Introduction to Process Modeling in Membrane Separation Process

Prof. Sirshendu De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

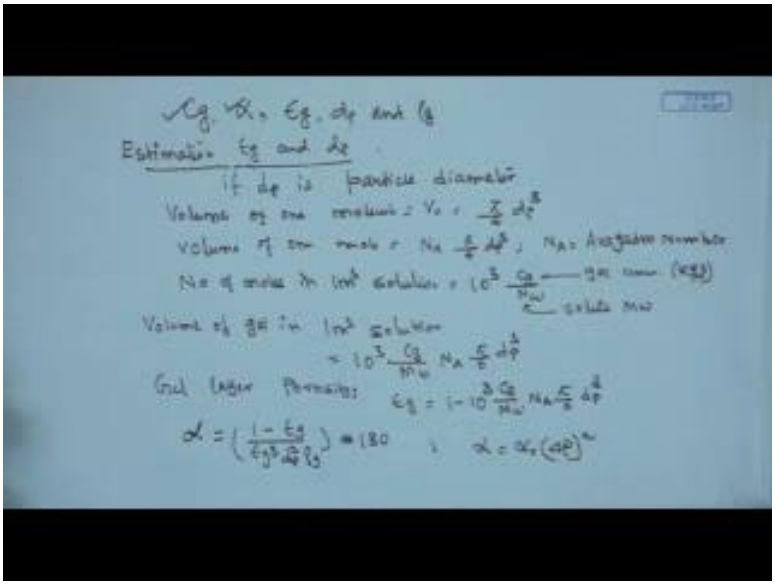
Lecture -15
Modeling of Gel layer Controlling Filtration (Contd.)
And Resistance in Series Models

Good afternoon every one. So, let us look into what we will doing in the this session in the last class, whatever we have done is that we are looking into the gel layer polarization gel layer controlling membrane filtration processes and as you have, as you are aware that in the last lecture I have told the gel can be formed in a system by two ways the filtration may be osmotic pressure control in initially and then it can convert into converted into the gel polarization range or there may be some solutes which will be forming a gel layer on a very beginning of the filtration purposes.

For example, naturally occurring gelling agents like pectin polyvinyl alcohol and things like that. So, we have seen the study state gel polarization model using the theory and then we have also seen that in a in a realistic situations. How the gel layers grows in time and permeate flux declines the function of operating conditions and we have seen its modeling as well now in the modeling we have taken a close similarity to the classical filtration theory and we using filtration theory we have model the gel layer control filtration by permeate flux goes down as you know varies a function of operating time of filtration.

Now, there are several parameters those who are you know appearing in the gel controlling filtration model. So, these among these parameters major parameters were gel controlling gel layer concentration.

(Refer Slide Time: 01:51)



Number 1 another 1 is the specific gel layer resistance alpha third 1 is the porosity of the gel particular diameter of the gel forming material and gel layer density. So, we have already seen how gel concentration will be evaluated we have already seen how alpha the specific gel layer resistance will be evaluated from the from by conducting experiment separate experiment in an un stirred batch cell and in this class we will be looking into how epsilon g and dp will be connected and estimated because these two will be connecting connected because as we already talk earlier that in membrane based filtration process, we will not be looking into the hazards feeds. Where the particles of defined diameter d p and they will be forming a type or gel type layer over the membrane surface will be you may having a polymeric solution we may be having a poly polysaccharide solution, which will be equivalent and this these may not be defined as well defined particles space particles.

So, they will be forming a gel whose resistance will be equivalent to a resistance offered by the particle diameter dp. So, there is the idea. So, let us look how to how we can estimate the gel porosity and gel forming or diameter with gel forming particle if dp is particle diameter in its molecular level then volume of 1 molecule is v 0 and this is pi by 6 dp cube then volume of 1 mole will be n a pi by 6 dp cube. Where n a is the Avogadro

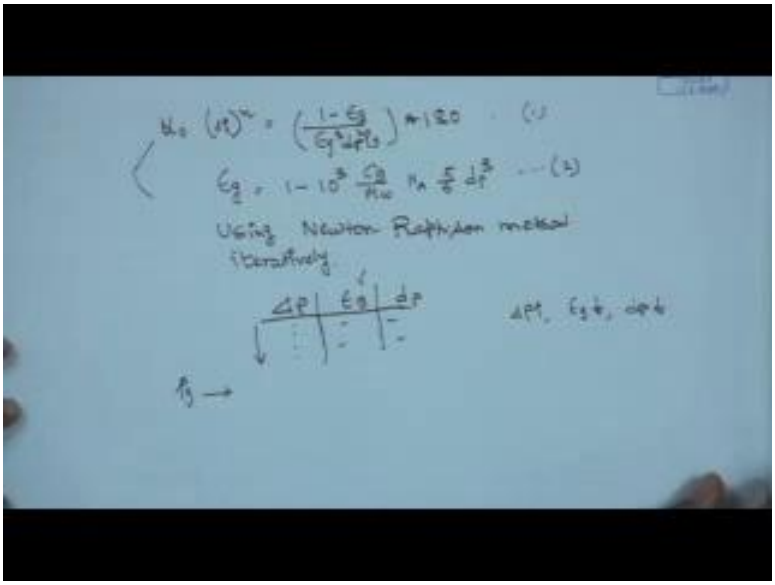
number now number of moles in 1 meter cube solution is 10 to the power 3 c g by m w c g is the gel layer concentration m w is the molecular weight.

So, this is nothing but gel concentration in k g per meter cube and m w is the solute molecular weight and does the volume of gel volume of gel in 1 meter cube solution becomes ten cube c g by m w n a pi by 6 d p cube does 1 can 1 can get the gel porosity, porosity as epsilon g is equal to the 1 minus ten cube c g by m w n a pi by 6 d p cube. Now, one can get the combine the specific gel layer resistance.

Now we know the alpha the specific gel layer resistance can be obtained from Kozeny Carman equation as 1 minus epsilon g divided by epsilon g cube dp square rho g multiplied by 180 and we know how alpha is varying a function of alpha 0 multiplied by delta p to the power n. As we have described earlier that we will be conducting the experiments in a un in a un stirred batch cell and from the plot of t by v verses b data we can get the value of specific gel layer resistance alpha from the slope of the curve and then, we will be conducting these experiments for different trans membrane pressure drop and can generate the correlation of alpha as a function of delta p.

So, we can equate this two and finally, we will be getting alpha naught delta p to the power n is equal to 1 minus epsilon g divided by epsilon g cube d p square times rho g into 180.

(Refer Slide Time: 06:46)



So, these will be giving an equation the second equation I will be getting from definition of epsilon g 1 minus 10 to the power 3 c g by m w n a pi by 6 d p cube. So, this is this gives me second equation. So, first equation connects me the variation of gel layer gel layer porosity and gel gel forming particle with the specific gel layer resistance. So, left hand side will be completely known for a particular delta p and the second equations gives me a fundamental relationship between the particle the gel porosity and the particle diameter. So, 1 can solve these two equations by using Newton Raphson methods iteratively and can guess can get an idea for different trans membrane pressure drop what are the values of epsilon g and d p. So, as for different values of delta p we can get we can solve these two equations simultaneously and we can get the values of gel porosity and gel forming particle. If we really do that then we will be finding out that as the trans membrane pressure drop increases gel porosity decreases this decreases as delta p increases gel porosity decreases and gel become denser and particle diameter, will be also decreasing particle diameter will also be decreasing.

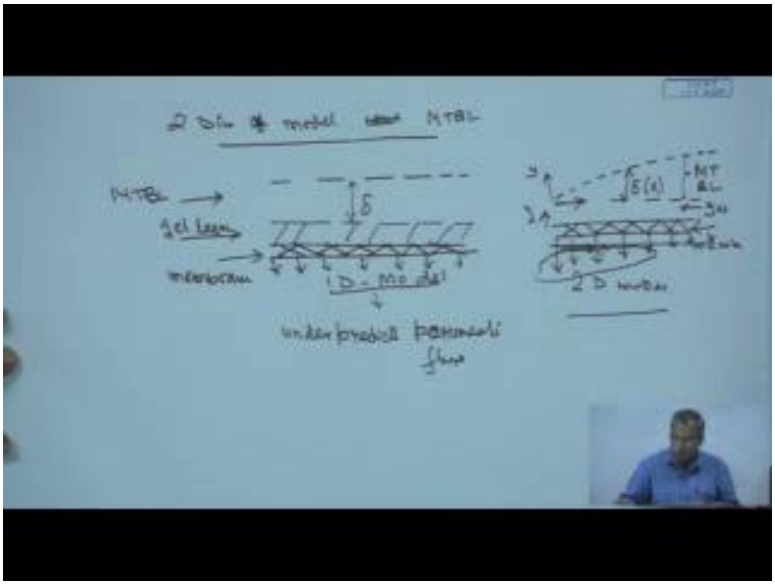
So, these will be give you an idea how these particular gel of you know highly viscous gel will be corresponding to the gel that is forming of equivalent space of diameter d p. So, these way the gel layer density and gel layer porosity and gel forming particle diameter of the gel forming particle equivalent particle will be obtained for different

operating conditions here the operating conditions will be delta p is a trans membrane pressure drop similarly.

So, what is what is left the only parameter that is left are estimated is gel layer density rho g and gel layer density is obtained by assuming by making the particles to be solid and then evaluating the density 1 can get the gel layer density and typically this particle the density of the this particle is slightly higher than the water density around 1000, 1051, 50 k g per meter cube around 1100 k g per meter cube these can be you know with some of the experimental conditions this can be used as an optimizing parameter as well and, then it can be evaluated and then can be incorporated in the model which will be used is a predictive mode.

So, these are the various ways the different parameters of a gel layer controlling model can be estimated. So, I have looked into the estimation of gel layer concentration we have looked at into the estimation of specific gel layer resistance gel layer the equivalent diameter of gel forming material as well as the gel porosity. So, that completes the gel layer controlling filtration only one thing is left that is, if we go for a two dimensional model in the gel layer that will also be useful and sometimes it will be very very useful in order to get the estimation of parameter flux.

(Refer Slide Time: 10:42)



So, two dimensional model of mass transfer boundary layer, as we have discussed earlier that 1 dimensional model, till now we have done. So, this gel layer you know formation and we are assuming a constant thickness of solutes is forming over the membrane surface. So, this is the gel layer this is the membrane. So, this is the membrane the next layer is gel layer next 1 is the mass transfer boundary layer.

So, the model that we have just discussed is the thickness of the mass transfer boundary layer is constant as we have seen earlier in 1 this. So, this nothing, but the features of the 1 dimensional model as we have discussed earlier that this 1 dimensional model suffers from a from under predicting of the permeate flux. Because, if we consider in actual scenario the mass transfer boundary layer is really developing in the system we are talking about because we are dealing with the solutes which will be having higher size and very low diffusivity high schmidt number and since the entrance strength of the, you know entrance strength of the mass transfer boundary layer is l e by d is equal to 0.05 reynolds times schmidt number since you are we are having a system of high schmidt number its entrance will be very high. So, therefore, most of the cases in the channel the mass transfer boundary layer will be developing.

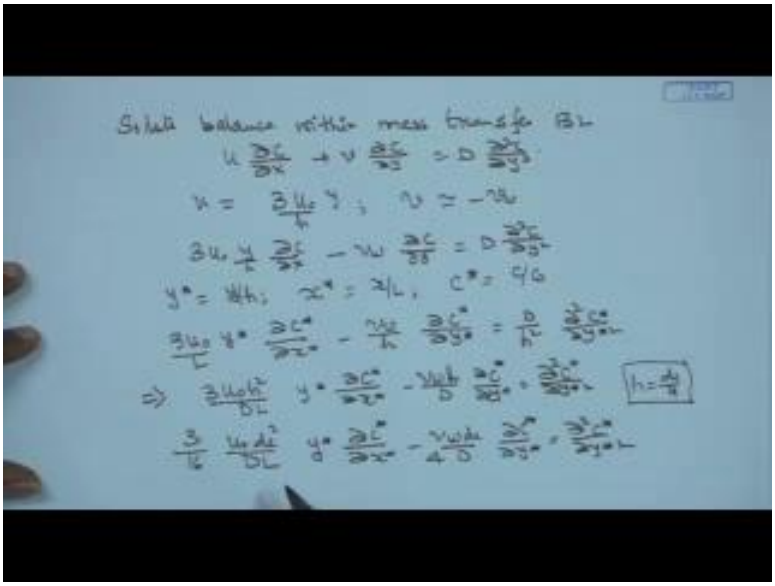
So, in case of 1 dimensional model since we will be considering the actually the constant thickness of mass transfer boundary layer, which will be basically appearing at the end of you know actual boundary layer probably after several meters in the downstream. So, we will be overlooking the initial portion of the mass transfer boundary layer where will be getting high permeate flux.

Therefore, in 1 dimensional model will be always under predicting the permeate flux in order to avoid that 1 can also go for a two dimensional model for gel layer forming gel controlling filtration now let us look into that. So, in two dimensional model if this will be the membrane let us say then there will be a formation of gel layer and then the mass transfer boundary layer will be growing like this instead it will be a instead of a constant. So, in this case this is the membrane this is the gel and this is the mass transfer boundary layer and where delta will be essentially a function of x in this case. So, this is the two dimensional model and in case of 1 dimensional model the permeate flux is remaining

constant all the time in case of two dimensional model will be having initially the higher permeate flux followed by decreasing it gradually then it remains constant.

So, therefore, in the 1 dimensional model we are considering the same flux throughout and in case of two dimensional model we are basically doing the actually scenario. So, in 1 dimensional model we were in not estimating of we are not accounting for this amount of flux. So, 1 dimensional model allows under predicts permeate flux. Therefore, let us solve the two dimensional model and what we will be doing will be fixing up coordinate systems here at the end of the gel layer, where the mass transfer boundary layer starts growing because gel layer mass transfer boundary layer will be always attach to the immobile to gel layer. So, mass transfer boundary layer we write down the solute balance equation.

(Refer Slide Time: 14:58)

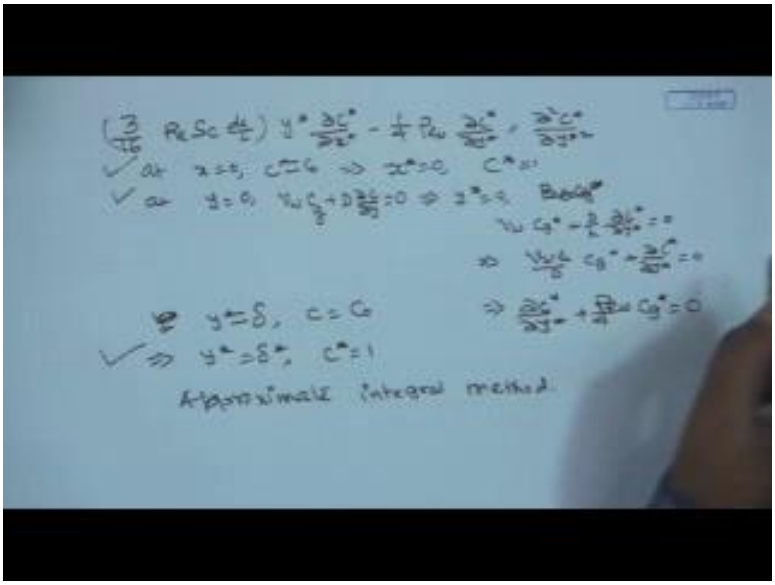


Balance within mass transfer boundary layer and as usual the at the steady state u the equation is u del c del x plus v del c by del y is equal to d del square c del y square with the same argument that we have we have done earlier that u will be basically the linear. So, it is a case of developing mass transfer boundary layer and fully developed hydrodynamics or velocity boundary layer.

So, you will be within the mass transfer boundary layer we will be considering a fraction of you know velocity boundary layer. So, at that part of the parabolic profile will be linear in the within the mass transfer boundary layer. So, you will be linearise the mass transfer the velocity profile and the linearised portion will be coming out to be expression coming out to be 3 u 0 y by h in the small you know close to the wall because the mass transfer boundary layer will be extremely small v will be almost approximately as minus v w. So, the equation becomes three u 0 y by h minus del c by del x minus v w del c del y is equal to d del square c del y square now we make this system non dimensional from the very beginning. So, y star will be y by h it is a x star will be x by l. Where h is half height of the channel l is the length of the model c star is equal to c by c naught, c naught is a feed concentration. So, let us say what we get. So, this becomes three u 0 y star del c star l del x star minus v w divided by h del c star del y star is equal to d over h square del square c star del y star square.

So, let us multiply both side by h square by d. So, what you will be getting is three u 0 h square by d l y star del c star del x star minus v w h over d del c star del y star is equal to del c star del y star square. Now we replace h over d by four equivalent diameter that we have already derived in the class earlier. So, it becomes three by sixteen u 0 d square by d l y star del c star del x star minus v w d by d times four del c star del y star is equal to del square c star del y star square now we identify that u 0 d square by d l is nothing, but a non dimensional number Reynolds schmidt d by l, when Reynolds define on equivalent diameter and v w d by d will be nothing, but the non dimensional parameter flux that is the e w. So, if we really put that in the governing equation let us see what.

(Refer Slide Time: 18:36)



You get will be getting 3 by 16 Reynolds schmidt d by l y star del c star by del x star minus 1 by 4 p w del c star del y star is equal to del square c star del y star square. So, this is my governing equation.

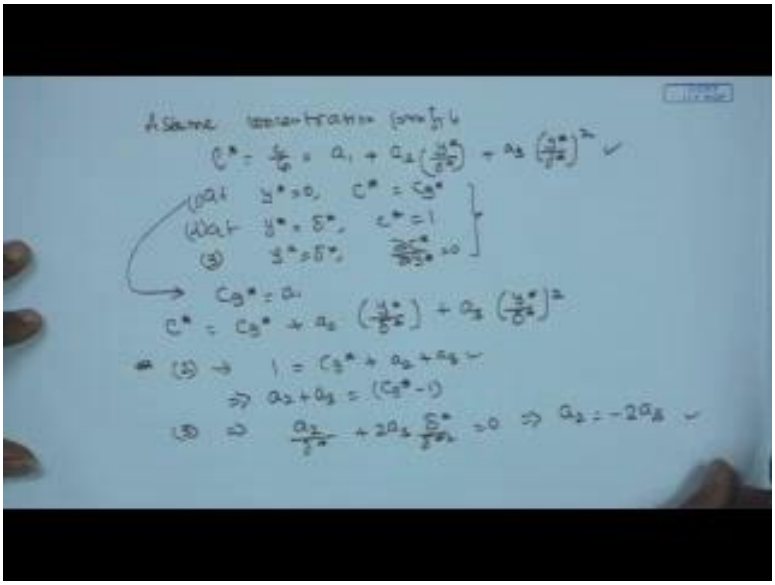
Now, let us see what are boundary conditions boundary condition would be at x is equal to 0 c star c is equal to c naught; that means, at x star is equal to 0 c star will be 1 at y is equal to 0 we will be having v w c plus d del c by del y equal to 0 the total flux was the membrane equal to 0 these are the convective flux to was the membrane diffusive flux was solute away from the membrane at y is equal to 0 c is equal to c g. So, if the boundary with the non dimensional form y star equal to 0 p w c g star, let us let us take a non dimensional that will be important v w c g star plus d by h del c star del y star equal to 0. So, this becomes v w h over d c g star plus del c star del y star equal to 0. So, will be having del c star del y star plus h is d by 4 1 by 4 p y by p w by 4 c g star is equal to 0. So, this will be the non dimensional form of the boundary condition at y star is equal to 0 and at y star y is equal to delta c is equal to c naught. So, therefore, at y star is equal to delta star c star is equal to 1. So, these three condition defines the boundary conditions for the governing equations and now will be will be demonstrating this problem how to solve using the integral method of solution.

So, it is also known as the approximate integral method of solution most of the all the boundary layer problems can be solved the approximate integral method why this method is called an approximate method. Because in this case we are going to assume a concentration profile which will be conforming the physical boundary conditions of the mass transfer boundary layer and then, we will be putting that particular profile into the governing equation and see what are the how the system behavior will evolve. Now where is the, why we are not doing a simulated solutions here we can do the problem by simulated solutions.

But we are doing the integral method analysis because it is another way of solving the boundary layer problems. The only advantage here will be getting is that if you remember that for the in the similarity solution. We have assumed that at y is equal to infinity c is equal to c naught because we are assuming that we are changing the condition from y is equal to delta to at y equal to delta c is equal to c naught to y equal to infinity c is equal to c naught. Because the delta is really very small and majority of the channel beyond the mass transfer boundary layer which will be equivalent to infinity c is equal to c naught.

But in this case we will be we can retain the boundary condition that at y is equal to delta c is equal to c naught. So, therefore, that will be an advantage of this of this method of solution, but the approximation is the approximated assumption of the concentration profile that will be prevailing within the mass transfer boundary layer. Now, let us try to find out what is that concentration profile.

(Refer Slide Time: 22:44)



We assume a concentration profile a parabolic 1 or second order c star is equal to c by c naught is equal to a 1 plus a 2 y star by del star plus a 3 y star by del star square of that, why we are taking a second order polynomial because, we know that three conditions these concentrations profile must satisfy what are the three conditions that first condition is that y equal to 0 c must be is equal to c g gel layer concentration at y is equal to delta the two conditions I was talking about in the in the earlier class that at least two conditions the boundary layer conditions must be satisfied by any kind of boundary layer, whether it is a hydrodynamic boundary layer thermal mass transfer boundary layer.

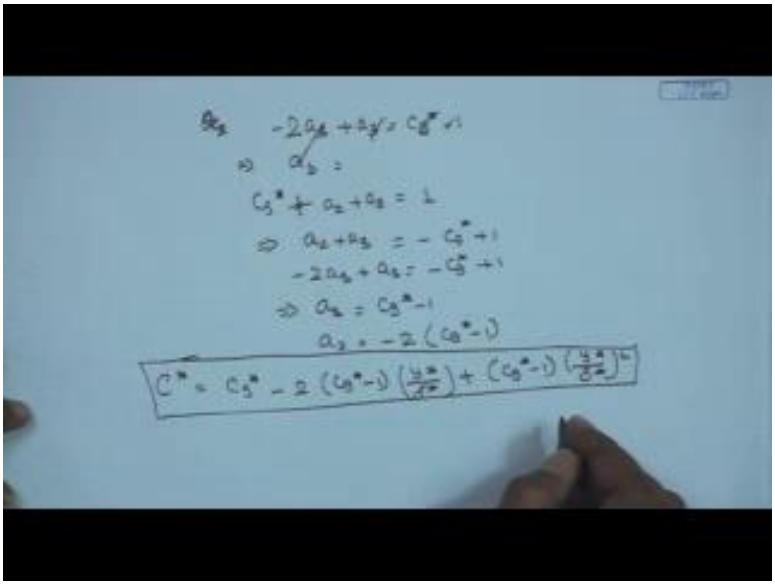
These are the conditions at the edge of the boundary layer at the edge of the boundary layer value of the depending parameter will be equal to the free stream condition and value and gradient of the dependent variable will be equal to 0; that means, at y star is equal to 0 my c star is equal to c g star that is c g by c naught and at y star is equal to delta star c star will equal to 1; that means, c is equal to c naught and y star is equal to delta star del c star del y star equal to 0.

So, I have 3 basic conditions that this mass transfer boundary layer should satisfy and therefore, I will be having 3 terms with three unknown coefficients a 1 a, 2 a 3. So, 3 times means it will be maximum a second order polynomial. So, that is why we have

considered a second order polynomial. So, next will be using these three boundary conditions in order to evaluate the unknown coefficients a 1, a 2 and a 3 once this coefficients are evaluated then, you will be getting a complete concentration profile within the mass transfer boundary layer.

So, let us do that, first boundary condition will give you c g star is equal to a one. So, the profile becomes now c star is equal to c g star plus a 2 y star by del star plus a 3 y star by del star square. So, now, let us put another boundary that at secondary boundary condition, this 1, this 2, this 3, where we utilized already 1. Now, from 2 will be getting 1 is equal to c g star plus a 2, a 3. So, a 2 plus a three becomes c g star minus 1 and from 3 will be getting del c star del y star equal to be 0, so a 2 at y star equal to del star. So, a 2 by del star plus 2 a three del star by del star square evaluated at del star. So, this will equal to 0. So, will be getting a 2 is equal to minus 2, a 3. So, therefore, using these 2 will be getting a 2 and a 3 and if you really do that. So, we will be getting a 2, a 2, I am writing minus 2 a 3 plus a three is equal to c g star minus 1.

(Refer Slide Time: 26:15)



So, therefore, you will be getting a three equal to. So, the first boundary condition we had c g star plus is equal plus a 2 plus a three is equal to 1.

So, a 2 plus a 3 would have been minus c g star plus 1 and a 2 is equal to minus 2 a 3. So, minus 2 a 3 plus a three is equal to minus c g star plus 1. So, therefore, a three will be c g star minus 1 once we get a 3. Then, we can get a two is equal to minus 2 a 3, minus 2 c g star minus 1. So, we completely estimate the concentration profile c star is equals to c g star plus a 2 a 2 is minus 2 c g star minus 1 y star by del star plus a 3 a 3 is c g star minus 1 y star by del star square of that. So, we have determined the concentration profile the parabolic concentration profile within the mass transfer boundary layer.

So, what we are going to do is that will be taking the derivative of this and inserting into the governing equation to replace the terms of the del c del x del y and del c square del y square from this concentration profile. Then will be inserting those into governing equation then will be integrating across the mass transfer boundary layer.

Thank you very much.