Introduction to Process Modeling in Membrane Separation Process
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Lecture – 14
Modelling of Gel Layer Controlling Filtration (contd.)

Welcome to this section. Now, as we have discuss in the last class that another typical physical phenomena occurring in membrane filtration is gel formation over the membrane surface. Now in the last class we have seen how the gel is formed over the membrane surface. There are two ways; one the filtration may be osmotic pressure control initially later on it can get transition into the gel layer control in phenomena. In second case there may be solutes which will be gel forming from the very beginning of the process like polyphenol alcohol (Refer Time: 00:54) and which are well known as gel forming agency.

So, in this class we will be looking into the various modelling approaches of the gel layer controlling filtration.

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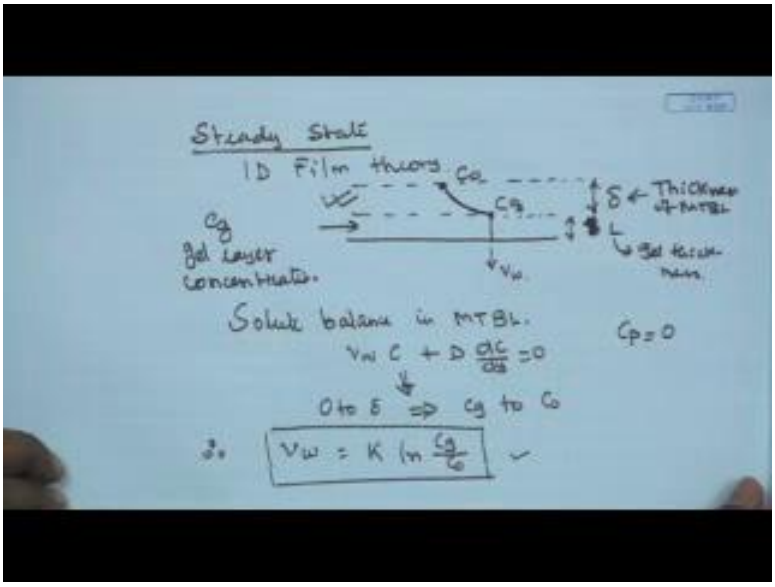

So, first one will be looking into the steady state model. And this will be same as one dimensional film theory model that we have considered in case of osmotic pressure control filtration. We assume a constant film of solutes of thickness delta is formed over the membrane surface and it will be having a solute concentration within the gel. So, cg

is known as the gel layer concentration and at the steady state there will be a constant growth of mass transfer bounder layer over this. These will be actually the delta the mass transfer bounder layer and these will be the gel thickness, so this is mass transfer bounder layer is a gel thickness. And we assume that the solute will be suffering a concentration gradient or polarization from the bulk at C naught to gel solution interface that is gel concentration. Then the concentration of gel remains same within the gel layer then permit you will be getting the permeate flux.

Now at the steady state we can write down the solute balance within these mass transfers boundary layer. If you write a solute balance in mass transfers boundary layer then will be getting Vw C plus D dc dy will be equal to 0. There is no solute concentration in the permeate stream in this case because in most of the cases the gel forming material are larger in size, they will be having bigger sizes and the larger molecular weight and typically they will be return by the membrane surface and the permit concentration is 0 in most of the cases.

So, if you really carry out this integral from 0 to delta with the variation of concentration from cg to C naught then we will be getting Vw is equal to K ln cg by C naught, so this is the steady state equation of the permeate flux in the gel polarization case. And mass transfers of coefficient can be estimated depending on the domain of the flow domain as well as the whether it will be a laminar flow and turbulent flow and or whether you will be working with a tubular module or whether you with a working with a flat sheet geometry or in a hollow fibre module.

So, depending on the module I realise the flow domain mass transfers coefficient will be evaluated whether it is a start cell, whether in a start cell unless than Reynolds from the less than 32000 about 32000. All those correlations of mass transfers coefficient we have already discuss earlier those are coming from the heat and mass transfers analogy will you valid. And depending on the flow domain and the flow geometry one will be estimating the mass transfer coefficient. Once the mass transfers coefficient will be estimated the gel layer concentration is known to you then one can estimate the permeate flux at the steady state.

So, what is the typical parameter in this case? The typical parameter in this case is the gel layer concentration; how to evaluate the gel layer concentration.

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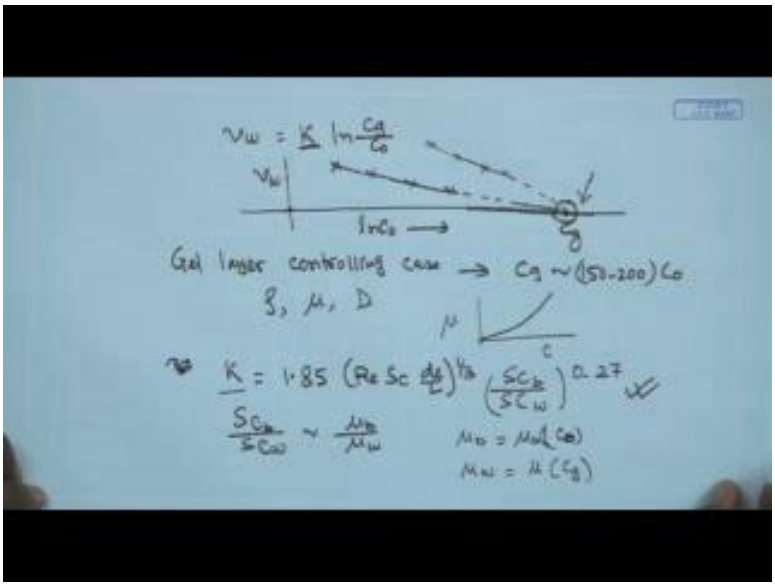

So, it is very easy to evaluate the gel layer concentration. The governing equation for calculating or estimating the gel layer concentration cg is this steady state equation. We conduct the experiment at different concentration of the feed solute and then measure the permeate flux at a particular K, K means at particular turbulence. So, we plot them Vw versus feed concentration in a lock skill, this will be a lock skill ln C naught. And then will be seeing that permeate flux will be decreasing as the feed concentration increases. So, it will be a straight line and we extra pull it this line and the gel layer in y equal to a where you cuts x axis that concentration is the gel layer concentration.

So, when C naught is equal to cg ln 1 will be equal to 0 so you will be getting the g o value of flux layer. But in an actual experiment you will be always getting some finite value of flux, so therefore this line will be extra polluted and we will be getting the gel layer concentration. If you conduct the same set of experiment with another value of turbulence or the mass transfers coefficient then you will be getting another. But if you extra pollute that will boil down into the same value of cg. That is how the gel layer concentration can be obtained experimentally and this parameter can be estimated.

So, once you estimate the parameter the gel layer concentration can be put, you know the unknown value of for a known value of C naught then you could be getting the value of permeate flux knowing the estimating the mass transfers coefficient. But remember in case of gel layer control filtration control in case cg must be very very high and

sometimes cg is around may be as high as 150 times to 200 times of C naught. That means, in a gel layer control filtration the concentration suffers maximum from within the mass transfer boundary layer from C naught to cg and sometimes cg can be as high as 150 times to 200 times of feed bulk concentration.

Therefore, variation of the transport coefficients will be extremely high in case of gel layer control filtration compare to the osmotic pressure control filtration. As we have discussed there are three major parameters which will be appearing in your system are the transport coefficients; one is a density, viscosity and diffusivity. Density is the weakest function of concentration, diffusivity is a little bit stronger, and viscosity is the strongest function of concentration. And typically viscosity varies exponentially concentration.

So therefore, correction by Sieder Tate correction is very very important in case of the gel layer control filtration. And one can get a correction factors something like this 1.85 Renyold Schmidt d e by L for the laminar flow in a rectangular channel Schmidt at bulk divided by Schmidt at the wall; so rise to the power 0.27. Sometimes this Schmidt number ratio can be approximated as mu at bulk divide by mu at wall, because if variation of diffusivity and density are minimal then it will be measure mainly the variation of viscosity. And mu at bulk means mu is evaluated at bulk concentration that is C bulk or C naught. As we have discussed C naught is the bulk concentrate in this case and mu at wall will be nothing but mu evaluated at gel layer concentration.

Then as we have discussed earlier that since this value is viscosity is increasing with concentration and C gel is always greater than C naught the denominator will be always greater than numerator and we will be and under (Refer Time: 09:35) you will basically the value of mass transfers coefficient will be decreased, because of the variation of the properties in the mass transfers bounder layer.

Hence, one has to invoke the Seider Tate correction factor in order to estimate mass transfer coefficient in case of gel layer control filtration, because here the viscous the concentration variation is a maximum. Therefore leading to the maximum variation of the thermo physical or you know transport coefficient or transport properties. So, this is a must in case of this correction factor, is a must in case of gel layer control filtration compared to the osmotic pressure control filtration.

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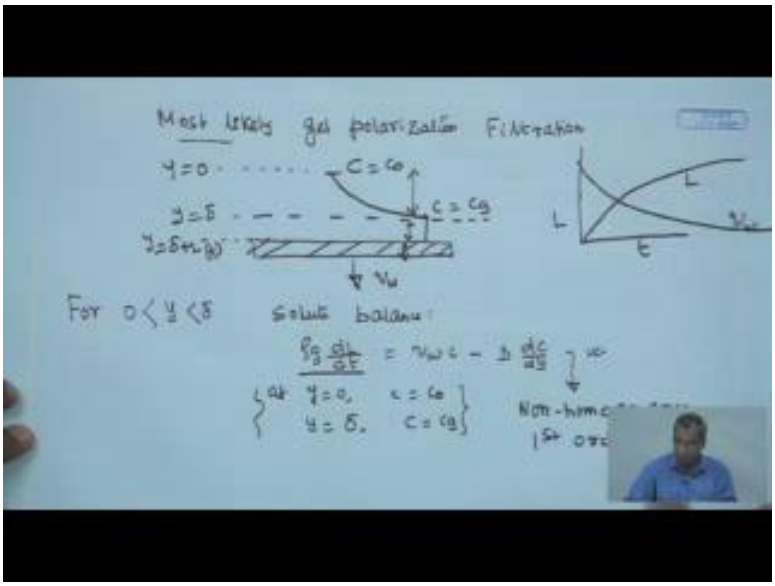

Now, next will be looking into a most likely gel polarized filtration cased filtration; so what is that? We will be having a membrane, there is a formation of gel over it and over their there will be a formation of mass transfers boundary layer because of the external flow. So, C is equal to C naught here, C is equal to cg here and cg remains same in the gel layer. And if you put your axis at y equal to 0 these will be y is equal to delta, but delta is a thickness of the mass transfers boundary layer and then these will be y is equal to delta plus l, where l is the thickness of gel layer at any point of time. And then you will be getting a permeate flux.

As time progresses these gel layer grows in the white direction and your thickness which will be essentially function of time. Therefore, this thickness l it increases with time, so if you look in to the value variation the gel layer thickness grows with time and then it will be reaching at steady state probably because of the existence of external flow. And as you have seen that there will be three resistances acting in random in this case; one is the mass transfer boundary layer resistance. Mass transfers resistance another is the gel layer resistance another is the membrane resistance.

So, these three resistance acting in tandem here and since the gel layer resistance is increasing as a function of time here, and these membrane resistance will be remaining constant and mass transfers boundary layer will be also will remaining constant and then you will be getting a constant decrease in permeate flux. This is how the permeate flux

varying with time, this is how the gel grows in time. When gel layer thickness becomes constants steady state permeates flux also becomes steady state.

So, we are talking about a system or let us say moving boundary, because l is growing as a function of time the value of l is increasing so if the boundary is moving, we are talking about a moving boundary system. Now, we write down the mass balance equation between a within the mass transfers transfer boundary layer. So, solute balance equation we write, so these becomes rho g dl dt this is the accumulation term is equal to Vw c minus d dc d y. And the boundary condition it should satisfy is that at y equal to 0 C is equal to C naught and evaluates at y equal to delta C is equal to cg.

So, within the two boundaries we evaluate this. And if you look into this type of this equation, now here l is a soul function of time soul function of t so l is a soul function of time and C is a function of y alone. Because C is the concentration that is solute concentration occurring in the mass transfers boundary layer. And mass transfers boundary layer is entirely depending on the hydrodynamics of the system. So, I can do this integration over y keeping left hand side is constant, so this can be treated as constant.

Therefore, if you can identify this equation this is a first order ordinary differential equation, but it is a non-homogenous first order ordinary differential equation and the non-homogenous term is basically these term rho g dl dt, rho g will be treated as constant during the integration over y.

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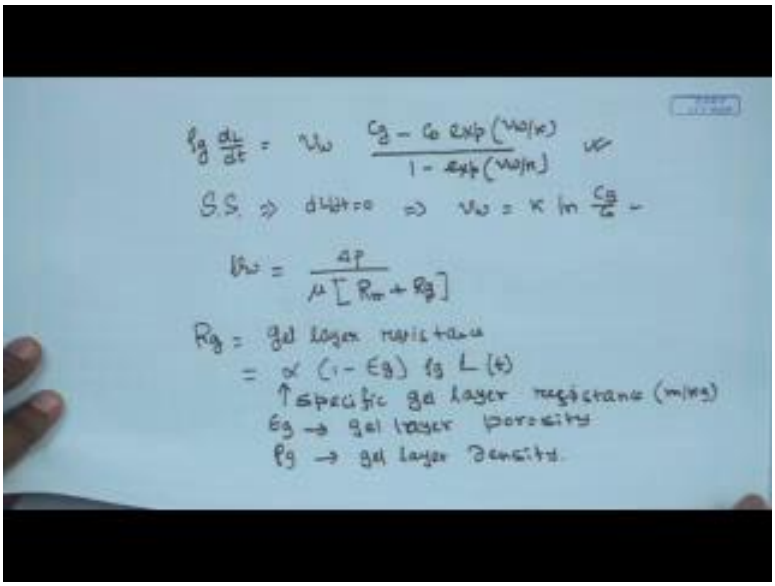

So, if you really you know carry out this integration then ultimately you will be getting rho g dl dt is equal to Vw cg minus C naught exponential Vw by K 1 minus exponential Vw by K. If you look into the steady state at the steady state dl dt will be 0 and will be getting back you steady state solution Vw equal to K ln cg by c naught. So, these will be the governing equation of the transient value of the gel layer thickness as well as the permeate flux. Now if you look into the expression of permeate flux as a resist from the phenomenological point of view this will be nothing but the driving force divided by the resistance. So, there will be two resistances Rm plus Rg, the gel layer resistance and membrane resistance and the mass transfers resistance is already taken care of in the mass transfer coefficient.

So, what is Rg? Rg is known as the gel layer resistance. What is gel resistance? This is nothing but alpha into 1 minus epsilon g rho g times l. What is alpha? Alpha is called as the known as the specific gel layer resistance, it is a gel characteristic is known as the specific gel layer resistance, it has a unit metre per kg. Epsilon g is the gel layer porosity; rho g is gel layer density. Typically this gel layer density slightly higher than the water density typical around 1015 1100 1050 kg per metre to like that. And l is the gel layer thickness which will be essentially a function of time.

So therefore, since l is a function of time rest all are gel characteristic Vw this Rg will be function of time. Since l is an increasing function of time Rg will be also increasing function of time leading to decreasing in flux as a function of operation.

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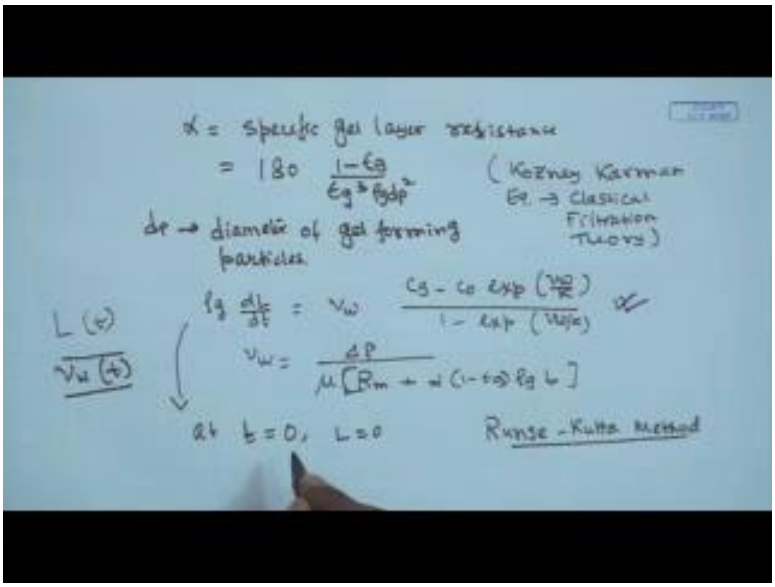

So, what is alpha? Alpha is specific gel layer resistance and these will be obtained from the Kozeny Carman equation as 180 1 minus epsilon g divided by epsilon g cube rho g dp square. So, this comes from Kozeny Carman equation classical filtration theory. So, rho g is the gel resist gel layer density, epsilon is the gel porosity, dp is the diameter of gel forming particles. Sometimes we are talking about the filtration of the polyvinyl alcohol the polymers, sometimes we are talking about the filtration of the fruit juice involve in pectin, but this polysaccharides or polymers they will be basically not be forming the heart spheres having a definite particle diameter dp. So, they will be forming a very viscous network over the membrane surface forming which we will be calling as gel.

Now, dp means it will be forming a gel layer which will be equivalent to the resistance offering by particles of averaged diameter dp, the it will be interpreted that way the Kozeny Carman specific gel layer resistance will be interpreted like that, Although you may not be having exactly diol defined particles of diameter dp, it will be the resistance that will be offered by the polysaccharides of polymers which will be a natural gel forming agent compared. Basically, they will be offering a gel layer resistance which

will be equivalent to a particle having a equivalent diameter dp. So, that will be the interpretation of the specific gel layer resistance.

Now if you look into the governing equation. We have the governing equation of gel layer thickness rho g dl dt is equal to Vw cg minus C naught exponential Vw by K divided by 1 minus exponential Vw by K and dl and Vw is equal to delta P by mu Rm plus Rg; Rg is alpha 1 minus epsilon g rho g times l, where l is a function of time and the governing equation of l is here. And these equation as to be solved at t is equal to 0, l is equal to 0. That means, at the starting there is no gel layer thickness.

So, this is the ordinary differential equation but these cannot be solved because Vw will be appearing in the right hand side. There are three (Refer Time: 20:46) places and Vw will be essentially a function of l. So, you will be getting an ordinary differential equation dl dt with a initial condition t equal to 0, l equal to 0. One can take request to the Runse-Kutta method to solve this equation as a function of time. After solution we will be getting l as a function of time. Once you get a get l is a function of time insert here so you will be getting the permeate flux profile as a function of time.

So, this is how the gel layer control filtration will be model in an actual scenario. But if you must have understood that there are several parameters those are appearing in the gel layer control in filtration. Next what will be looking into this how to estimate these parameters? Once these parameters are estimated then only I will be able to solve the governing equation for the gel layer control in filtration.

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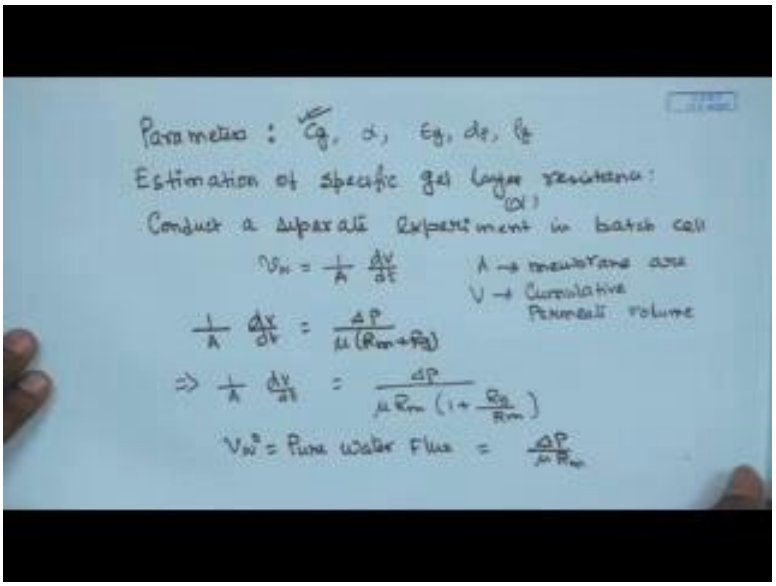

Now, let us look into how various parameters are estimated. Now first will let us list down what are the parameters those are appearing in the gel layer model. One is the gel concentration, and I have already seen how the gel concentration is estimated. And specific gel layer resistance alpha epsilon g dp epsilon g gel porosity dp is a gel forming particle and rho g is the gel layer density. So, the estimation of gel layer concentration I have already done. Now let us look into the how to estimate the specific gel layer resistance alpha. For that we have to conduct a separate experiment in a batch cell batch cell. And we develop a theory for permeate flux profile how to obtain in a batch cell. Then we will see how the specific gel layer resistance can be estimated quite easily by conducting a separate set of experiment in the batch cell.

So, in a batch cell if you write the expression of Vw permeate flux it will be nothing but 1 over A dv dt. Where, A is the membrane area or filtration area, V is the cumulative permeate volume. We can write 1 over A dv dt is equal to delta P divided by mu Rm plus r g, so this is also known as the series model; so two resistance there are acting in series. So, 1 by A dv dt will be is equal to delta P by mu I take Rm outside so these becomes non-dimensional Rg. What is Vw naught? This pure water flux that is nothing but delta P by mu Rm, if there is no solution only pure water there is no osmotic pressure, there is no gel layer resistance nothing so we will be getting delta P by mu Rm or l p delta P. So, this is a pure water of flux.

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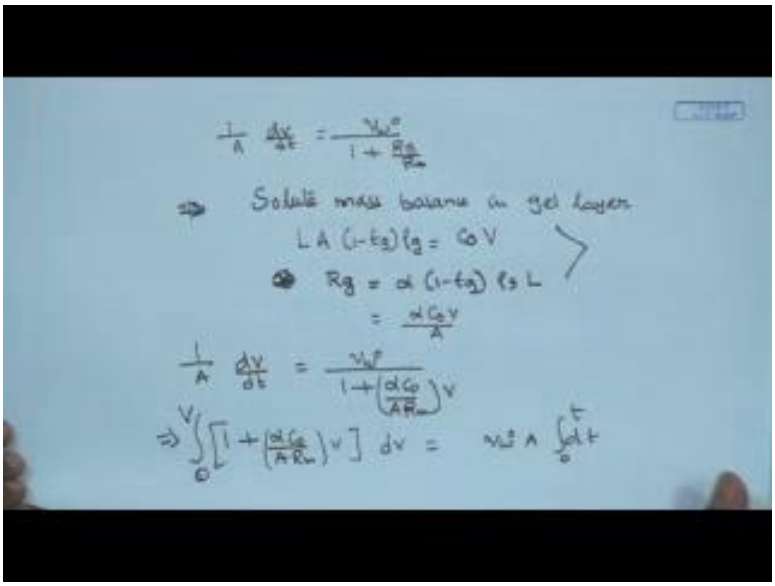

So, we can write 1 over A dv dt is equal to Vw naught divided by 1 plus Rg by Rm. So, next we write down a solute mass balance in gel layer. If you really do that this becomes L A 1 minus epsilon g rho g terms C naught terms V, V is the cumulative volume of the filtrate. At a particular time this much volume has been filtered out so that will be carrying C naught concentration of the solute. So, total mass that will be deposited as the gel is C naught terms v. In terms of the gel properties it will be gel layer thickness multiplied by l cross section that will be a gel layer volume multiplied by 1 minus epsilon g there will be actual material that is there in the gel layer multiplied by gel layer density is the amount of gel that is presented.

So, Rg is basically as we have written earlier Rg is alpha 1 minus epsilon g rho g terms L, so by combining this two we can get Rg is equal to alpha C naught V by A. Once we get that we can write the governing equation of cumulative volume as 1 over A dv dt is equal to Vw naught divided by 1 plus alpha C naught by A Rm terms V. So, we take it on the other side and do the integration, so it will be 1 plus alpha C naught divided by A Rm times V dv is equal to Vw naught A dt. Where you integrate from 0 to t and these will be integrate on left hand side from 0 to V. So, let us write down what do we get after the integration.

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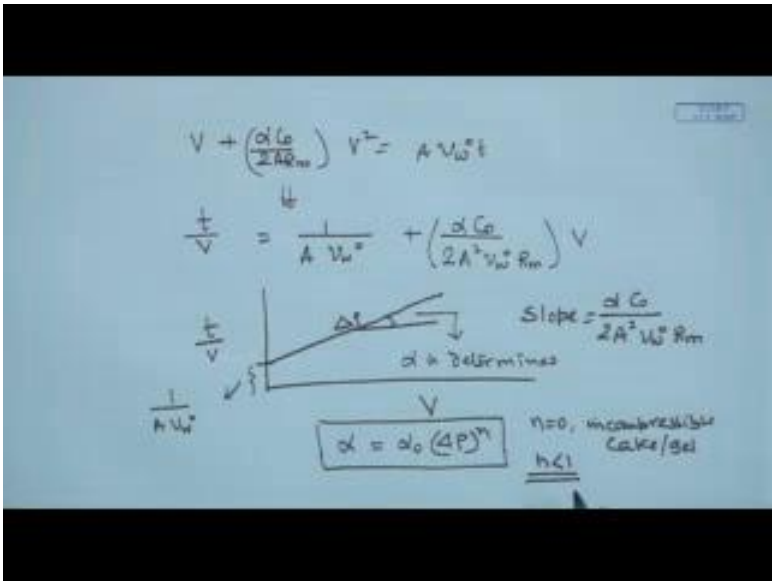

After the integration we will be getting V plus alpha C naught divided by 2 A Rm times V square A Vw naught dt. And then we do a rearrangement to make it linear form of equation t by V is equal to 1 over A Vw naught plus alpha C naught divided by 2 A square Vw naught Rm terms V. Then what we do? We measure the cumulative volume we conduct batch cell experiment and measure cumulative volume as a function of time, and then plot t by V as a function of v so y equal to m x plus c; so we will be getting value these for a particular delta P.

So, from the intersect will be getting 1 over A Vw naught that we can verify and from the slope we can get alpha C naught divided by 2 A square Vw naught Rm. So, if you remember that C naught is known to us there is a fit concentration membrane resistance is known to us, Vw naught is known to us that is delta P by mu Rm so that is also known to us. Area is the membrane area so membrane filtration areas as well known to us. We know everything in this case slope we have determine, so will be getting the value of alpha.

So, alpha is determined from the slope. Now we conduct the experiments for different pressure drop and then estimate the value of alpha then put a correlation of alpha is equal to alpha naught delta P to the power n. So, if n is equal to 0 then the cake or gel is incompressible; n is equal to 0 incompressible cake or gel. Otherwise the gel is compressible and typically the value of n is less than 1; generally it is compressible. So, it gets compressed its epsilon g or the porosity decreases as we increase the trans

membrane pressure drop. So, this is how the parameter specific gel layer resistance is estimated as in case of gel controlling filtration.

In the next class, we will looking into the estimation of gel layer porosity gel layer particles size equivalent particle diameter for a typical polymeric or polysaccharide gel and gel density. Then that will completely windup the filtration of gel controlling membrane separation system.

Thank you very much.