Introduction to Process Modeling in Membrane Separation Process

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Lecture – 18
Design of Membrane Module (contd.)

Welcome to the session of our class. So, we are looking into the module design of a membrane module design of an actual case, where osmotic pressure is quite significant and we are looking into the spiraling module that is a flow through a channel. And as we have seen we have derived the governing equation of governing by equation of axial pressure drop, governing equation of velocity in the retentive as channel and governing equation of concentration.

We have seen that these are basically already differential equation for the axial pressure drop will be a second order ODE for the velocity will be a first order ODE and the concentration it is a first order ODE. So, three ordinary differential equations are to be hooked up, but there are membrane surface concentrations that will be coming all the time in the picture. So, this membrane surface concentration has to be related to the bulk concentration, and as we have seen that this is related to the definition of mass transfer coefficient, which will result in you can result in an algebraic equation. So, ultimately you will be getting up landing up into a set of ordinary differential equation coupled with algebraic equation it is a DAE system differential algebraic equation system. So, let us just continue with that and finish of this problem.

So, we have already seen that mass transfer coefficient the algebraic equation that will be connecting the membrane surface concentration and bulk concentration.

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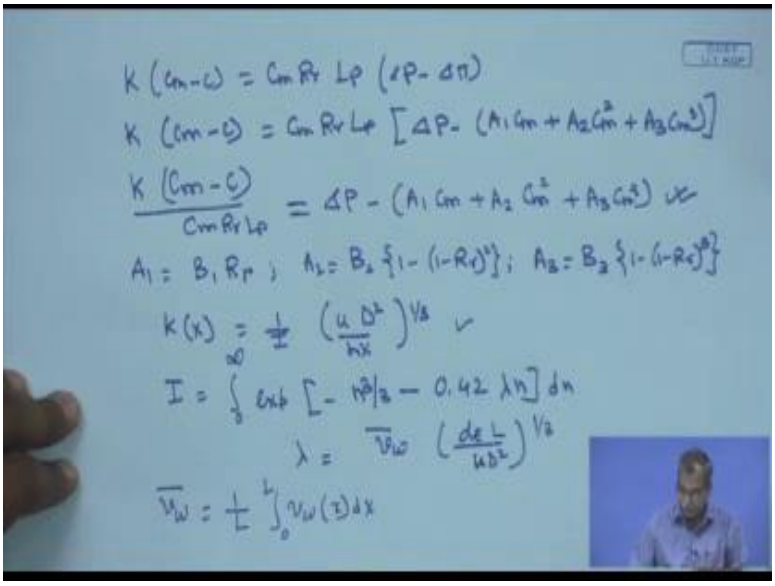

Through this equation, but A e is also a function of x. So, this is the relationship between K and x and let us look into what is I? I is the definite integral zero to infinity exponential minus eta cube by 3 minus 0.42 lambda theta d eta. And lambda is the length of rich permeate flux d e is equivalent diameter l u d square etcetera, u u is the velocity and d is the solute diffusivity to the power one-third and v w bar is nothing, but one over l, zero to l v w d x. So, d is the length of it is permeat flux. So, this has become. So, will be having three ordinary differential equation and one algebraic equation as to be solved. So, this is very very complicated situation now because the integration I also invoke the length of it is permeate flux. So, what is the algorithm? So, we will be having three let us let us just summarize what we are having, this can also be solved analytically like the earlier cases.

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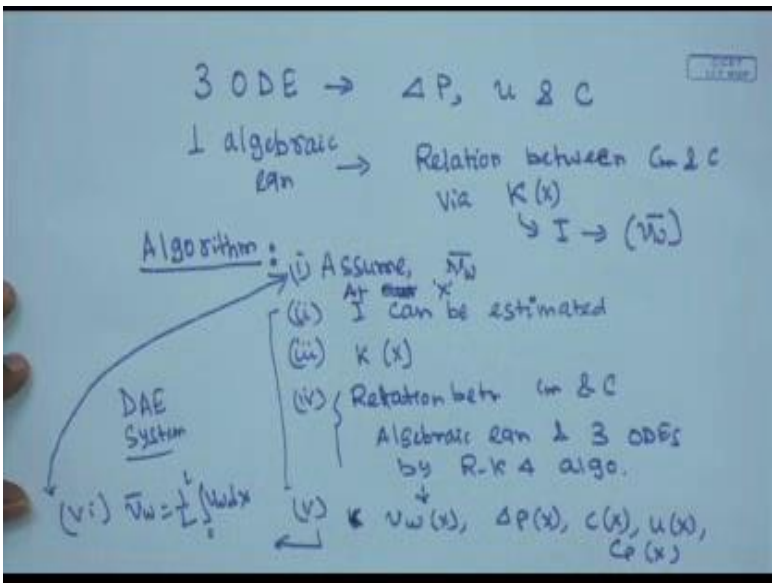

So, will be having three ordinary differential equations, and these equations are basically governing equations, for delta p governing equation of u, and governing equation of C concentration, bulk eutectic concentration and one algebraic equation. The algebraic equation is the relationship between C m and C via definition of mass transfer coefficient k, which will be essentially a function of x and that, will be a function of I, and I will be basically a function of v w bar, or the length average permeate flux.

So, what we will be doing. So, the algorithm goes like this the algorithm will be like this. We assume v w bar, a length average permeate flux once the length average permeate flux is assumed, and then I can be estimated. Once I can be definite integral I can be estimated by using a trapezoidal rule, than I will I can get K as a function of x, once I get the K as a function of x then I can go to the relation between C m and c. And estimate the value of C m and, then you will be hooking up this equation, with the with the DAE system. So, this four will be basically the algebraic equation and three ODE s lets say by r K four algorithms. So, three ODEs and one algebraic equation this will be formulating a DAE system the differential algebraic equation system, and this has to be solved simultaneously, and then we will be in the in the process what will be getting the output will be will be getting K as a function of will be getting v w as a function of x, delta p as a function of x, C as a function of x u as a function of x and C p as a function of x.

After that what we will be doing we will be finding out the average length; average permeate flux as one by l zero to l v w d x. and we will see whether these length average permeate flux is coming close to this value or not. If this is not coming close then you have to curb you have to assume another value of length average permeate flux redo this calculation. So, this will be very complicated. And it will be basically this at every step of r K four this algebraic equation has to be evaluated by using Newton-Raphson method. So, Rangekutta four nested in Newton with Newton-Raphson method at every step of r K four. And then the whole loop will be with there is an outer look which will be carrying at a particular value of x. So, this will be continued till it goes for the. So, it goes for the full length of the channel at a particular x. So, these whole calculations has to be calculated as the particular value of x. and then it has to be you have to go forward x to plus delta x. It will be converged and likewise we have to do the calculations at every x location we have to calculate the permeate flux. Then you have to do a length average permeate flux using Newton-Raphson. And then you will check whether this calculated average permeates flux will be coming close to the case value or not. If not then it is you have to alternate if yes then you will be getting a converged solution.

So, likewise the in an axial module, module system the whole calculation to be done or a module can be designed. So, at the end of the design what you will be getting. You will be getting a length average permeate flux length average permeate concentration. Axial pressure drop across the module. So, once you know the axial pressure drop across the module one can select a pump which will be suitable to pump the fluid to overcome these axial pressure drops across the module. So, that completes the module you know design in a linear actual scale. And similar thing can be translated to a tubular channel or a tubular module as well.

So, next we will be going to the going to our next topic. That is you know dialysis operation and dialysis becomes very very important in most of the membrane separation process, but the there is a basic difference of dialysis between the pressure driven processes.

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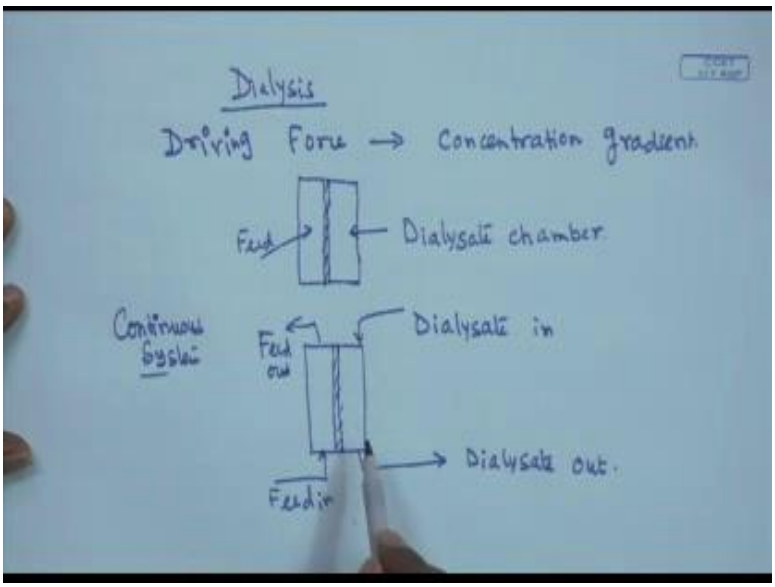

And the major driving force in dialysis is concentration gradient concentration gradient. So, we will be having a feed chamber. There will be two chambers those will be separate by a by the membrane will be having a feed side or feed chamber. And then will be having a dialysate side dialysate chamber. So, in a continuous system, feed is going into the feed side. Feed in and feed is going out. And from here it is a counter current you have seen counter current is always effective dialysate in and here you will be having the dialysate out. So, the t solute which will be having a very small size and molecular weight they will be having you know zero concentration here and they will be going from feed size to the dialysate side.

We are maintaining a dialysate flow rate. So, therefore, whenever the targeted solutes of smaller molecular weight or size they will be coming permeating through the membrane coming to the dialysate side immediately they will be washed away. So, the concentration maintained in the dialysate side will be will be will be almost equal to zero. So, therefore, we are maintaining the maximum concentration gradient of solute targeted solute from the feed side to the dialysate side.

So, therefore, one can expect a very high mass transfer across the membrane module now. Let us see what are the different you know the transfer resistances those are

involved in dialysate process, diffusion flux. Diffusion is the main mechanism is the because of the concentration gradient.

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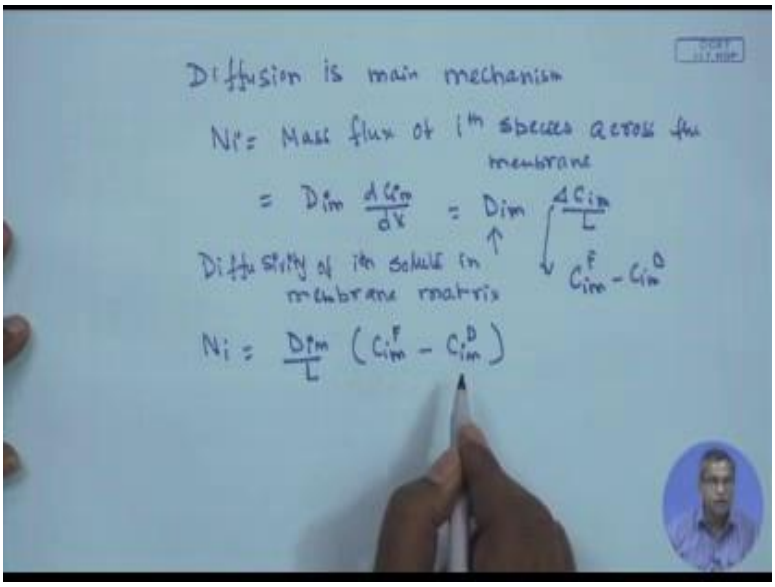

So, let us say N i is the mass flux of ith species across the membrane. So, d will be nothing, but D im d C im d x. So, so d is nothing, but D im delta C im by l D im is the diffusivity of the ith solute in membrane matrix. And delta C im is nothing, but the concentration difference in the feed side and the of ith species from the in the feed side and the dialysate side. So, these will be N i be nothing, but D im over l C im in the feed side minus C im in the dialysate side. So, determination of D im is very very important. So, if we can determine the concentration the diffusivity of solute in the membrane matrix. Then by measuring the concentration in the feed side in the dialysate side we can really look into we can really find what is the total molar flux or the mass flux across the matrix.

Now, let us look into the different type of resistances those will be occurring appearing in the dialysate process.

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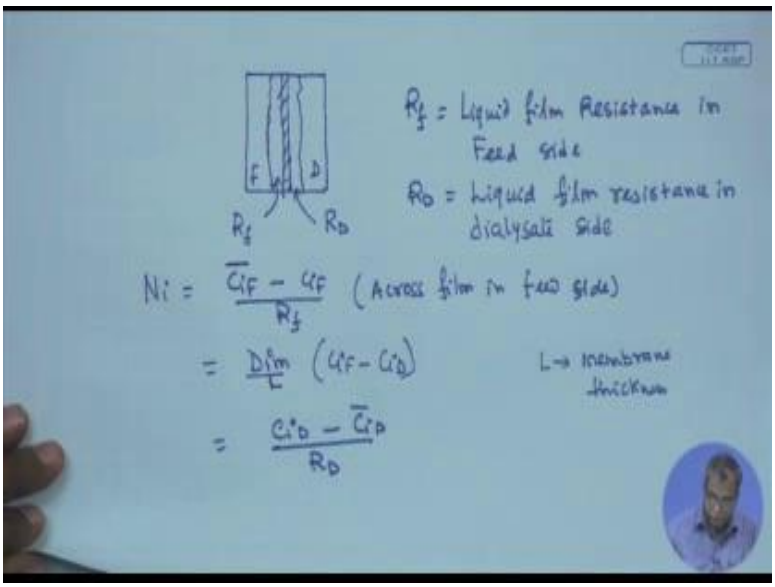

So, there will be apparently three resistances one can come across during dialysate process. So, one will be the feed side resistance, mass transfer coefficient you know mass transfer of boundary layer in the feed side, and there will be a master from the dialysate side, and there will be the membrane resistance there are three resistances you will be encountering one is R f in the feed side, and the I is R d in the dialysate side and is the membrane. So, R f is liquid film resistance in feed side R d is liquid film resistance in dialysate side and d I and this will be he diffusivity in the membrane. So, these are membrane resistance we must write down the mass flux of ith species which will be nothing, but C if bar minus C if divided by R f. This is across film in feed side this bar representing the concentration in the bulk. These will be equal to D im divided by l C if minus C id. C if is the ith species concentration in the feed in the dialysate side D im is the diffusivity of ith species in the membrane matrix. And l is the membrane thickness. and there will be dialysate resistance C id minus C id bar divided by R d.

So, now we can we can add this all up and see what we get. So, we will be getting an overall resistance. So, from the three equations we will be getting C if bar minus C if is equal, to N i R f C if minus C id is equal to N i l over d I m, and then C id minus C id bar will be giving you N i R d.

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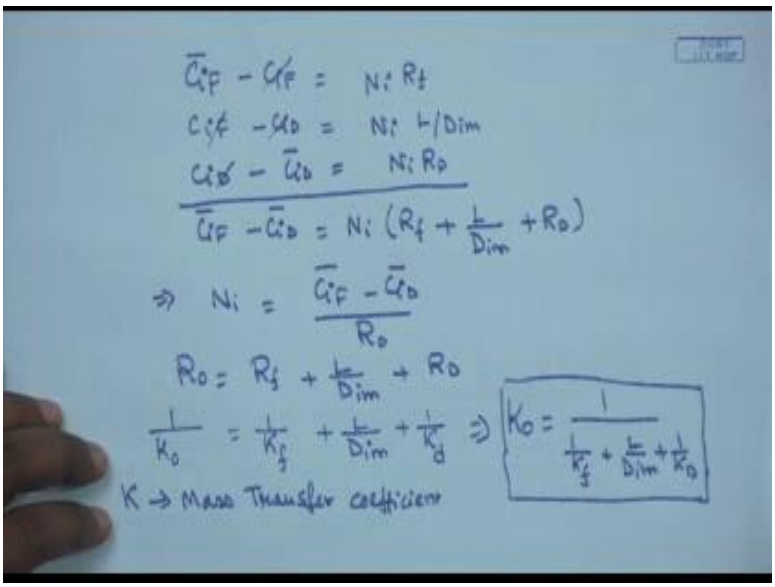

If you add all them up then these will be canceling out, and you will be getting C if bar minus C id bar is equal to N i. All the resistances are added in series. So, we can get an overall resistance N i is equal to C if bar minus C id bar, and divided by r overall. So, overall resistance can be written, as R f plus l over D im plus R d and you can you can write the overall mass transfer coefficient. So, K overall is equal to 1 over K in the feed side plus l over D im plus one over K in the dialysate side. This K's are basically the mass transfer coefficient. So, one can get an expression of overall mass transfer coefficient as one over K f plus l over D im plus, 1 over K d this will be the expression of overall mass transfer coefficient. So, once we get the overall mass transfer coefficient we will be able to get the net mass flux across the dialysate module from the feed side to the permeate side or the dialysate side.

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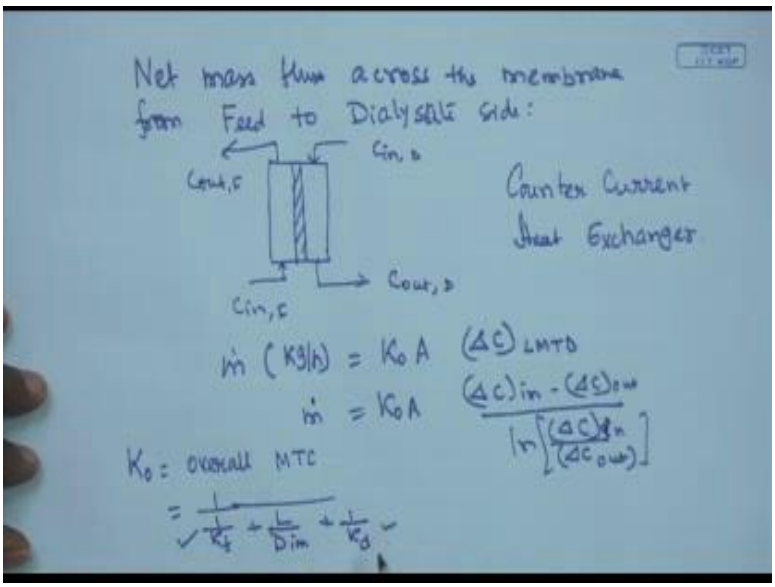

So, net mass flux, across the membrane from feed to dialysate side. So, that will be giving you a system in for a counter current flow. So, C in, C out in the feed, C in - in dialysate C out in dialysate typically in the dialysate we will be giving the pure solvent. So, typically C in - in the dialysate side will be zero it will be some final variant C out. So, this will be exactly like a counter current heat exchanger. The analogy is counter current heat exchanger. And in our heat transfer course we have already found out that how the overall in the heat transfer heat transfer that is taking place in a counter current heat exchanger. Now in the case of dialysis it is instead of a counter current heat exchanger it is a counter current mass exchanger. And we can write down in instead of overall heat transfer coefficient we can write the overall mass transfer coefficient delta t log min temperature difference can be replaced by delta C LMTD.

So, we can write down the mass flux mass flow rate kg per unit time, kg per hour, will be nothing, but K zero a delta C LMTD what is delta C LMTD K 0 a delta C out delta C in minus delta C out, divided by l n delta C out divide by delta C in divided by delta C out. So, this will be the expression of mass exchange or mass transfer across the channel and K zero is the overall mass transfer coefficient. and we have already obtained we have already seen the expression the expression of overall mass transfer coefficient, it will give one over K f plus l over D im plus 1 over K in the dialysate side.

So, therefore, we know the mass transfer coefficient in the feed side, we know the mass transfer coefficient in the dialysate side, and if we can determine what is the diffusivity of the of the solute we are taking about in the membrane phase we can calculate the resistance occurred by the membrane l, by d m l is the membrane thickness this can be determined if we put a cross section under the scanning electro-microscopic. We can estimate the thickness of the membrane, but D im may not be possible to know directly. So, there are if somehow we can estimate the value of diffusivity of ith solute within the membrane matrix, we can evaluate the overall mass transfer coefficient.

Once we can evaluate the overall mass transfer coefficient we can find out the concentration steam all the steams coming out of the system going into the system. Once we get that we will be able to estimate the net delta C l t d term and once if you know the value of D im we and if you know the membrane area we can find out what will be the mass that will be transferred through the dialysis or unit time. If the concentration is fixed mass exchange is fixed we can evaluate what is the area of the dialysis membrane that is required for designing such purposes. So, once we know we can you can determine D im these equation can be utilized for the design purposes, for the given design these equation can be utilized, for the symbolization purposes as well.

Now, in the next what we are going to do, we are going to do a how to evaluate or the method of procedure how to evaluate the diffusivity of ith species in the membrane matrix. For that exactly like the case of permeability and real retention you conduct an independent set of experiment in a small batch dialysis process or a small batch dialysis cell. So, let us look into the batch dialysis analysis.

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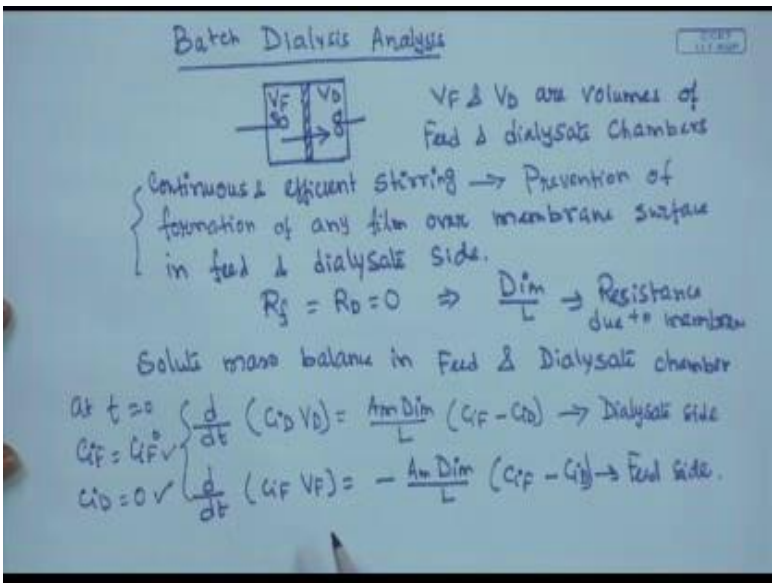

In a batch dialysis analysis, there will be no continuous flow. The cell is exactly the same like before. The two chambers have to be separated by a semi permeable barrier. That is the volume of the feed chamber is v f volume of the dialysate chamber is d. You put a dialysate solution in the dialysate chamber in most of the cases it is pure water and we will put a solution in the feed chamber, and there will be mass exchange from the feed side to the dialysate side.

Now, v f and v d are volumes of feed, and dialysate chambers, and the solute will be permeating from the feed chamber to the dialysate side. Now we should have stirring here as well as here. So, a very good stirring will be will be preventing formation of any film over the membrane surface. So, continuous and efficient stirring is required, that will lead to for prevention of formation, of any film resisting the solute movement. Any film over membrane surface, both in feed side and in dialysate side, if that is done we do not have R f and we do not have R f and we do not have R d in this case. So, there is a stirring should be provided. So, what resistance is existing is the membrane resistance D im over l. So, membrane resistance should be there. Resistance due to membrane on it

So, next we what we are going to do, we are writing the solute mass balance, in feed chamber and in dialysate chamber. If you do that the accumulation should be the material

that is going out of the system. So, accumulation will be d d t of C id, and v d is equal to a m d I m, divided by l, C if minus C id. So, d is the solute balance in the dialysate side. d is the accumulation of solute in the dialysate chamber. And that is the material this accumulation is because of the material that is the solute that is coming in from the feed side. Similarly you will be writing the solute balance in the feed side. So, that will be equal to the amount of solute that will be going out of the feed side. So, it will be a depletion in the in the case of feed chamber. So, d d t should be equal to C if v f is equal to minus a m D im over l the amount remain same C if minus C id this is a feed side solute balance. So, dialysate side will be enriched by the solute transport and the feed side will be stripped by the solute transport.

So, what will be the boundary initial conditions at for both of them at t is equal to 0. we have C if is equal to C if naught. And t is equal to 0 C id equal to 0. So, it was a pure dialysate initially there is no solute present there. So, C id was 0 and in the feed side there is a feed concentration corresponding to C if naught. Now these two equations have to be solved simultaneously. These two ordinary differential equations initial way initial volume problem. So, they are since their initial conditions are specified, these are basically initial value problem. These two equations have to be solved simultaneously in order to get the concentration profile in the dialysate side. So, we take a sample from the dialysate side and monitor the concentration of the solute in the dialysate side and then, we will be comparing with the if the theoretical value of the experimental profile of the concentration in the dialysate side, should be compared with the predicted or the theoretical values, and from that comparison the value of D im will be evaluated. So, I will stop in this class.

In the next class, I will we will show how the value of D im, it will be coming out from this analysis.

Thank you very much.