Plantwide Control of Chemical Processes
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Lecture - 7
Frequency Domain Analysis
Nyquist Stability Criterion
Gain margin and Phase margin
Maximum Closed Loop Log Modulus Tuning

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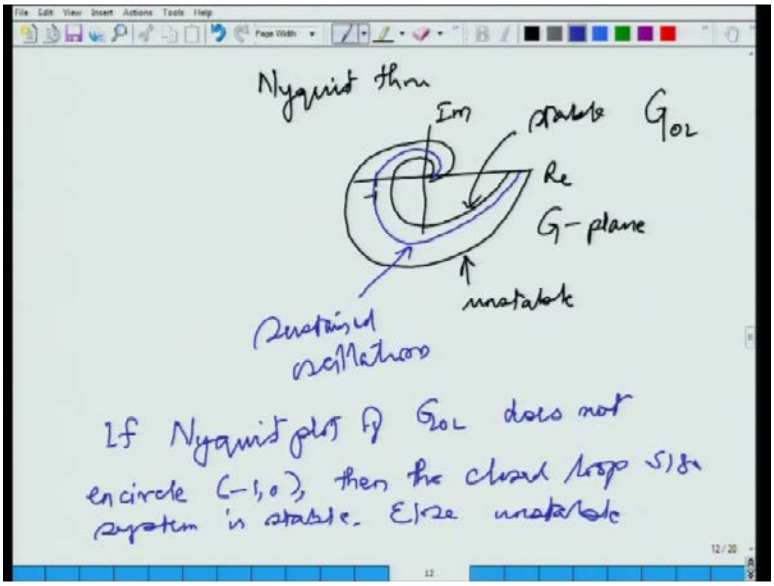

So, this leads us to the Nyquist theorem and what the Nyquist theorem says is that if your Nyquist plot of G open loop, if your Nyquist plot of G open loop, does not encircle minus 1 0, it is a real axis, this is the imaginary axis on the G plane. This is the G plane does not encircle minus 1 0, then the closed loops single input, single output feedback system is stable. On the other hand, if it encircles minus 1 0. So, this one is stable and this one is unstable, because it is encircling minus 1 0. And of course, if you got 1 that goes like this passés though, this 1 is this corresponds to sustain oscillation.

So, the Nyquist theorem says, if the Nyquist plot if the Nyquist plot of G open loop feedback, loop is not yet closed does not encircle minus 1 0, then the closed loops single input, single output system is stable. Of course, the opposite is also true. So, it is a necessary and sufficient kinds of consequence else unstable. And of course, it is a there is an assumptions there, that the open loop system by itself a stable, you do not have a open loop and stable system all right.

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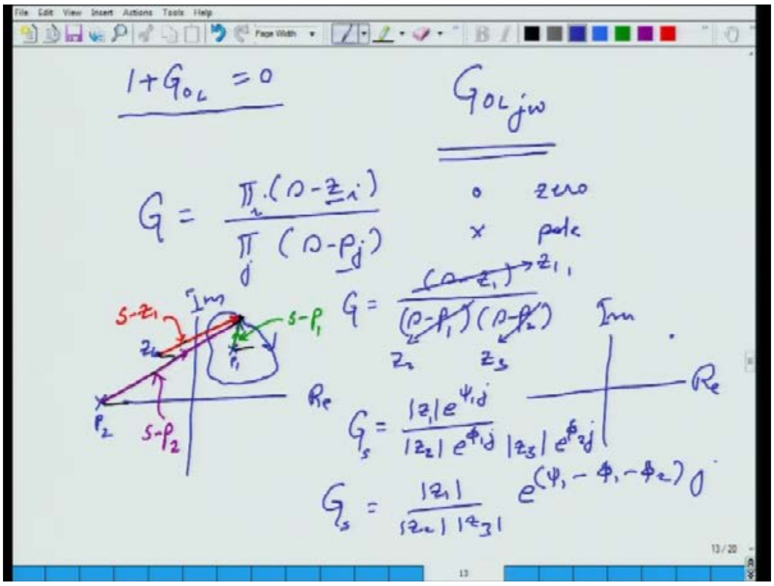

So, this is the Nyquist theorem, and this sound like magic. You look at G open loop, j omega makes its Nyquist plot and if he open loop Nyquist plot, does not encircle minus 1 0 close system is stable. So, this sounds like magic, but where does, this magic come form, I will quickly go through, that and it comes from complex, basically a bit of a complex analysis. So, let say I have got G is the transfer function, which is product of product of, what the 0's over I, and product of poles over plus, should I make it plus or minus, I do not know will let say, this is G.

Let us say, if we talk about the characteristics equation and if the characteristic equation roots are in the right half plane, then you get you get an unstable system for a for a feedback control system. The characteristics equation is 1 plus, G open loop is equal to 0. So, it actually all boils down to figuring out, if any of the roots of this equation are in the right half plane or not.

So, I will just tell you, how it works out. So, let us say this is the s plane, and this is the G plane; s is a complex number and s is well, s is equal to sigma plus j omega, G is also a complex number, which is a function of a. So, as s changes G, which would be complex number, would also change, all right. Now, let me lets have got s, which is here for this values of s, when I substitute this value of s into this expression up, there into the expression of G, I will get another complex number a complex number for G. Let us say, I am plotting the complex number; and that complex number turns out to be somewhere here somewhere. So, this is G corresponding to this s all right.

All right, now let us say, this s is moving in a closed contour, let us say it moving like this, what I do is, I also plot these poles and 0, and this poles and 0 usually be real, but some time, you can also have complex conjugates poles and 0s all right. If I do that, this is the real axis, this is the imaginary axis, this is the imaginary axis. Now, what I do is, I also show, where the 0s are and the poles are, and the 0's I will show with a circle, that is a 0, and the poles I will show with a… So, this is called the pole 0 maps on the s plane, I will also show, where the pole 0s are.

And, let us just say that, some of the 0s are outside, some of the poles are outside, but then may be 1 pole is inside, let us make a 2 poles are inside and 1 is 1 0 is here and then of course, there are let say, complex number. Well, you do not really need complex conjugate also, actually it could be it this is in general, anyway how it will be made, I will be made like this. So, let us say, for the time being the we are not talking, let us say this is what a things this is what things look like for just to keep it simple, some of the poles are outsides, some of the 0's are outside and may be let us say, one pole is inside inside the contour ok.

Then, so you got 1 0, which is here and you got 2 poles; one of the pole is here and other pole is here, let say you got a system like this, then when s is here, let us say I have got a point s, which is here, this is s, the black the black one. Then I draw a straight line here, I also draw a straight line here, I also draw a may be a straight line, here this is the angle, then this is the angle all right. Now, so when I look at G, G would be s minus z 1 and this s minus z 1, so this plus z 1, this is the vector, which is s minus z 1, this is the vector which is. So, we will call this p 1, p 2, z 1.

So, maybe I will should color code of few things here. So, let us say this is the z minus p 1 and let say, the purple is this is z minus p 2, red is z minus s s minus z 1, purple is s minus p 2, and this let us, make this green this guy is s minus p 1. So, you can see that the red curve is this guy is s minus z 1, the green Chap is s minus p 1, and the magenta chap, this guy is s minus p 2 all right.

Then, the complex number G will turn out to be, well of course, s minus p minus z 1 divided by s minus p 1 into s minus p 2, now this guy would be a complex number with a magnitude; I will call it z 1; let me call this complex number z 1, let me call this complex number z 2, and let me call this complex number z 3. So, then in that case, G will turn out to be magnitude of z 1 times exponential of angle of z 1, and I will call the angles of 0's si si 1 and j, and the angle of the poles, as j 2 times e to the power phi 1 of j, and the last guy would be z 3 times e to the power phi 2 of j all right.

And now, if I solve it further, what I will get is that, what I will get is that G G of s is actually equal to z 1, z 2, z 3 times e to the power si 1 minus phi 1 minus phi 2 j, note that the angles subtended by the 0's are positive, while the angles subtended by the poles become negative, because when you take the angle, form positive angle form denominator to numerator, you know e to the power phi j, become s e to the power phi j. So, if you generalize this, what we get is that if you got, so many 0 and, so many poles. So, many 0's and so many poles, the angles of the 0's will get add it, the angles of the poles get subtracted their much is cleared. So, now as s is moving on the contour, as s as s moves on this couture in this direction, do you see that man, maybe I should do it on the board.

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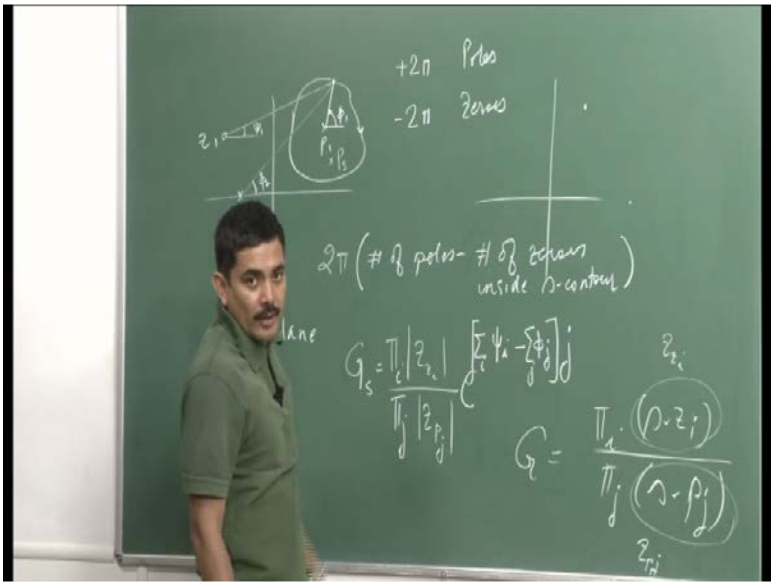

So, this is the s plane, and I have s moving on a contour, I have a got a pole inside, and I have got 0 here, and may be may be a pole here. So, this one is p 1, this was p 2, and this was z 1, and what I was saying was, this is s minus z 1, then this is si 1, this is s minus p 1, and this is phi 1 and this is s minus p 2, and this guy is phi 2 and corresponding to this is this s plane, corresponding to this, when I take G, is equal to product of s minus z i divided by product over j of s minus p z.

So, let us say this guy a map somewhere here, then what I just showed you, was G s gone be, let me call this number by a z, let call this complex number be called z 0 i, and let this complex number be called z pole z, then what I have is G s is product of the magnitudes of all those s minus z i terms. So, z i divided by over i product of z times product of product, over z of the s minus p terms. So, z p j this should be the magnitude and the angle would be e to the power summation si i, which is the angle subtended by this is the angle subtend by the s minus z i terms minus summation phi z. So, this is e i and he is j, then you combine it, and this is actually multiplied by the square root of minus 1, which is j all right.

So, this is what we have. So, now we get the complex number G of s corresponding to a single s. Now, let us say I am going this way, I am going around the contour, I go around the contour as I go around the contour, do you see that; this angle increase; I mean this angle si 1, actually is right, now it is positive; then it goes negative and then it turns back again, and when you reach back here, then it goes those way, and when you reach back here. The angle, total angle swept, you see this is the angle, the total angle swept is 0, this line actually goes like this and then. So, angle is decreasing becoming negative, then it starts to you know increase again, when it is increase and when you get back, the angle is same. So, the net angle swept but si 1 as s moves around the contour is actually 0.

Similarly, the angle swept by this guy, again it is a positive angle, then it goes negative, then it starts to increase again, then it when you comes back to this point, it goes back 0 back to the same values. So, as s moves along this contour, the angle swept by z 1 and you know by this 2 vectors, the angle swept is actually 0 on there. Other hand, when you going along the contour, the s minus p 1 terms swept an angle of 2 pi 360 degree, the angle as you go around increases by 2 pi yes. Or now, once you see this, suppose you had another pole here, let say there was a p 3 here, the angle of this guy would also have increase by 2 pi.

So, any 0’s and poles, which are inside the contour, so this angle swiping would be actually. Well, if you are saying that, the angle anticlockwise is positive pole is swiping minus 2 pi, then when you take it up, because the pole terms are in the denominator, when you take those angle in a to the top, the phi will become positive. So, pole sweep an angle of plus 2 pi 0 swiping an angle of minus 2 pi, and this poles and 0, which are inside the contours closed contour.

So, as s moves in a closed contour, the angel of G, the net angle of G will change by 2 pi times numbers of poles minus number of 0’s inside s contour, because poles changes the angle by plus 2 pi, each pole inside the contour will change the angle by 2 plus. Each 0 inside the contour, will change the angel by minus 2 pi, inside the s contour. And s is moving clockwise, that is the kind of thing, that we see form here, now how does this translate, how does this translate to the Nyquist theorem; that is the next thing, we will show.

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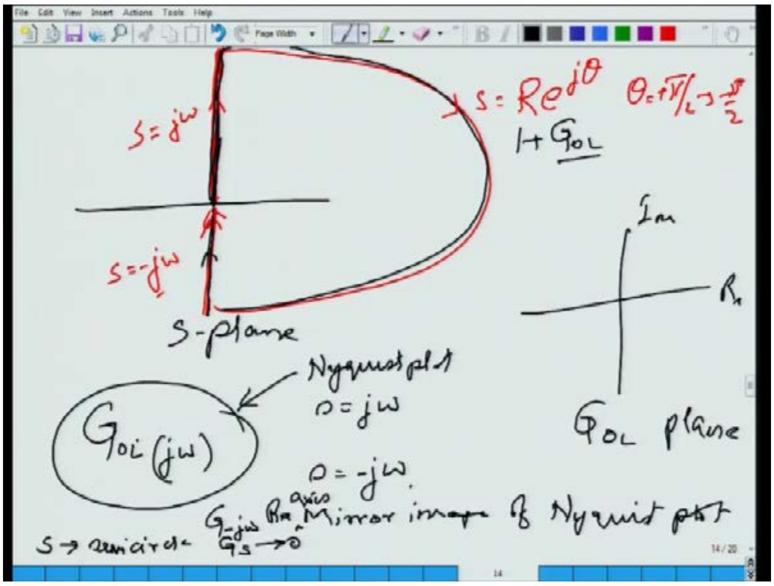

Now, let us talk about the Nyquist theorem, to decide if you have this is the s plane, you got G to decide, if G is stable or unstable, you need to find out, how many how many poles are? So, 1 plus G, so what I can do is, 1 plus G open loop equal to 0, is the characteristic equation. And, I want to find out, whether any of the roots of this 1 plus G open loop equal to 0 on the right half plane or not? I am assuming G open loop, is well behaved no 0s or poles with no control; that means, the open loop systems has no 0’s or poles in the right half plane. Everything is in the left half plane, when there is no control for the open loop system. So, G open loop by itself has a all poles and 0’s in the left half plane, that is well behave system, well behave open loop system.

Now, what I can do is, I can plot the Nyquist plot of just G open loop, and then I want to it yeah, I can make you know, I can plot Nyquist plot of I can make the polar plot of G open loop, and add one to it. Now, let me take in the s plan, a contour that goes like this, I am change the color; that goes like this, it is an infinite semicircle; that is enclosing all of the right half plane, I want to find out, whether any of the root of this guy is the right half plane or not?

So, let it enclose all of the right half plane all right, then what I do is as s changing from 0 to… So, this is actually G j omega, this is actually this guy is G with minus G omega and what is this guy the semicircle part, this is actually, sorry this is s equal to j omega, this should be s equal to minus j omega, and here s would be equal to a radius time e to the power j theta; theta going form plus pi by 2 to minus pi by 2, that is the contour covering the right half plane, as s is going along this along this curve as s is going this straight line, s equal to j omega in the G plane, this is the G open loop, plane p L a N e real part, imaginary part as s is going alone G omega, this part what I will get is G open loop will be noting, but G open loop s equal to j omega. So, this part is nothing, but the Nyquist plot.

Similarly, as Nyquist plot, when s equal to j omega going from 0 to infinity, that is what this is omega going from 0 to infinity, yeah. Similarly, when I am going alone, this part of the curve s equal to minus j omega, then what I will get is G open loop, s is minus G omega. So, I am replacing G omega by minus G omega all. I will get is the complex conjugate, because I have replace G omega by minus G omega or j by minus j. So, what I will get, the complex conjugate; complex conjugate means the Nyquist plot gets mirror image about the real axis.

So, when s equal to minus j omega G would noting be, G would be mirror image of G minus j omega, is actually mirror image of real axis are about the real axis; real axis mirror image of Nyquist plot and then finally, when I am going around this semicircular part of infinite radius, when I am going around this semicircular part of infinite radius, please note that all real system, will have the order of the denominator greater than the order of the numerator, and that is a consequence of causality. If I make a change in the input, it will take time of for the output to responds, that is causality; it will never be that I change input now, and its effect happen in the past, that is simply not possible then causality is violated.

So, all real physical system have G open loop will have a G open loop, what we are trying to say, well all real physical on the semicircle, because well on the semicircle, because all real stable open loop systems have order of numerator greater than order denominator.

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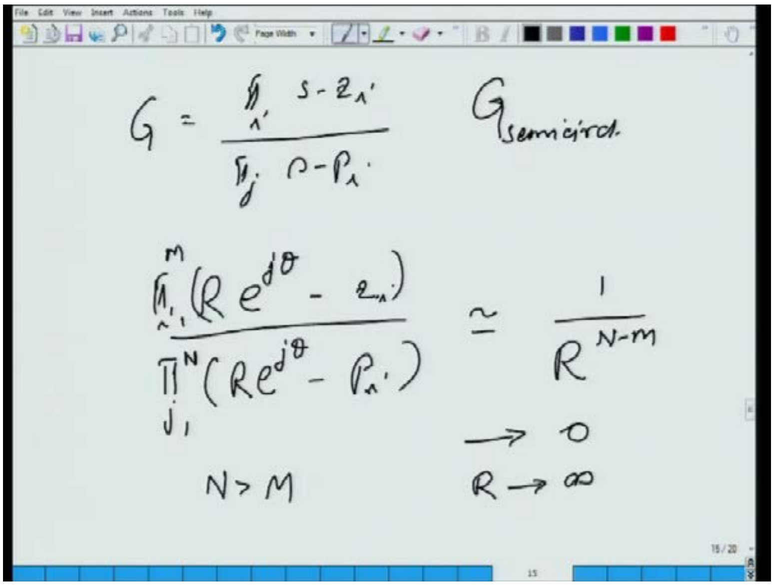

Basically, what I will have is, if G is equal to product over i product over i of s minus z i divided by product over j or s minus p i, then what I will have is, if I substitute G is equal to capital R e to the power j theta minus z I product divided by product over j into capital R into e to the power j theta minus p I, please note that z and p are much much smaller than capital R. So, this will essentially boil down to the z being negligible compare to this guy, and p being negligible compare to that that guy.

So, when you take the limit R tending to infinity, what you essentially end up with is R to the power, let say this is I going from 1 to m, and j going from 1 to N, N has to be greater than m, because of Causality. So, therefore, what you will get is essentially, the limit will turn out to be 1 over R to the power N minus m, N is greater than m and therefore, this would tend to 0 for R tending to infinity. So, what; that means, is G on the semicircle on the semicircle tends to 0.

So, let us get back to this figure. So, when s equal to G, s equal to j omega, you get the Nyquist plot, when s equal to minus j omega, you get the Nyquist plot mirror about the real axis and, when s equal to semicircle. So, s semicircle, you get G s maps to 0 yeah all right. So, if I have G open loop, I can makes it Nyquist plot and as I makes it Nyquist plot, 1 plus G open loop would be noting, but that Nyquist plot with the origin with 1 added to everything.

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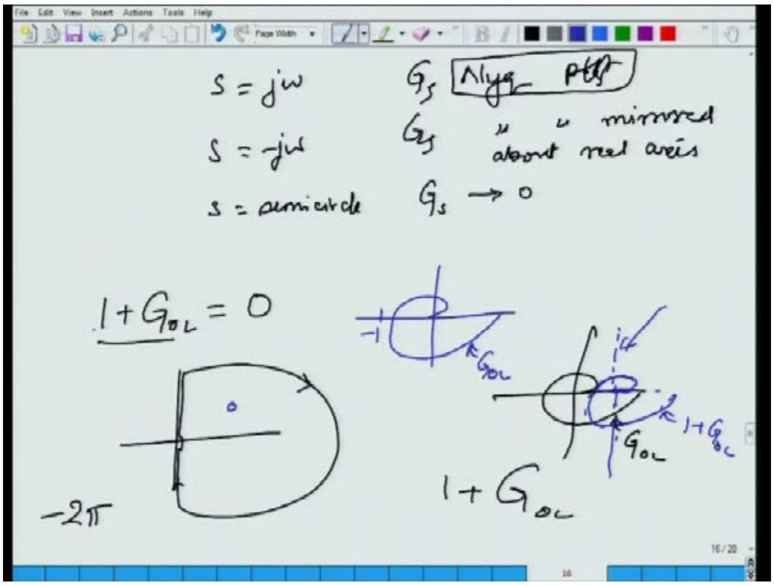

On s equal to j omega, you get Nyquist plot on s equal to G, s is essentially the Nyquist plot on s equal to minus j omega G, s is Nyquist plot mirrored about real axis, and s equal to semicircle. Usually what will happen is G s maps to 0. So, you can see that for more realistic system, just the Nyquist plot contains all the information and now getting back to, what I am actually looking for is the encirclements, the right half plane pole; the right half planes roots of this equation yeah.

If my s contour is this guy, and i plot G open loop, the Nyquist plot of that guy, what I want to know is how much the for each pole for each root, that is inside this contour in the s plane, what I should get is that the angle should change by, because this pole is the numerator the angle should change by this is going like this. So, the angle should change by clockwise by 2 pi. So, clockwise by 2 pi is minus 2 pi for every root of this equation in the right half plane, the angle will change by minus 2 pi. Now if I am plotting G open loop, then 1 plus G open loop, let say this is the Nyquist plot of G open loop, then this is the Nyquist plot of G open loop, then 1 plus G open loop would be, what I would simply be adding 1 to the real part.

So, my everything will have 1 added, and I would just have shifted, I would just have shifted everything to like this from 0 to 1; this would be 1 plus G open loop yeah. So, the blue curve would be 1 plus G open loop; now if the blue curve is encircling, what the hell man, 1 plus G open loop would be this. Well, now if 1 plus G open loop, if I redefined my axis has blue one, where the origin is shifted to this blue curve, then if this guy is encircling the origin; that means, if G is swiping an angle of well, this is anticlockwise minus 2 pi. If G is swiping an angle of minus 2 pi, that is means at least; that means, exactly 1 root of 1 plus G open loop is in the right half plane is encircled by the N s contour right. If it is encircling the origin twice; that means, there are two poles or two roots of this 1 plus G open loop equation in the right half plane.

So, now instead of plotting G open loop, and then shifting the origin to 1 0 and calling that the new origin, what is generally done is, you just plot G open loop; this is G open loop and instead of adding 1 and shifting the origin to 0 and looking for encirclements of 0 0 in the shifted axis, in the in the blues axis, this is the shifted axis, what you do is, you basically plot G open loop, the Nyquist plot of G open loop and look for encirclements of minus 1 0, these 2 are same thing, this is way the Nyquist plot or the Nyquist theorem comes form and it is all about, how many roots are in the right half plane.

So, take an s contour, that covers the right half plane and if 1 plus G open loop has any root, which is in the right half plane or as many roots are in the right half plane; that many number of encirclement, you will get off the point minus 1 0, because each root this is actually 0, I am this is actually a 0, because one plus G open loop has numerator. So, well that is that. So, this is where, Nyquist theorem, where did we do the Nyquist theorem; this is where the Nyquist theorem comes form. If the Nyquist plot of G open loop, does not encircle minus 1 0 well what; that means, is there are no roots in the right half pane of the equation, that I am looking at which is 1 plus G open loop equal to 0 on the other hand, if the Nyquist plot encircle minus, the number of roots in the right half plane would be as many encirclements of minus 1 0 as I have, so I hope his clarify, where does the Nyquist theorem comes from, how do we use it.

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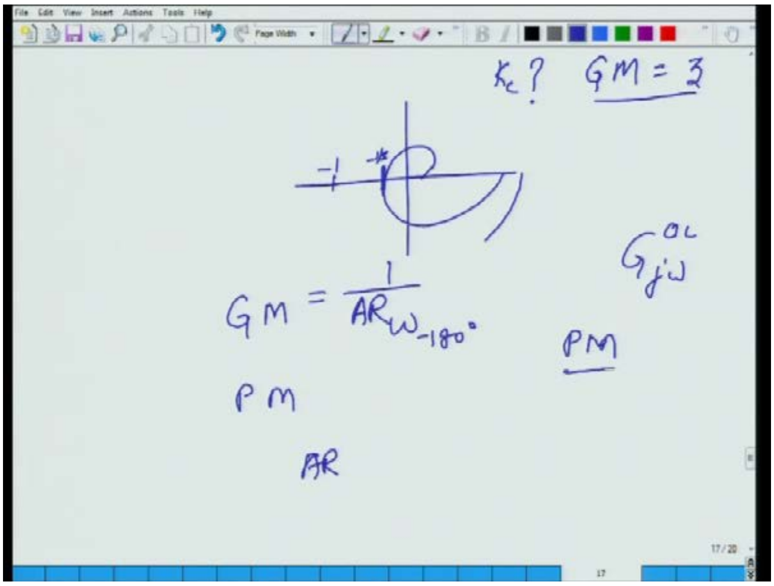

Well, it is use as follows, you want basically what we have is, if I get too close to minus 1 0, this is the G z omega. if I get too close to and, since I am G open loop, G omega if I get too close to minus 1 0. If pass though minus 1 0, I have sustain oscillation, if I encircle minus 1 0, I have instability if I pass close to minus 1 0, I will have oscillation that die down very slowly. So, my close loop system will keep oscillating and you know those oscillation will take long time to die down. So, what do we do, to design controller, we basically say, I have to be sufficiently away from minus 1 0. So, there are not too many oscillation.

So, I do not know, how to show this part? yeah. So, what we do, if you look at text book, this what is call gain margin and there what is call phase margin? what is gain margin? Well, if you look at the Nyquist plot, you can see that its quite far away from minus 1 0, and then you are saying is well, if the amplitude ratio at the frequency, where the phase becomes minus 180 degree; this particular frequency amplitude ratio at this particular frequency 1 by; that is equal to gain margin.

So, let us say I am passing though 1 by 3, then gain margin is 3. Let say, this point is minus 1 by 3, then I am passing though then my gain margin is 3. So, what we can say is gain amplitude ratio, you should design a controller. So, when I am designing a controller, typically what I will do is, I will write just the k c. So, the as I increase k c, this curve will blow at higher k c, I will get blown up curves yeah. So, I adjust the k c.

So, that my curve for, what k c does my curve pass though minus 1 by 3 that is basically saying that, I am designing my controller.

So, that I get a gain margin of 3, what should my k c be. So, that my Nyquist plot passes though minus 1 by 3; that gives me a gain margin of 3; this is gain margin, design there is also what is called phase margin? there is also called phase margin, what do I do in gain margin? I look at the amplitude ratio at the frequency, where the phase is 180 degree and I am adjust my k c. So, that that amplitude ratio is sufficiently smaller than 1 all right.

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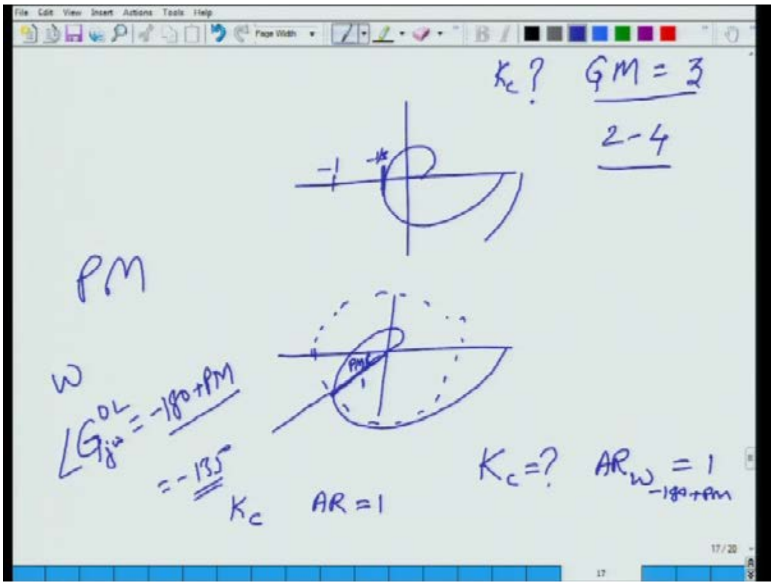

What do I do, in phase margin; in phase margin what I do is, I look at that frequency, at which I look at that frequency, at which the angle of G open loop is equal to minus 180 plus phase margin. So, let say I want a phase margin of 30 degree 35 degree or 45 degree, then what I am looking for is, what is the frequency at which the angle of G open loop, G omega? Of course, G open loop is phase margin of 40 degrees means, minus 135 degrees. So, what I am now trying to do is, this is the angle, which is minus, which is minus 135. Now, I adjust my k c, such that at this angle the amplitude ratio is equal to 1. So, what I am trying to do is, what should the k c be such that, the amplitude ratio at the frequency, where angle is minus 180 plus phase margin, that I desire is exactly equal to 1. So, basically let say this is minus 1 0. So, this is my unit circle, what I am trying to do, then is this is, where the amplitude is 1. So, amplitude is 1 unit amplitude and I have adjusted my gain, such that you know, this angle is phase margin. I just said phase margin is 45 degree, typical rule of thumb is 45 to 60 degree of vocalize phase margin, typical rule of thumb for gain margin is anyway from 2 to 4 k c is adjusted. So, that you get the phase margin, which is between 2 to 4 or a phase margin, which is between 45 to 60 digress this is a typical design method, that is recommended in text book.

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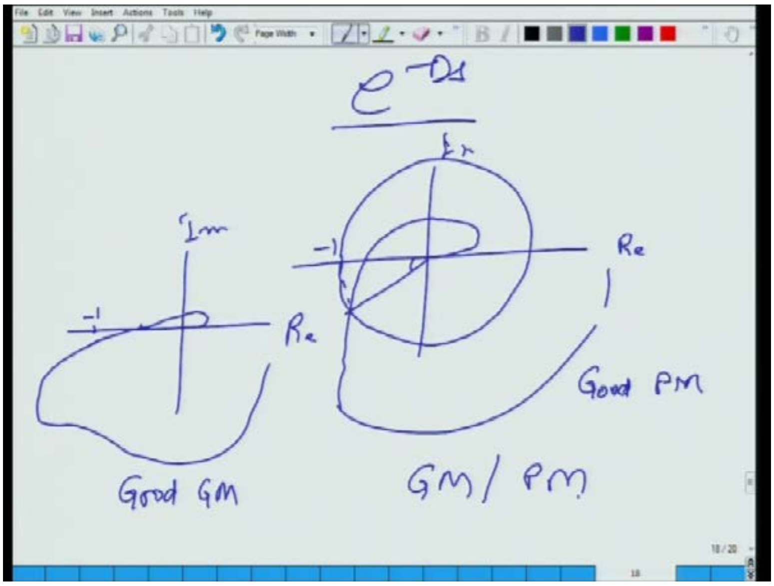

However, in chemical system; there is always a problem and what is it; that cause that problem, it is this guy called date time sometime, we also get inverse response and. So, forth and if you have these yield behave dynamic system.

Then, you do for example, you can get the Nyquist plot this is minus 1 0 right, you can get a Nyquist plot, which look for example, like this I mean you get large phase margin. So, I do not know something like this, if you look at the unit circle, if you look at the unit circle, this is the unit circle; this is 1 0 real axis, imaginary axis, you can see that I have sufficient phases margin, but then my curve is passing very close to minus 1 0, my Nyquist plot is passing very close to minus 1 0.

So, even though I am not unstable because, my curve is passing very close go minus 1 0, I get, what is called? I will get not sustain oscillation, I will get oscillation; that take a very long time to die out even though, I have sufficient phase margin, you can also have other situations, where the Nyquist plot is funny and therefore, even though you have sufficient gain margin for example, I get sufficient gain margin, let say, this is what the Nyquist plot looks like.

So, I have got this is minus 1 0, there is sufficient gain margin you can see that here at this frequency the amplitude ration is I do not know may be half may be half or may be less than half; however, my Nyquist plot is passing very close to minus 1 0, this one again will, even though I have sufficient in this case I have good phase margin, and in this case I have good gain margin, and yet when I do the closed loop, when I closed the loop I will find that the this two system are such that, even though I have sufficient phase margin or gain margin the response is very oscillating, all right.

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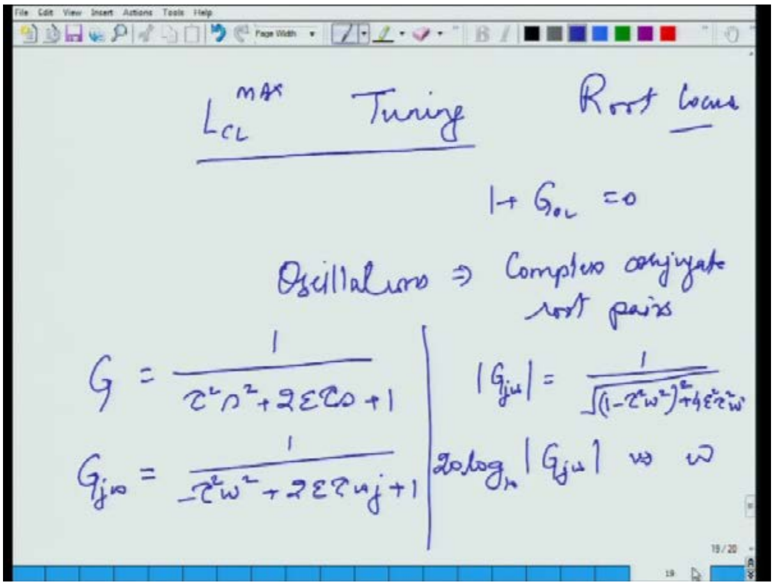

So, clearly gain margin, phase margin is not going to work here, so then what do, we do then, what do, we do is we take recourse to, what is called closed loop maximum log modulus tuning, and this is especially useful for chemical systems, and what is the basic idea; well the basic idea is I will just go through the basic idea very quickly, the basic idea is straight forward, in that it is motivated from root locus, and if you look at root locus, what you will find is that, the well I see how do I put it, think of a real system is without any control, has a very smooth exponential response, or has a very smooth slow sluggish response to change in the input, that is typical of chemical system.

Now, you got an open loop system like this, let say, you are trying to control the output using input. So, you so you putting the controller, now if you start cranking of the gain, and if I if you actually take a third order system, if you start cranking of the gain, what you will find is that, initially the response remain sluggish, as you start keep cranking of the gain, the closed response, starts to show some oscillation, as you crank of the gain those oscillation starts taking long time to die out, as you crank of the gain, further those oscillation becomes sustain, and as you crank of the gain, further those oscillations actually blow up, yeah, this is what happens for any for most 90 95 percent of the real system.

Now, what I want is, that the fact that there are oscillations tells us, that there are at least two roots, which are complex conjugate, two roots of the characteristic equations 1 plus G open loop is equal to 0, oscillation imply complex conjugate root pairs and if you look at well may be a need to cover this also. So, so if you have, what is the simplest transfer function, that can give you oscillation, that is the second order under damped system.

What is second order under damped system? second order under damped system, let say gain is unit unity, it is actually tau square s square plus 2 epsilon tau s plus 1, if you should do G j omega, you will find that this is actually 1 over tau square j square omega square, that will make it minus tau square omega square plus 2 epsilon tau omega j plus 1, and now if you want to find out, its magnitude, which is the amplitude ratio, you will find that G j omega, what would it be just by inspection, it would be 1 over square root of real part square plus imaginary part square. So, the real part is 1 minus tau square omega square plus imaginary part square, so imaginary part square gone be 4 epsilon square tau square omega square and this guy also square.

Now, if you plot this guy against omega, if you plot for example, this is what is typically done in electrical engineering, which is called the bode plot. So, log on base 10 of magnitude G j omega. So, if you do if you plot this versus omega, what you will find is if epsilon is of course, under damp system wait a second 19 20 under system is this guy epsilon is less than 1, I have forgot to tell you that.

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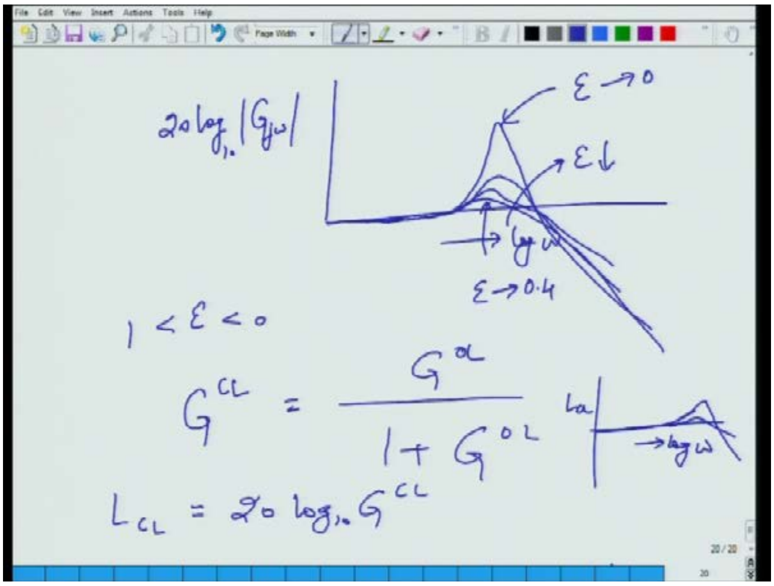

If you plot 20 this is call the log modulus on base 10 of absolute value of G j omega, this is on the y axis, on the x axis is where usually it is logarithm of omega, what you will find is as epsilon reduces. Let see me let me argue it out with you look at this expression this guy, when omega is 0 magnitude is 1, if magnitude of is 1 log of 1 is 0. Now, as omega starts increasing from small values, because you are taking omega square this omega square term will be negligible.

So, if epsilon is smaller and smaller, then what will happen is this term, the first term it will become slightly less than 1, but however, if epsilon is tending to 0, that means epsilon is becoming smaller and smaller, then the positive term that you have will become much smaller, and because the positive term is becoming much smaller. And what you will have is essentially this term is negligible, the second term is negligible as omega is increasing slightly above 0, the second terms is negligible, the first term has become less than 1, if the first terms has become less than 1, 1 over square root of that will become greater than 1.

And therefore, your log modulus will become slightly above 0, however, as omega is increasing further of course, this term will start to blow up the second term will start to blow up, and then the magnitude amplitude ratio will again go down. So, you see that there is a hump that comes in the magnitude. So, what you will find is as epsilon is going down you get a slight hump as epsilon goes further down toward 0, the hump becomes more prominent, more prominent much more prominent and so on, so forth. So, the fact that you get a hump in the log modulus indicates, that you got under damping or significant under damping on your transfer function, significant oscillatoryness is indicated by this hump.

This would be epsilon tending to 0 this for example, would be epsilon tending to may be 0.2, 0.4, 0.5, something like that. So, as epsilon decrease that your oscillatoryness increase epsilon is going down, in that direction your under damping is increasing; epsilon is tending towards 0, the hump in the log modulus curve will become larger and larger, so what that means, is if you have a closed loop system, which has a s is o, transfer function as G open loop over 1 plus G open loop. If you take the log modulus of this guy between what we are doing is closed loop log modulus is equal to 20 log on base 10 of G, closed loop if you have taken the log modulus of this guy, then if this log modulus is showing a hump. If you plot this log modulus, if this guy is showing a large hump, that means, you have got too much oscillatoryness, this is L C L, and this is of course, log omega. On the other hand, If it is showing a small hump, that means hump implies oscillatoryness is there, but since the hump is small the under damping is not too large. So, the response is oscillatory, but not to oscillatory. So, we want a hump, but not too large hump, and this is what is called the maximum log modulus tuning rule.

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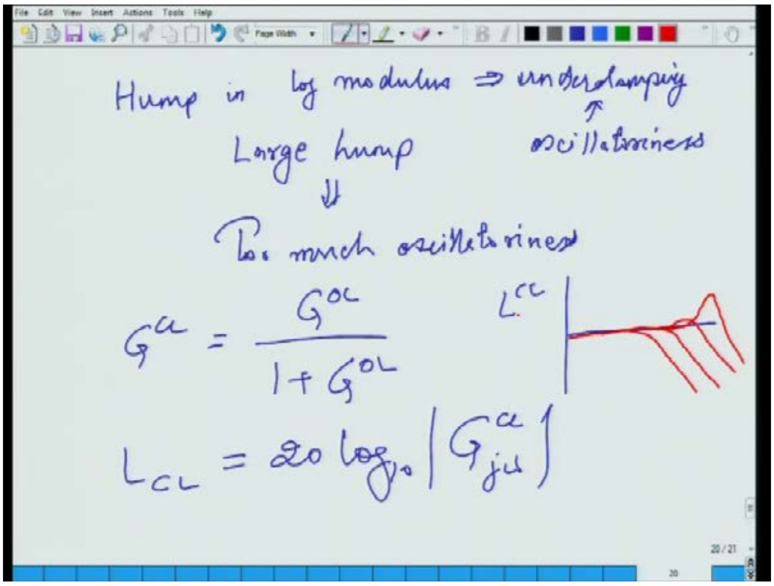

So, the basic idea is basic idea is hump in log modulus implies under damping, and under damping is a sign of oscillatoryness, you want an under damped response you want some oscillation. Because, that means the close loop system is tune for a fast response, but if you tune it, if the gain is too much. If you tune it too tight, then those oscillation would not die out, so oscillatoryness is good, but too much oscillatoryness is not good. So, large hump implies too much oscillatoryness; that means you are close to instability, too much oscillatoryness, you define G close loop is equal to G open loop upon 1 pulse G open loop, that is by the transfer function and then you define close loop modulus s 20 log on base 10 of absolute value of G close loop j omega, this is L C L.

And now I reward back, if you are tuning is very sluggish, you do not have a large gain, what you will find is L C L, if I am plotting L C L versus log omega, what you will find is that the L C L curves looks something like this, no hump, no hump means well it means there is no oscillatoryness. If you crank up the gain a little more, what you will find is the controller gain a little more, you will find that the curve moves toward the right; that means the response is becoming faster band width, is increasing response time is close loop response time is decreasing. If you crank up the gain further, you will find that sum of the closed loop characteristics equation roots have gone conjugate, you get a pair of complex conjugate roots, and then as you are cranking up the gain further, you will start getting hump, you crank up the gain further well you will get a larger hump.

So, the fact that a hump is coming in L C L that is indicating oscillatoryness, and the basic idea is if I tune such that, that hump is too large magnitude, then what I will have is an oscillatory response with the oscillation dying down quickly. So, the response is fast and not too oscillatory.

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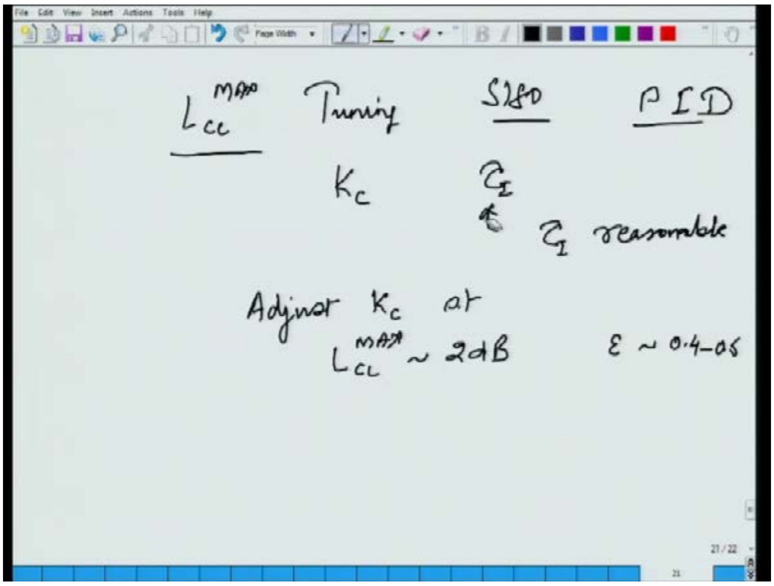

So, that is the basic idea L C L max tuning of SISO system, let us say it is a PI controller let say it is a PI controller, if it is a PI controller, what do I want I will set there are two unknown K c and tau, I set tau I to a reasonable value, which is equal to dominant time constant of the open loop system, or you can say you can, you can for example, get tau I form a ziegler type of tuning method. So, let say tau I to a reasonable value, said tau I to a reasonable value and then adjust K c such that, L c L max is not too large. What is the reasonable value of the about 2 decibel, and 2 decibel corresponds to close loop damping coefficient which is about 0.4 between 0.4 and 0.5. 0.4, 0.5 means the response is not to oscillatory and your over suit will also be of the order 5 percent, 10 percent, no more than that.

This is a typical way of tuning of SISO controller using L C L max tuning, where the dynamics is such that gain you know, the Nyquist plot is such that is not so well behave. So, that if you use gain margin, if you know you get a situation like this, where gain margin or phase margin even though you have good margin and or good phase margin. Yet, your response is too oscillatory for these kind of system, this closed loop log modulus tuning is actually a pretty good way of doing things. Now, let us say I want to tune a PID controller, what is the purpose of the action the purpose of the action is to introduce sufficient anticipation, or as much as anticipation not too much, but appropriate anticipation; so that I can crank up the K c for the same amount of oscillatoryness.

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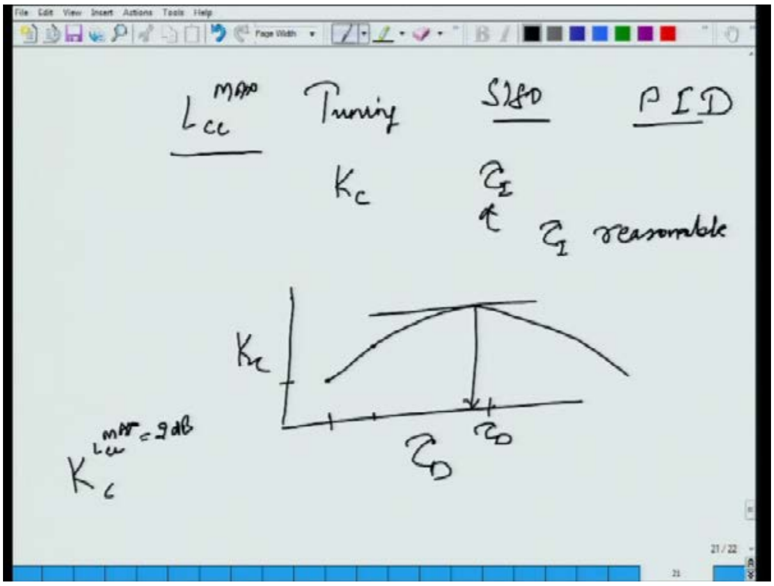

So, what do I do? I calculate as before, I calculate tau I, set tau I to a reasonable value, choose different values of tau D let say I choose a value of tau D, which is here calculate K c for 2 d B, L C L max is equal to 2 d B. So, for this values of tau D and the fixed value of tau I which is reasonable, let say I get a gain like this K c, that I calculate for 2 d B was this I vary the tau D to a different value. Recalculate, you will find that the curve will actually look something like this, so this is the value of tau D, that allows me to maximize my K c without too much oscillatoryness.

Because, that each of these values of tau D, I am calculating K c. So, that the L C L max is 2 d B, so this is my appropriate value of tau D, so this is called close loop maximum close loop log modulus tuning, and you can use it for PI, PID controller. You can also use it for PIC controller, there is no tau I, there just adjust K c. So, that your log modulus turns out to be 2 d B, close loop modulus turns out to be 2 d B.