Introduction to Process Modeling in Membrane Separation Process

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Lecture – 10
Osmotic Pressure Controlling Filtration (Contd.)

Welcome to the session of this course. So, we have in the last class we have already seen how the system performance in osmotic pressure controlling filtration can be obtained in case of ultra filtration system and then we have looked into the reverse osmosis nano filtration system by replacing the definition of real intention by the solute mass flux to the porous membrane in here to connect the membrane surface constriction and the permeate constriction of the solute. So, we are then looking in to the various (Refer Time: 00:53) of the solution diffusion model and how to include those in our calculations in order to get a system performance. So, next model the modification.

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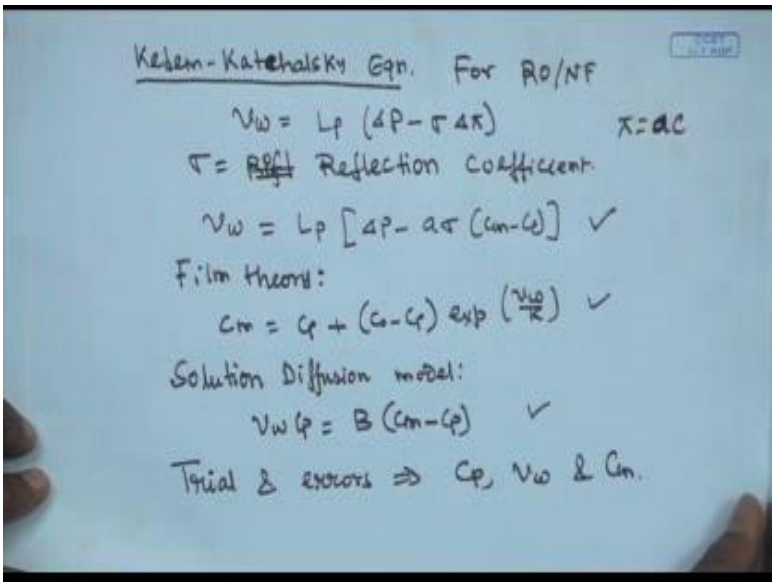

we will be looking into the Kedem-Katechalsky model for the RO-NF system for reverse osmosis and nano filtration system. In this case the osmotic equation module of the Darcy's law is modified as Lp delta P minus sigma delta pi where, sigma is known as the reflection coefficient.

So, one can get the expression of solvent flux as Lp del p minus a sigma Cm minus Cp. So, we are assuming that pi is a linear function of constriction a times c in order get a simplification in our calculation. So, sigma is a reflection coefficient and then will be using the film theory model. Film theory model as Cm is equal to Cp plus C not minus Cp exponential Vw by k. Then we have the solution diffusion model that is Vw times Cp is equal to B times Cm minus Cp. So, we have 3 equations 3 are knows Vw Cm and Cp. We have 3 independent equations involving these 3 parameters. Gain 3 equations 3 unknown system this is highly solvable in by using a trial and errors method this set of equations can be solved to obtain the permeate constriction permeate flux and membrane surface constriction.

And it has to be just noted that, if we are talking about a completely retentive membrane then reflection coefficient sigma is equal to 1. We will be looking into the next variant of solution diffusion model.

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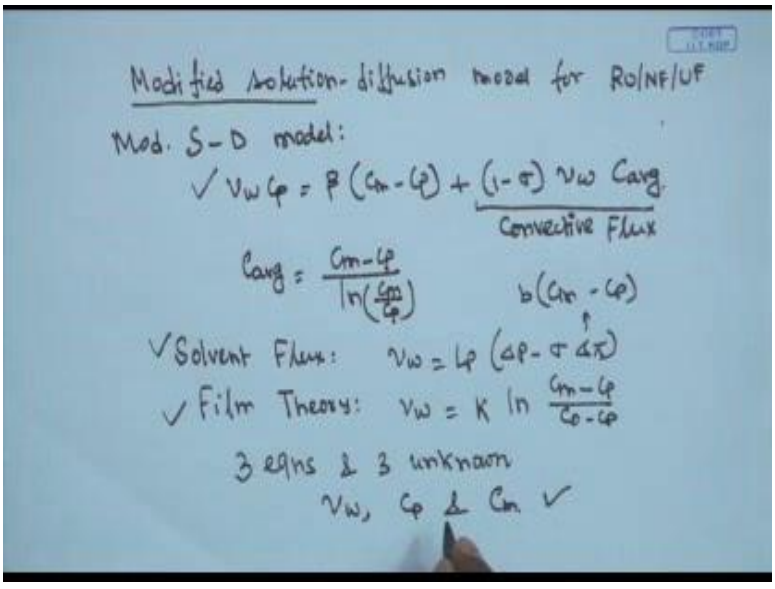

This is known as the modified solution diffusion model for RO/NF and it is used sometimes for UF also. So, what is the modification? Modifications here is we are invoking a term because if we are go for a more open membrane for example, reverse osmosis membrane is assume to be almost an impermeably imperiousness membrane and

it is such a dense membrane. In case of nano filtration lower cutoff membranes the pore size is increased compared to the reverse osmosis membranes. So, there are appearances of well defined small post inside the membrane matrix.

Now, if you go for the higher cutoff of nano filtration membrane for example, 600, 800, 900 molecular cutoff, then the pore size of the membranes will be more and the defined pores will be appearing more in number in the membrane matrix. So, therefore, if the defined pores will appear then in addition to the diffusive flux of solute through the membrane the convective flux through the membrane will be also predominant. So, later in the later monoculate cutoff of nano filtration membrane both convective and diffusive flux will be equally important and there will be correction factor or modification factors will be appearing in the solution diffusion model, in order to incorporate the convective flux to the membrane.

So, therefore, the modified equation will be the modified solution diffusion model can be writ10 as Vw Cp is equal to beta Cm minus Cp plus 1 minus sigma Vw C average. So, this is the additional term that will be taking care of the convective flux, through the membrane. What is the C average have? So, sigma is the reflection coefficient Vw is the permeate flux, C average is the average solute constriction in the membrane matrix and C average is basically a log main concentration difference between the Cm and Cp. It will be Cm minus Cp divided by ln Cm divided by Cp. This is a more realistic model for the solute flux through the membrane surface in case of open nano filtration membrane or ultra filtration membrane. So, in case of ultra filtration membrane the pore sizes even larger and. So, therefore the convective flux will be sometimes predominant compared to the diffusive flux.

So, in that case the one has to take care of the convective flux as well. Therefore, we can modify the solution diffusion model in this fashion and if you really look into the calculations you have the film theory by in the solvent flux through the membrane matrix by the Darcy's law. So, that will be nothing, but Vw Lp delta P minus sigma delta pi and we have the film theory model for the flow through the channel outside the membrane. Vw is equal to K ln Cm minus Cp divided by C not minus Cp and delta pi can be expressed in terms of Cm and Cp, if we going to consider this pi is a linear function of

concentration, again 3 equations and 3 unknowns. Vw Cm and Cp this 3 equation 3 unknowns systems can be solvable 3 equations and 3 unknown and they can be solvable and one can get system perdition Vw Cp and Cm.

So, these are the various variants of solution diffusion model in order to get the system performance in a very predictive mode. Only some of the parameters like Lp real in10tion B which is basically the solution diffusion constant for the solution diffusion model. This has to be estimated separately by conducting separate set of experiments and once these are done then if the operating conditions, if the transport co-efficient, if the geometry and the solution properties then one can have a system performance in a predictive mode. Next will be looking into the. So, that is all about the one dimensional film theory model for modeling the transport concentration polarized in the mass transfer boundary layer. Combining it with the transport processes those are occurring within the membrane surface membrane matrix. Combination of these will be really going in to predict the performance of the system completely.

Next will be looking into the short coming of one dimensional modeling or the film theory model and then will be adding completion to the system by considering a 2 dimensional model. On before that we are to just look into the short comings or limitations of the one dimensional model.

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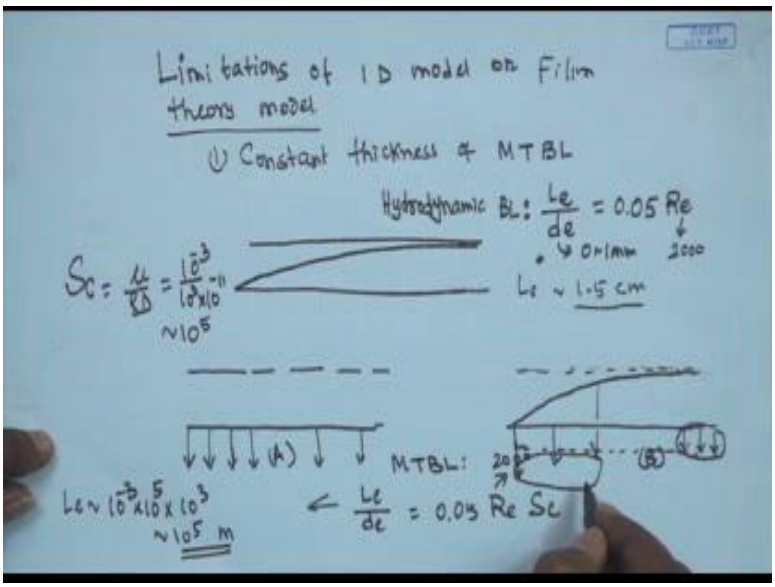

So, the first major limitation of the one dimensional model is the constant thickness of mass transfer boundary layer. So, as you have described earlier that if there is a membrane surface there will be a development of mass transfer boundary layer over the membrane surface. In case of film theory we are assuming a constant thickness of the membrane surface. Now it let us let us look into 2 cases separately it should not super pose on each of them. So, this is a constant thickness of film or solute over the membrane surface this is case (A) and in the second case there is a developing mass transfer boundary layer over the membrane surface.

So, as we have seen earlier that, the thickness of mass transfer boundary layer will be offering a resistance against the solvent flux and against the solvent flow. If the thickness is more the less self permeate flux we are going to get. If the thickness of mass transfer boundary layer is less than we are going to get a higher permeate flux or the solvent flux across the membrane. So, in case of constant thickness of mass transfer boundary layer, the permeate flux we going to get as same we are going to get same permeate flux through the membrane as you go along the length of the module.

But in case of developing masters, So, actually this is a very unrealistic case when were you are considering the thickness of the mass transfer boundary layer to be constant.

Why? Because, let us look into the entrance length of any boundary layer. For entrance length of a hydrodynamic boundary layer is 0.05 times Reynolds number. So, this is a case of hydrodynamic boundary layer or velocity boundary layer. Now if you considered the equivalent diameter is in the order of millimeter and Reynolds number is in the order of 2000, that in the laminar.

So, if it is the order of millimeter 10 to the power -3 meter and Reynolds number is in the order of let say 2000 then entrance length will be order of few centimeter, 1-5 centimeter; that means, if you are construing a channel a module which will be having a length of one meter. Initial apart from 5 centimeter or 4 centimeter the full channel will be having the developed fully developed hydrodynamic boundary layer or velocity boundary layer. Let us look into the entrance length for the mass transfer boundary layer. for mass transfer boundary layer the entrance length will be Le by d e is equal to 0.05 Reynolds times Schmidt.

So, what is Reynolds number? Reynolds number we said laminar 2000 and d equivalent is one millimeter in the order of one millimeter let us look into the Schmidt number. Schmidt number is mu by rho d. Now in case of membrane filtration, we will be selecting solutes which will be higher pores size I mean I know larger size compare to the pro size of the membrane. So, that there will be separated out of the membrane by the physical separation processes. Now let us if you taken a viscosity of water. So, 10 to the minus 3 and density of water that is 10 cube and the diffusivity typical diffusivity is in the order of 10 to the power -11 to 10 to the power -12 meter square/second.

If it is 10 to the -11 then the Schmidt number will be order of 10 to the power 5. If diffusivity is in the order of 10 to the power -12 then the Schmidt number will be in the order of 10 to the 6 So, therefore, in case of membrane separation, we are talking about the solutes which will be having very high Schmidt number. If the Schmidt number is very high and then if you really calculated the entrance length the entrance length will be in the order of d e will be one millimeter means 10 to the -3, Schmidt number will be 10 to the power 5 or 10 to the power 6 into Reynolds number, it will be 10 to the power 3.

So, this will be in the order of 10 to the power 5 meter. So, entrance length for the mass

transfer boundary layer in a membrane channel will be very very high. So, it is not that if you are considering 2 meter 5 meter even 10 meter channel length or the tube length the maters of boundary layer will be still developing. Now let us look into the interpretation of developing boundary layer. If the boundary layer is developing, then at the begging of the channel the thickness of the boundary layer is less. So, therefore, it offers less resistance against the solving flow, the solvent flux will be maximum. As the thickness increases it offers more resistance and solvent flux is lower. So, it will be less. So, like that it will be less.

So, as you go down across the channel length and later on when it will be growth will be almost sluggish, and then you will be getting the constant value almost constant value of permeate flux. So, at the form downstream of the channel where growth of the mass transfer boundary layer will be almost sluggish; that means, its thickness is almost equal, then you are going to get a constant thickness of the film over the membrane surface where you are getting the constant permeate flux, but the majority of the channel length will be 95% of the channel length will be having a developing mass transfer boundary layer where the permeate flux is very very high.

So, now if you assumed a constant thickness of mass transfer boundary layer is formed over the membrane surface, you will be assuming a constant permeate flux over the membrane surface. So, the excess flux due to the developing mass transfer boundary layer is already world looked. So, therefore, if ones goes for an assumption of film theory which assumed a constant thickness of film, one has to under predict or under consider the permeate flux.

So, what are permeate flux one will be getting that will be always less than whatever in the actually scenario. So, therefore, it is essential to go for a 2 dimensional model where mass transfer boundary layer is a function of both x and y and one will be getting an actual realistic picture for a developing mass transfer boundary layer. So, there is the first assumption. So, what is the limitation of the one dimensional model? The limitation of one dimensional model is that if you want adopts the one dimensional model then, the one will be under predict permeate the flux.

What will be the second assumption?

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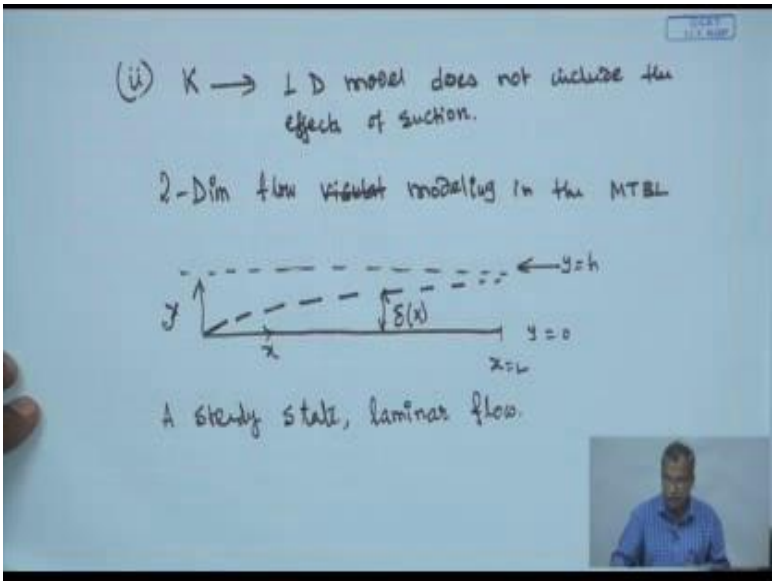

The second assumption is masters for co-efficient that we have used in one dimensional model does not include the effects of suction. Typically the schedule number relations are obtain from the heat mass transfer (Refer Time: 17:27) for example, this heat mass transfer (Refer Time: 17:30) and the heat transfer Nusselt number relations are basically for the non porous convict. there are all for impervious convict.

Now in case of membrane separation system will be having the wall or we having the conduits which will be having the porous walls. So, therefore, the effects of the porosity will be really dominant in some cases. For example in case of reverse osmosis the wall porosity will be not that much the effect may not be very important for reverse osmosis and nano filtration for ultra filtration and micro filtration the pores sufficiently large and one will be the mass transfer boundary the masters co efficient will be a function of wall suction in that case. The effects of all porosity will be cannot be neglected. So, this 2 are the limitations of the one dimensional film theory model for modeling mass transfer boundary layer outside the membrane surface inside the flow channel.

Now, in order to (Refer Time: 18:26) will be considering a 2 dimensional flow situation

flow visualization or modeling 2 dimensional flow modeling in the mass transfer boundary layers. So, we assumed the boundary layer which will be the practically the actual case, is developing over the membrane system over is the membrane channel this y direction this x direction. So, this is the half height of channel. So, y is equal to h here y is equal to 0 here x is equal to L here at any point of time delta is the thickness of mass transfer boundary layer and delta will be a growing function of x. Now let us consider a steady state laminar flow. So, let us right down the mass transfer solute balance equation within the mass transfer for boundary layer and see what we get?

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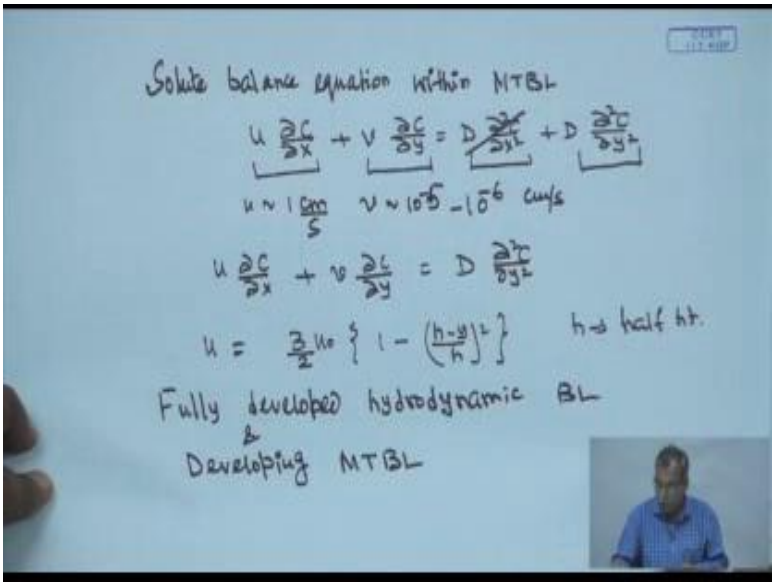

If we write down a solute balance equation within mass transfer boundary layer let see what we get at the study u del c del x plus v del c and del y is equal to d del square c del x square plus d del square c del y square.

So, first term represent convention in the x direction, second term reflects the convective flow in the y direction the first term on the right side is the diffusive term diffusive flux in the x direction and this is diffusive flux in the y direction, but we already know that convection u is in the order of centimeters and v is in the order of 10 to the power minus 6. It is in the order centimeter one centimeter per seconds v is the order of 10 to the power minus 5 to 10 to the power minus 6 centimeter per second. So, therefore, the

convective flux in the x direction will be much dominates compare to the diffusive flux in the x direction.

So, we can neglect this term compare to the convective term x direction. So, u del c del x plus v del c del y will be is equal to d del square c and del y square. So, now, let us look into the velocity profiles. So, what is u? u is equal to for a laminar flow in rectangular channel 3 u zero by 2 one minus h minus y divided by h square. So, will be having a typical the parabolic velocity profile as a laminar flow within the rectangular channel and h is the half height and u not is the cross sectional average velocity. So, now, as we have already seen that in our case, our case is a developed hydrodynamic boundary layer because entrance length required for hydrodynamic boundary layer we completely developed is few centimeter compare to the channel length of in orders of meter

Similarly, but on the other hand the mass transfer boundary layer will be really developing throughout the whole channel length. So, therefore, it is a case of fully developed master hydrodynamic boundary layer and a developing mass transfer boundary layer. So, it is case of fully developed hydrodynamic boundary layer and developing mass transfer boundary layer. So, now, let us again go back to the geometry of the system.

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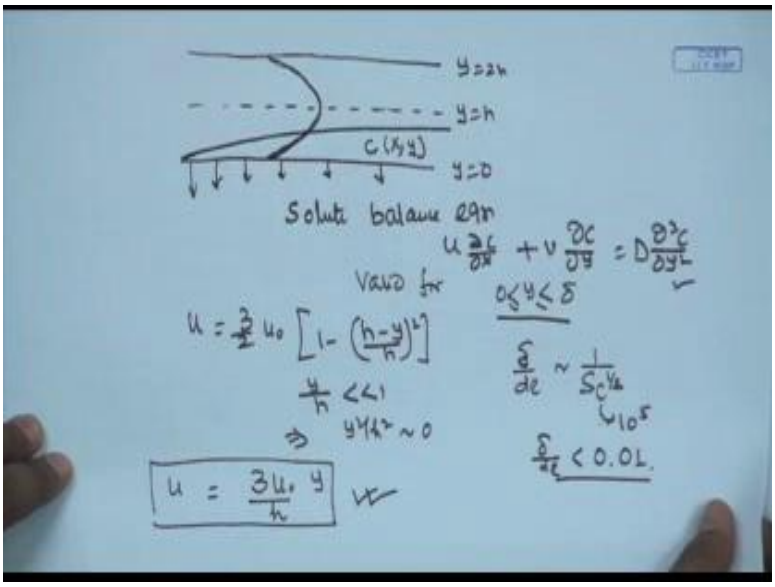

If you really go back to the geometry of the system this is the channel we are considering and this is the middle line of the channel y is equal to h this is y is equal to zero this is y is equal to 2h and membrane is placed at the bottom and these are completely developed hydrodynamic boundary layer and the mass transfer boundary layer is still developing. So, beyond the concentration within the mass transfer boundary layer is changing as a function is the function of both x and y. Beyond the mass transfer boundary layer concentration is same it is equal to the bulk concentration within the boundary layer it is a function of x and y.

So, therefore, let us. So, about the governing equation whatever we have written of the solute mass balance equation that is valid for solute balance equation that is u del c del x plus v del c del y is equal to d del square c del y square is valid for y is lying between zero and thickness of mass transfer boundary layer. Now if that is the case and the velocity profile you have taken the complete velocity profile is a parabolic velocity profile is 3/2 u not one minus h minus y divided by h square of that parabolic one we are talking about the this complete velocity profile, but if you look into the mass transfer boundary layer which is the dominate of validity of this equation we are only considering only this part of the complete velocity profile and this part is a linear one so; that means, the mass transfer boundary layer is really very very small compare to the actual dimension of the channel.

So, if you look into that the thickness of the mass transfer boundary layer can be roughly assumed approximately by this equation delta by d e will be inversely proportional to the Schmidt number and delta if you talk about if you put at this Schmidt numbers around 10 to the power 5 this will be delta by d e will be less than 0.1 or 0.01; that means, with respect to a actual dimension geometry dimension of the channel the thickness of mass transfer boundary layer is only 1 percent of the whole channel length.

So, therefore, he can really neglect; that means, y by h is very very small and we can neglect y square by h square and higher order terms. If you really do that we can expended this quadratic term you can find out the after simplification this will becomes a linear one. You can neglect y square by h square term by opening up by opening up this square is one will be canceling out and y square by h square can be neglected and it will

boil down to a linear velocity profile and that will be corresponding to this figure that the part that that lies the part of the velocity profile that lies within the mass transfer boundary layer is really a linear part of the total parabolic profile of the velocity profile. So, this linear part within the mass transfer boundary layer is replaced by this.

Similarly, we can now let us look into what is replaced by v?

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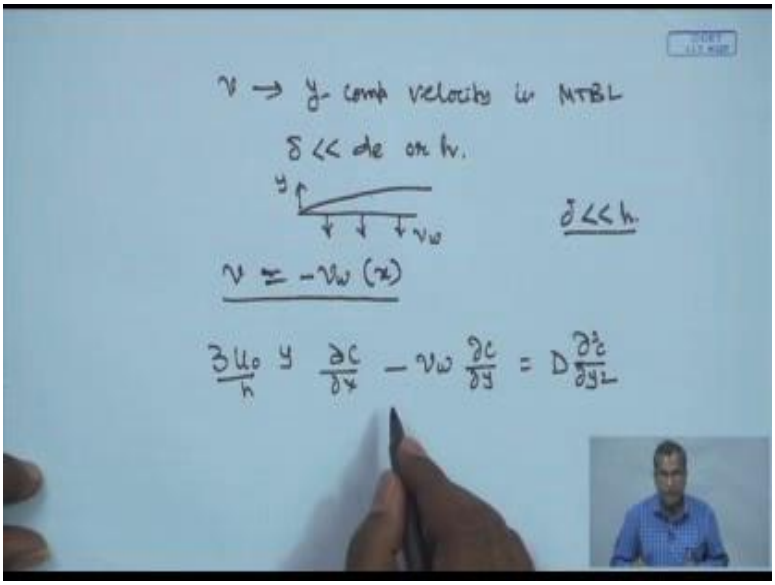

V is the y component velocity in mass transfer boundary layer and we assumed that since we have already founded that delta is much-much less than effective diameter or channel half height. So, therefore, the thickness of mass transfer boundary layer is probably less than 1 percent of the channel dimension and it is really very very small.

So, in order to have continuity we can assume that within this small channel the y dimensional. So, this is basically Vw which will be essentially coming to the negative y direction. So, we can simply assumed v is equal to minus Vw which will be a basically function of x. So, why this profile is can be a velocity profile in the y component can be assumed because the thickness of mass transfer boundary layer is much-much small compare to h. So, we are assumed even if there is a variation of v as a function of x and y within mass transfer boundary layer, but since the thickness of the mass transfer

boundary is very very really very small then, it can be assumed that at the wall that the velocity we know that it is minus Vw. So, we can assume if it is really small, every where it is minus Vw in the channel.

So, therefore, the solute balance equation is now looks like 3 u zero y by h del c del x minus Vw del c del y is equal to d del square c del y square. So, I stop in this class in the next class what will be doing will be sitting up the boundary conditions of this to solve this equation and we will be looking into the complete solution of this equation along which is boundary condition and how the system performance can be calculated in this case of 2 dimensional analysis and that will be giving a very very realistic solution of an in actual what is happening the physical phenomena that were those are happening in an actual membrane channel.

Thank you very much