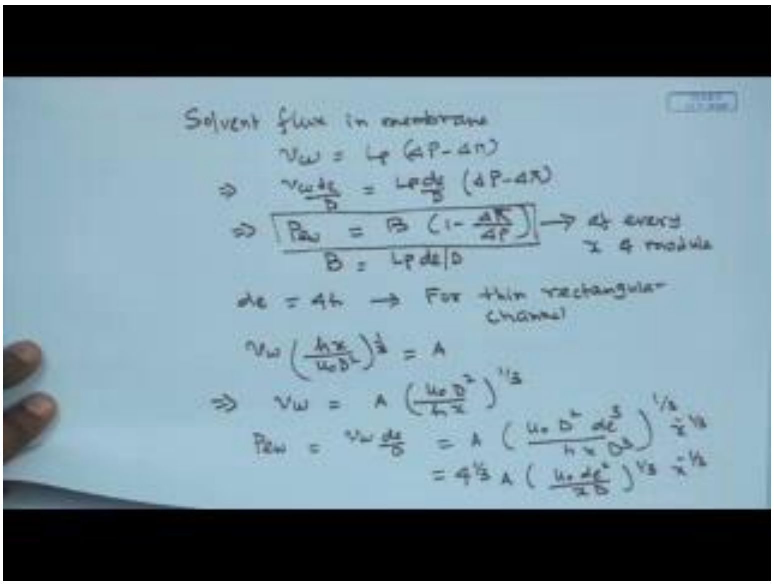
Introduction to Process Modeling in Membrane Separation Process

Prof. Sirshendu De
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 12
Osmotic Pressure Controlling Filtration (Contd.)

So, welcome back to this session of the class. So, we will be basically now looking into the solution of cross flow filtration system, these are only valid for reverse osmosis ultra filtration as well as the micro filtration. We are looking in today two-dimensional mass transfer boundary line analysis. And in the last class, we have seen how the concentration profile in the mass transfer boundary line will be solved and then these concentration will be evaluated at y equal to 0, you have to get an idea of c m where is the membrane surface concentration and how it varies with the hydrodynamic of the system. And then once we get that then we will be looking at up with the transport loss to the lost membrane.

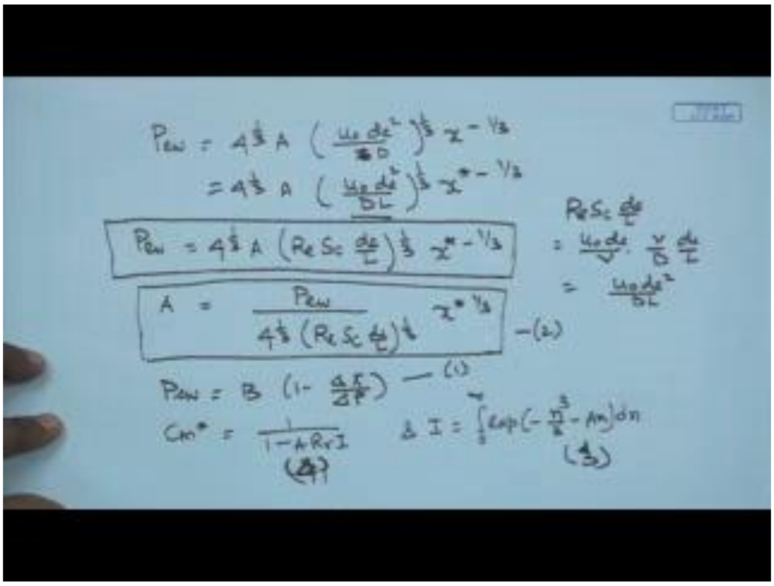
(Refer Slide Time: 01:04)



Let us look into that if you look into the solvent flux in the membrane. In membrane, we get v w is equal to L p del p minus del pi and we make it one-dimensional by defining v w de, we multiplied both side by d by d so it will be L p d by d delta p minus delta pi. So, therefore, these will be nothing but one-dimensional permeate flux P e w is equal to let us say this parameter is b these in so we write everything in the form of one-dimensional parameter. So, p w is v w d by d is one-dimensional, it is a (Refer Time: 01:56) number it is basically one-dimensional permeate flux and b is a one-dimensional parameter L p d by D. So, these equations must be valid at every location of module or channel. So, this will be the equation that will be satisfied everywhere.

Now, let us look into the other equations so if you remember that what is the equivalent diameter we have we have defined earlier that equivalent diameter is nothing but four of aside for a thin rectangular channel. Now, if you remember that we have defined v w as v w h x divided by u 0 D square to the power 1 upon 3 is equal to A. And let us write it take it on the other side a u 0 D square divided by h x to the power 1 upon 3. Make it one-dimensional, so this becomes P e w is v w d e by D, so A u 0 D square d e cube and h x D cube to the power 1 upon 3. And we will just put h is equal to d e by 4, so this becomes four to the power 1 upon 3 A u 0 this D will be cancelling out d e square divided by x D to the power 1 upon 3 . So, now, if you convert this into Reynolds and Schmidt number, this will turn out to be this will be raise to the power x to the power 1 upon 3 on the other side, x to the power minus 1 upon 3.

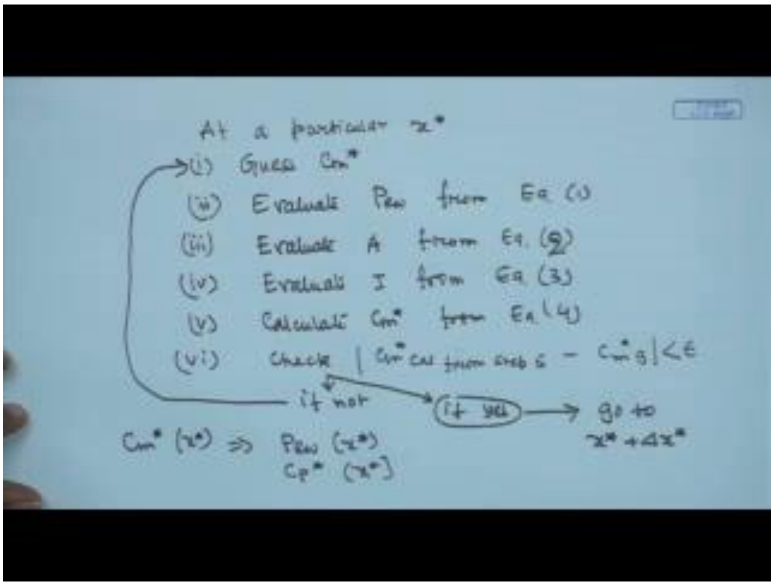
(Refer Slide Time: 04:27)



So, now, let us put it in a one dimensional form, so P e w will be to the power 1 upon 3 A u 0 d e square by x D. In this we it is x is already included so you can take x out of this so it becomes if we make it make x one-dimensional then this becomes this x will be outside. So, x to the power minus 1 upon 3, so 4 to the power 1 upon 3 A u 0 d e square by S times L or L is the channel length this becomes x to the power x star minus 1 upon 3 there will be a square. Now, what is this number d is again a one-dimensional number this Reynolds-Schmidt d e by L. If you can open it up u 0 d e by nu, nu is the nu by rho so this nu by d this is d by l this becomes u 0 d e square by D L. So, this becomes P e w is now becomes four to the power 1 upon 3 A Reynolds-Schmidt d e by L to the power 1 upon 3, x to the power minus 1 upon 3.

So, this will be the relationship, how permeate flux is varying as a function of x and one can get an expression of a now as P e w divided by four to the power 1 upon 3 Reynolds-Schmidt d e by L to the power 1 upon 3 x to the power 1 upon 3. So, now, let us write down what are the different equations we are having different equations we are having is they will be expression of P e w will be osmotic pressure model b times one minus delta pi by delta p. And you will be having the expression of i and C M star will be one over one minus A R r I and what is the expression of I, I is 0 to infinity exponential minus eta cube by 3 minus a eta d eta. Now, let us write down the different equations number 1. This will be then 2, this will be 3, and this will be 3 and this will be 4.

(Refer Slide Time: 07:52)



So, let us write down the so these four equation. Now let us see how we do the computation of the system performance and predict the system performance. Now, next step is we will be doing a you know numerical calculation. So, at a particular x location at a particular x star we guess a value of C m star guess CM star; once CM star is obtained then is guessed then from equation one evaluate P e w from equation number one this is the osmotic pressure relationship. Once P e w is known then one can get the value of A evaluate A using the P e w at that particular x location from equation 5.

So, what is our equation 5? In the previous slide, we write this is the equation 5. So, once we get the value of A, we will evaluate a from so 5 is not required this is obtained from there only. So, you will be getting the value of a from equation 2, so because we have obtained the value of P e w from the osmotic pressure of model. We insert that here Reynolds-Schmidt d e by L at the operating conditions d e by L is geometric factor, 4 is a numerical value, x star we are evaluating at the particular x-location.

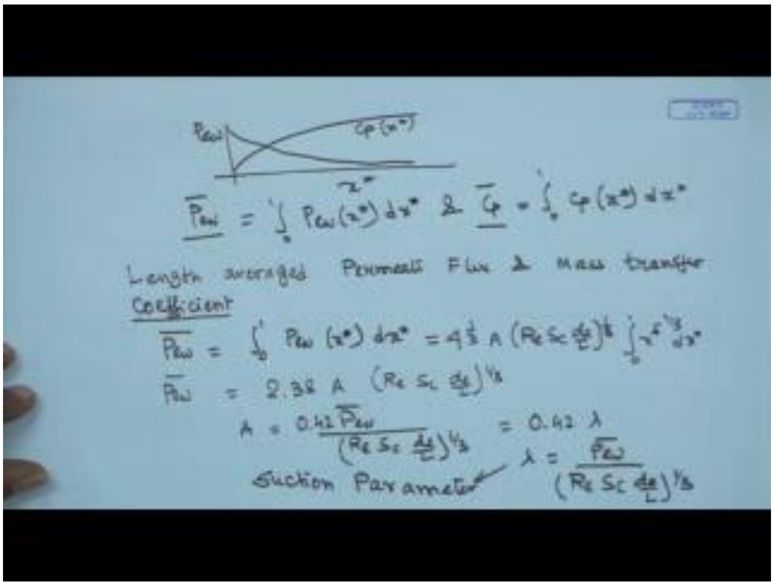
So, therefore, we will be evaluating A from equation number 2. And once you evaluate a then we are in a position to evaluate the definite integral I from equation 3. So, once we evaluate the definite integral I from equation 3, then from we can we can evaluate you can be recalculate calculate C m star from equation 4. So let us look into let us go back to the equations at a at a particular x location we have guessed the value of C m star. Once you guess the value of C m star, we can evaluate P e w because delta pi will be function of C m star only all the other parameters are known to us. So once P e w is evaluated, we can get back to equation 2 and evaluate the value of A; once we evaluate the value of C, we will be getting back to equation three and we will be evaluating the value of I. And this integration has to be evaluated numerically using a trapezoidal rule or a Simpson’s rule.

So, this infinity will be replaced by a higher value let say 5 or 10, so let say put a value of 10 and evaluate the value of integral then we will be putting another value of 15 or 20 and we will be evaluating the value of I numerically using trapezoidal, Simpson. If the answer of the two values will be different will be different in the decimal places let say one or two decimal places then we can take the infinity as the value of 10. So, once we evaluate the value of i we have already evaluate the value of A then you can calculate C m star from equation four and then we will check whether this C m star is coming close to the guessed value or not.

So, six is check absolute of C m star calculated from step five minus C m star guess is less than epsilon. This epsilon can be a small number. If it is not then we have to guess the value of Cm. So, if not then we have to guess another value of C m and carry out this loop. If yes then we have to again go to x star plus delta x star. The next x location and redo this calculations iteratively.

So, in the process, what we will be getting, so there will be two loops one will be the inner loop where we will be calculating the C m star from the algorithm at a particular x location then we will be going to the next x location x plus delta x and redo the calculation iteratively. And likewise ultimately what you will be getting ultimately you will be getting C m star as a function of x star. So, once you know the C m star as a function of x star then you will be getting the P e w as a function of x star and C p star as a function of x star.

(Refer Slide Time: 12:45)



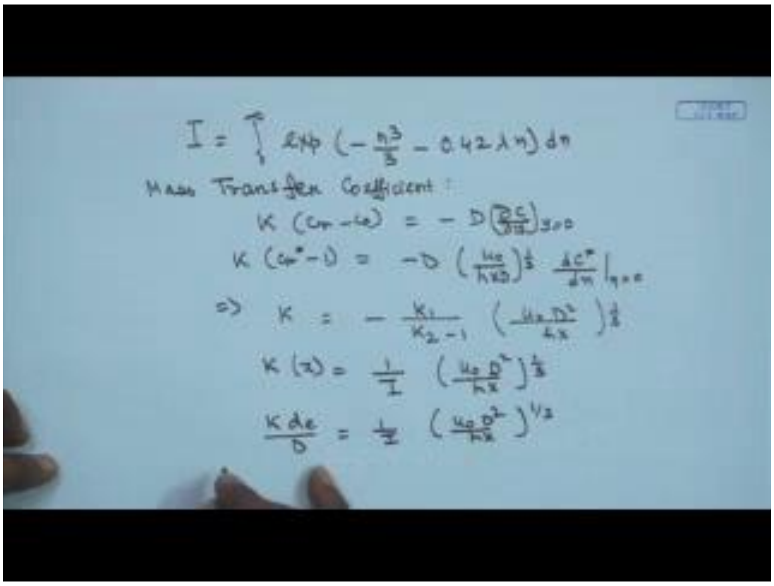
So, we will be getting the profile of permeate flux non dimensional permeate flux as a function of x star and permeate concentration as a function of x star. And then these will be again left to do a Simpsons rule and doing a length averaging, so average length average permeate flux will be nothing, but 0 to 1 P e w x star dx star, and C p length average permeate concentration will be 0 to 1 C p x star dx star. So, therefore, one will be getting the when the length of which permeate flux and permeate concentration and get you can get the system performance as a function of operating conditions so this way the osmotic pressure control can be can be predicted to a to a completely a can be can be utilized completely predictive way.

So, for a well defined system when where we know the solute diffusivity solute the geometry of the system perfectly everything is known then this osmotic pressure control filtration model can be utilized to get a system performance. Now, let us go a little bit ahead. And let us look into the mass transfer coefficient and then we will see how this mass transfer coefficient will be estimation of mass transfer coefficient will be helping us in reducing the rigor of calculation further.

So, let us look into the length average permeate flux, and evaluation of mass transfer coefficient. So, length average permeate flux will be given as we have described here 0 to 1 P e w x star dx star. So, we have the expression of P e w as the function of x star. we insert there that and see how much we are getting 4 to the power 1 upon 3 A Reynolds-Schmidt d e by L to the power 1 upon 3 0 to 1 x star to the power minus 1 upon 3 dx star. And ultimately, we will be getting as P e w bar is equal to 2.38 A Reynolds-Schmidt d e by l to the power upon 3. So, we will be getting the expression of A as a function of length average permeate flux as A is equal to 0.42 Reynolds-Schmidt d e by L to the power 1 upon 3. So, we call this as lambda 0.42 lambda where lambda is equal to P e w bar divided by Reynolds-Schmidt d e by L to the power 1 upon 3. This lambda is also known as the suction parameter.

It is a non-dimensional suction parameter. It indicates that that wall is really porous. And if the wall is impervious P e w equal to 0 and 1 will the suction parameter will be zero. So, therefore, for an impervious conduit when there is no wall porosity P e w that will be indicated by P e w equal to 0. On the other hand, if we have wall suction, there will be definite values of P e w bar the length average permeate flux, and there will be definite value of lambda not equal to 0. So, in case of no suction, there is no membrane place is placed in the conduit in the flow conduit lambda will be equal to 0 in presence of membrane lambda will be assuming a finite value.

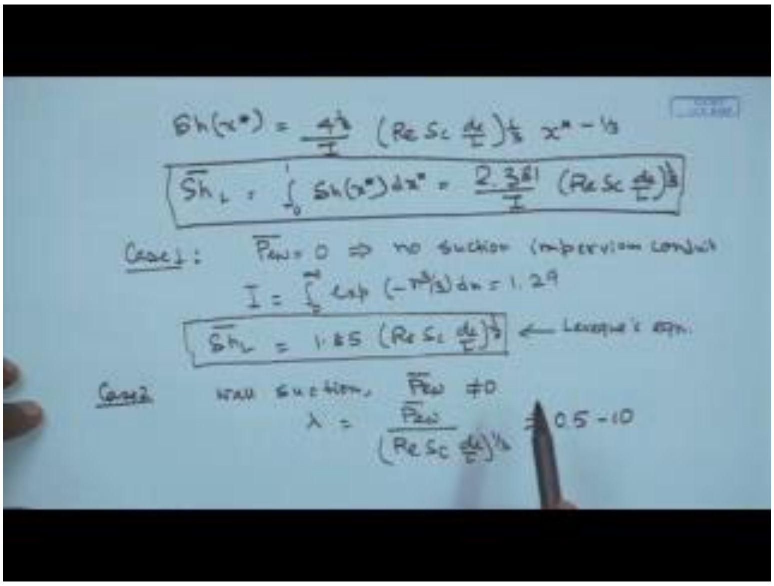
(Refer Slide Time: 17:00)



So, next we will be looking into the expression of mass transfer coefficient. So, if we go back to the expression of the indefinite integral 0 to infinity, the definite integral infinite integral 0 to infinity as exponential minus eta cube by 3 minus A eta. So, I am replacing A by 0.42 lambda eta d eta. And now we define the mass transfer coefficient as k C m by C naught is equal to minus D del c del y at y equal to 0 that will be a typical definition of fluid mass transfer coefficient and k comes. So we make it one-dimensional k C m star minus 1 is equal to minus d we replace del c del y in terms of one-dimensional in terms of similarity parameter eta. So, this becomes u 0 h x D to the power 1 upon 3 d c star d eta evaluated at eta equal to 0.

Now, if you really do that and we have already seen that C m star is equal to nothing but the parameter K 2 and this. This is the d eta at eta equal to 0 will be the integral constant k 1. So, mass transfer coefficient can be replaced as minus K 1 divided by K 2 minus 1 u 0 d square divided by h x to the power 1 upon 3 and k, as if you will be if you replace the value of k 1 and k 2. This becomes 1 over I u 0 d square h x divided raise to the power 1 upon 3. Now, we will be defining the Sherwood number in terms of K d e by D, and this becomes the Sherwood number becomes 1 over I u 0 d square over h x to the power 1 upon 3.

(Refer Slide Time: 19:36)

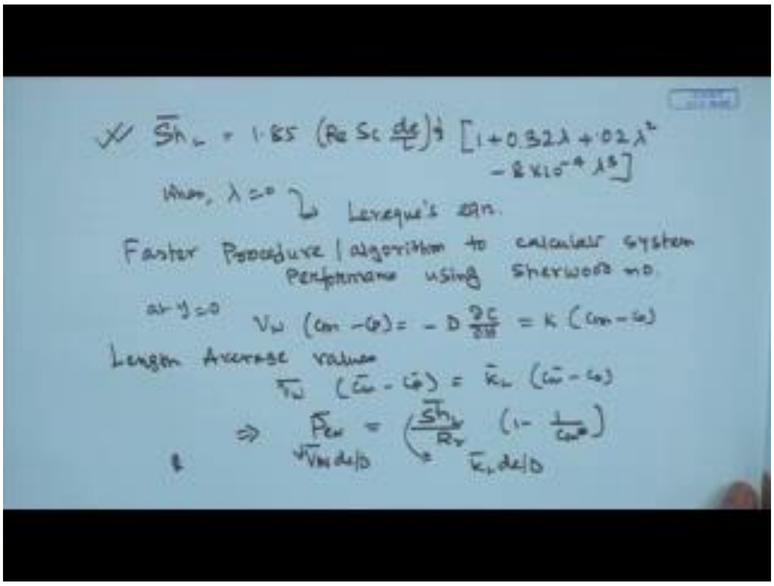


And if we now replace everything in the non-dimensional form in the non-dimensional form this becomes Sherwood as a function of x star is 4 to the power 1 upon 3 I Reynolds-Schmidt d e by L to the power 1 upon 3 x to the power minus 1 upon 3. So, this factor 4 comes from conversion of (Refer Time: 20:01) into the equivalent diameter and L comes from one-dimensional version of x star x star is defined as x by L. So, we can get the length average Sherwood number, length average Sherwood number will be nothing, but 0 to 1 S h x star d star and these becomes 2.381 divided by I Reynolds-Schmidt d e by L to the power 1 upon 3. So, this is the expression of Sherwood number length average Sherwood number in the case of membrane channel where the walls are really porous.

Now, let us look into the various you simplified cases. For case 1, P e w equal to 0, then P e w bar equal to 0 that means, there is no suction and impervious conduit. Now, if we can really find out the expression of I, now this will become zero to infinity exponential minus eta cube by 3 d eta this becomes 1.29, and if you really put it there then average Sherwood number becomes 1.85 Reynolds-Schmidt d e by l to the power 1 upon 3. Now, if you can you know recognize this equation; this is nothing but the Leveque's equation so for that is coming from the heat and mass transfer analogy for the impervious conduit.

So, we get back to the Leveque's equation, if we put wall suction is equal to 0 or the suction parameter lambda is equal to 0, when there will be no permeate flux coming out of the system. So, next case - case 2, when wall suction is not equal to 0 P e w bar is not equal to 0 then that means, that you will be having a membrane channel an into the system. So, what we will be doing, if you look into the value of lambda, lambda is nothing but P e w bar divided by Reynolds-Schmidt d e by L to the power 1 upon 3. For a typical value of lambda in membrane, system for microfiltration system P e w bar will be will be quite high. So, in that case we will be having a high value of lambda. In case of reverse osmosis, P e w bar will be quite less, so we will get less value of lambda and typically this lambda rise from 0.5 to 10.

(Refer Slide Time: 23:03)

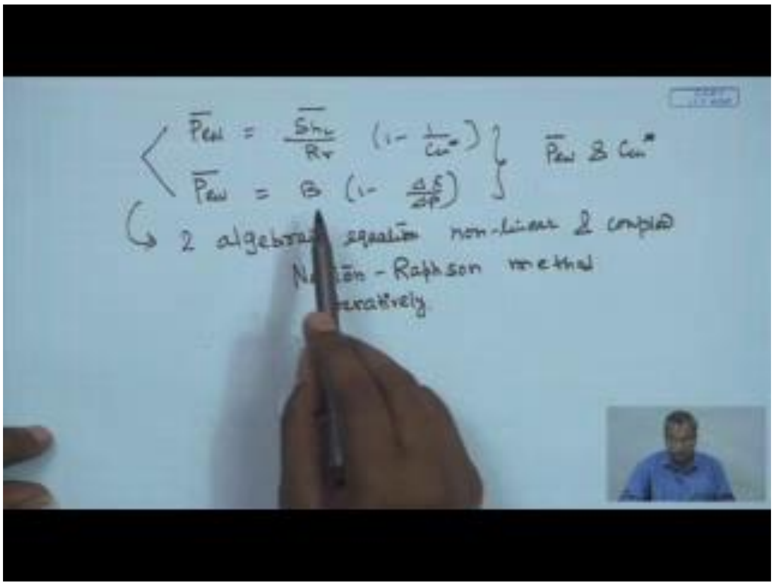


Next, what we will be doing, we will be evaluating the integral I as a function of lambda numerically, and then we fit a curve of 1 over I as a function of lambda. If we really do that then we will be getting Sherwood length average Sherwood is equal to one point eight five Reynolds-Schmidt d e by L to the power 1 upon 3 1 plus 0.32 lambda plus 0.02 lambda square minus 8 into 10 to the power minus 4 lambda cube. Now, this will be the expression of Sherwood number in a membrane channel when the walls are really porous and now this these expression if we put the no suction case no suction means when lambda equal to zero when lambda equal to zero we will be getting down getting back the Leveque's equation. So, this is the expression of Sherwood number by incorporating the value of we know we know suction parameter in the membrane wall.

So if you remember the Leveque's solution that was developed in 1885 and the modification of the porous wall the Sherwood number will be is has been developed recently in 1997, in late 90s. So, it will be what it indicates that the suction parameter increases the mass transfer coefficient of Sherwood number. And what is the extent of increase the extent of increase will be given by these factors and depending on the value of lambda, the porosity of the wall or permeability of the wall. The suction parameter will be enhanced the Sherwood number compared to the no-impervious conduit or the when the suction parameter will be equal to 0.

Now, next we will be seeing how to utilize the Sherwood number relationship in order to find a quick calculation of the system performance quite easily. So, we will be looking into the faster procedure or algorithm to calculate system performance using Sherwood number or average mass transfer coefficient. So, if you look into the boundary conditions at y equal to 0 at y equal to 0, we had v w C m minus C p is equal to minus d del c del y and at the same point we have the definition of mass transfer coefficient as k times C m minus C 0. So, if we consider the all average values length average length average values will be having that v w bar c m bar minus C p bar is equal to k l bar c m bar minus c naught. And in terms of non-dimensional parameter you will be getting P e w bar is equal to s h l bar R R 1 minus 1 over C m star. So, P e w bar is equal to nothing but v w bar d e by d and Sherwood is nothing but k l bar d e by d.

(Refer Slide Time: 26:54)



So, this relation will be giving you the expression of average value of wall suction parameter or the permeate flux with respect to with respect to length average permeate flux. So, you will be having the length average permeate flux this is expression P e w bar is equal to bar divided by R r 1 minus 1 by C m star and we have the osmotic pressure model P e w bar is equal to b times one minus delta pi by delta p.

Now, we have the expression of s h l bar average Sherwood length average Sherwood number as a function of lambda which will be nothing but p w bar divided by Reynolds-Schmidt d e by l d e by l raise to the power 1 upon 3. And we have delta; delta pi which will be function of C m star now we have two so essentially we will be having two equations and two unknown systems, what are the unknowns P e w bar and C m star. So, now you have to solve two algebraic equations, which are non-linear; and they are coupled again this can be solved by using Newton-Raphson method iteratively.

Therefore, now using the mass transfer, what we have seen what we have seen now in these class, we have seen how to solve a two detailed two-dimensional model for osmotic pressure control filtration. And once you get the concentration profile then if you hook it up with the transport phenomenon through what as you know membrane then you will be getting a system performance. If you adapt to an algorithm which will be giving you ultimately the value of permeate flux and permeate concentration as a function of x, but if you remember up to that point we have not used the definition of mass transfer coefficient.

Next, what we have done we have derived a fundamental relation of length average Sherwood number or mass transfer coefficient from the first principle and obtain an expression of Sherwood number. Now, using the definition of mass transfer coefficient we have formulated the problem once again but at this time it will be in the terms of lay all length average quantities. So, therefore, we will get ultimately landing up with two algebraic equations.

Now, in this case, we are not going to evaluate the profile of dependant parameter permeate flux and permeate concentration as a function of x location and then doing one more length averaging by adopting Simpsons will not trouble your trapezoidal rule. In this case, we have landed up into two algebraic equations directly having two unknowns and one can get the length averaged permeate flux and permeate concentration directly from this equation using the Sherwood number relationship, so that is why people use to find out the mass transfer coefficient in any transport problem. And then, that will immensely simplify the solution of the problem in terms of the mass transfer coefficient, so that will complete the predictive mode of how to predict the osmotic pressure filtration.

And osmotic pressure filtration, if you remember will you will come across with the in case of the solutes which will be having a low molecular weight. For example, salt, sugar, polyphenols, dyes etcetera where so we have seen how to in a cross flow system which will be quite frequently applicable for an industrial scale to model the system in an entirely predictive method. In the next class what we will be doing, we will looking into the importance of the back-shell and how to un-start back shell and how to model the un start back shell quite accurately in a predictive manner using the similarity transform from the first principles.

Thank you very much.