous Dittus Boelter relationship correlation. Now this correlation is obtained from the heat and mass heat and mass transfer analogy and it may be noted that for the Levegues equations these are basically the relations or equations and these relations are obtained theoretical calculations from the fundamental theory from basic principles first principles.

On the other hand in the case of turbulent flow Dittus Boelter relationship is obtained from the from the correlations. So, it is not a relationship it is a correlation and it is generally obtained from the analogy between heat and mass transfer next will be looking into the how the Sherwood number or mass transfer coefficient are estimated for stirred cell.

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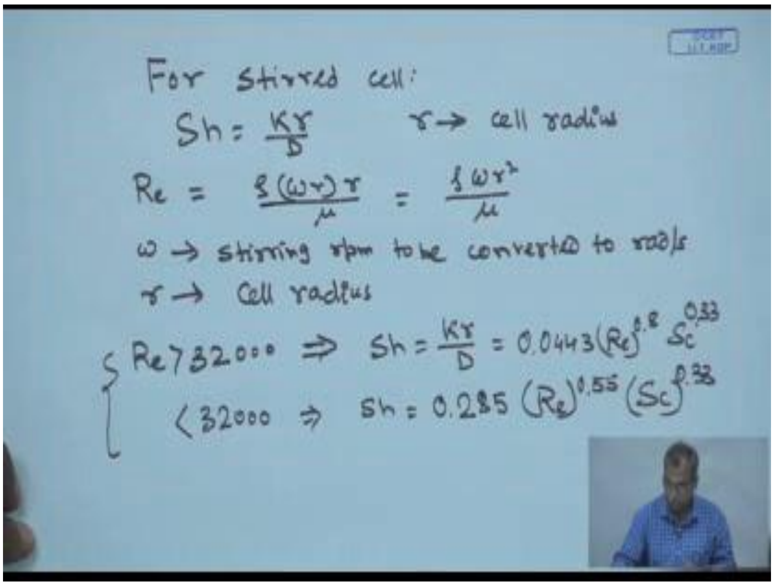

Because in most of the membrane operations the in laboratory scale operations they are they are conducted either in stirred cell. So, in case of start cell stirred cell the Sherwood number defined as s h is equal to k r over d and r is basically k is the mass transfer coefficient as usual d is the solute diffusivity r is the cell radius cell radius and we have two domains of you know Reynolds number Reynolds number is defined as rho omega r r divided by mu. So, omega is the angular velocity of stirring stirring r p m. So, it will be ultimately converted into radiant per second to be converted to radiant per second. So, this will be having a unit of velocity. So, rho v r divided mu r is the radius of the cell. So, this will become rho omega omega r square by mu, where r is the cell radius.

So, now we have defined Sherwood number in domains for Reynolds number greater than you know 30,000 and Reynolds number less than thirty thousand in case of Reynolds number greater than 30,000 the relationship is s h is equal k r over d equal to 0.443 Reynolds to the power 0.8 Schmidt to the power 0.3 three and in case of Reynolds number less than 32,000 this will be Sherwood equal to 0.285 Reynolds to the power 0.55 and Schmidt to the power 0.33 these two relationship will be will be used in order to estimate the mass transfer coefficient in case of flow through a start cell next what will be looking into will be looking into a system of cross flow filtration.

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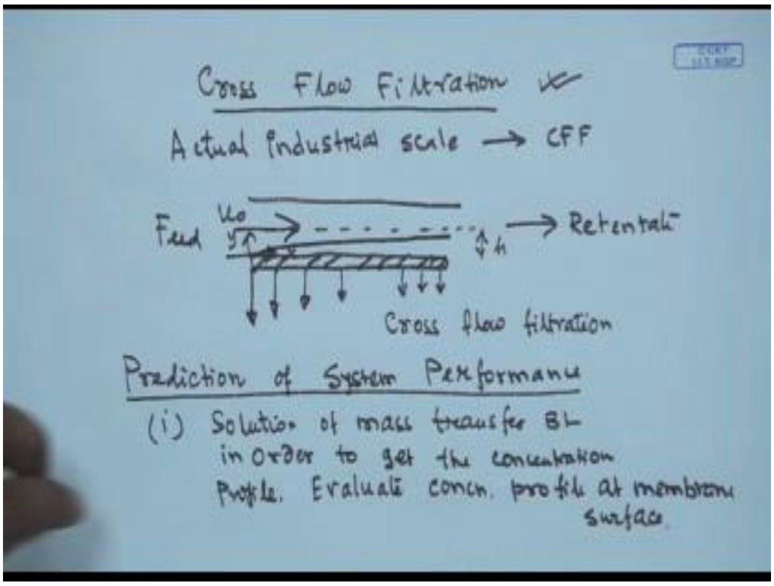

What is a cross flow filtration cross flow filtration is basically a system which will be which is generally used under in an actual industrial scale actual industrial scale we have the cross flow filtration. Why it is called as a cross flow filtration because as we have discussed suppose there is a flow occurring through a channel and membrane is placed at the bottom. In fact, in an actual module there is a in the top surface also you place an membrane. So, at the surface area of membrane is to be is to be more in the in a in a small channel.

So, therefore, one can get a very compact design can realize high surface area in a small volume and the design becomes very very compact. So, let us just for the time being we assume a membrane is placed at the bottom surface and this is middle of the channel. So, this is half height of the channel now when the flow is occurring the concentration of the solute will be deposit over the membrane surface and it will form a mass transfer boundary layer as we have discussed earlier.

Now, the problem is this mass transfer boundary layer can grow unlimitedly, but if it grows unlimitedly it offers more resistance against the solvent flux. So, therefore, the beginning of the mass transfer boundary layer the resistance is minimum and one will be getting a very high permeate flux as the thickness of mass transfer boundary layer increases it will be offering more resistance gradually and the solvent flux will be decreasing slowly and slowly. So, later on the downstream of the channel when the mass transfer boundary layer will be growth of the mass transfer boundary layer will be very very less. So, one will be getting a constant flux at the in the in the downstream of the channel.

So, therefore, hot cross flow, what is cross flow? So, actually the feed is pumped into the channel and the bulk flow is occurring in this direction. So, therefore, the feed is entering into the channel permeate is coming out and the retented stream is going out now in this case feed is following in this direction in horizontal direction in the x direction and in the if you if you if you call this x direction and this is y. So, feed is flowing in the x direction and permeate is following in the y direction which will be at a 90 degree or.

So, that is why this flow is called a cross flow filtration why the cross flow term is coming because the filtrate or permeate is coming at 90 degree compare to the direction of flow in the feed chamber. So, therefore, be in this cross flow filtration what is the advantage the advantage is we are not allowing the mass transfer boundary layer to grow unlimited the mass transfer boundary layer is limited in the growth of the mass transfer boundary layer is limited or arrested by the forward by the by the convection post convection caused by the velocity of the feed in the channel.

So, the growth of mass transfer boundary layer is controlled or arrested by the feed flow and that is why in the in the cross flow filtration system will be very very efficient and will be curbing the growth of the mass transfer boundary layer therefore, is the resistance against the solvent flux therefore, one can realize very high permeate flow flux or the througput in a very in a cross flow. So, cross flow system cross flow filtration are the quite important and popular and they are widely used in order to get in an actual membrane filtration system. In fact, all the spiral wound module the tabular module flow flow through hollow fibers they are the cross flow systems. So, now, we will be looking into the system prediction of system performance of cross flow system using the film theory one dimensional model film theory and osmotic pressure model.

So, as we have discussed there are two aspects in the modeling of this one is the you know solution of mass transfer boundary layer in order to get the concentration profile we evaluate this concentration profile at the membrane surface why because then it will be hooked up with the flow through the porous membrane evaluate concentration profile at membrane surface once this is done then will be will be having another transport flow domain.

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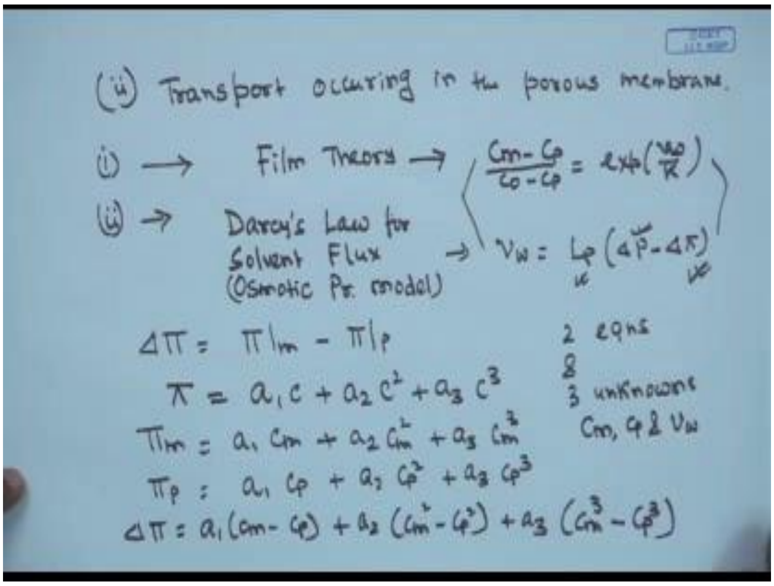

The next domain is transport occurring in the porous membrane in the first for the first case for the flow through membrane channel we have got the concentration profile and obtained the mass transfer obtain the concentration profile evaluate the evaluated the concentration profile in the membrane at the membrane surface. So, that is obtained by the film theory. So, film theory is the theory that was applied external to the membrane surface in the flow channel as Cm minus Cpdivided by c naught minus Cpis equal to exponential v w by k.

Next will be looking into the transport to the porous membrane one is first one is the darcys law for the solvent flux solvent flux and this is also known as the osmotic pressure model this is identical osmotic pressure model now what is that v w is equal to l p del p minus del pi now in this equation l p is the membrane performance parameter it is a membrane characteristic. So, will be as we have discussed earlier we have already evaluated this permeability from the separate set of experiments delta p is the trans membrane pressure drop let us look what is delta pi is delta pi is pi osmotic pressure at the membrane surface minus osmotic pressure at the in the permeate side. So, what is the osmotic pressure osmotic pressure can be expressed as a function of concentration like this a 1 c plus a 2 c square plus a 3 c q.

So, what is pi m pi m will be nothing, but a one Cm plus a 2 Cm square plus a 3 Cm cube and what is pi p pi p will be in the permeate side a one Cpplus a two Cpsquare plus a three Cpq. Once we quantify these we can really quantify what is delta pi delta pi will be difference of these two. So, this becomes a 1 Cm minus Cpplus a 2 Cm square minus Cpsquare plus a 3 Cm q minus Cpq. So, now, these if you if you if you look.

So, this equation the second equation the darcys law or the osmotic pressure model we have a constant value of l p we have the operating pressure delta pi and delta p and delta pi osmotic pressure difference will be expressed in terms of Cm and c p. So, you have two governing equation one equation for solute flux in the mass transfer boundary layer another equation is the solvent flux for the flow through the membrane. So, we have how many unknown let us see we have three unknowns VWCm and Cp and we have 2 governing equations. So, two equations and three unknown and these three unknowns are Cm Cpand v w.

So, I stop in this class in the next class we will see how we invoke one more equation in order to solve these 3 unknown and three equations system.

Thank you very much.