

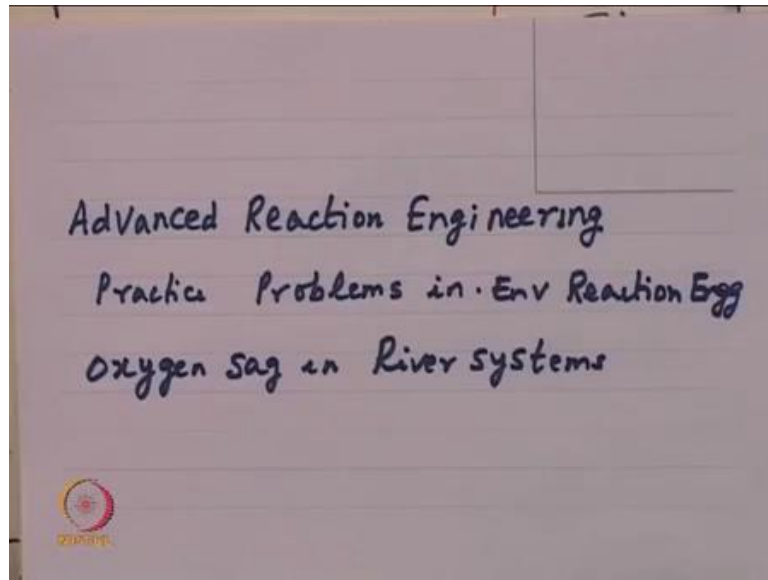
Advanced Chemical Reaction Engineering
Prof. H. S. Shankar
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture - 40

Shrinking Core Ash Diffusion Model and Combination of Resistances

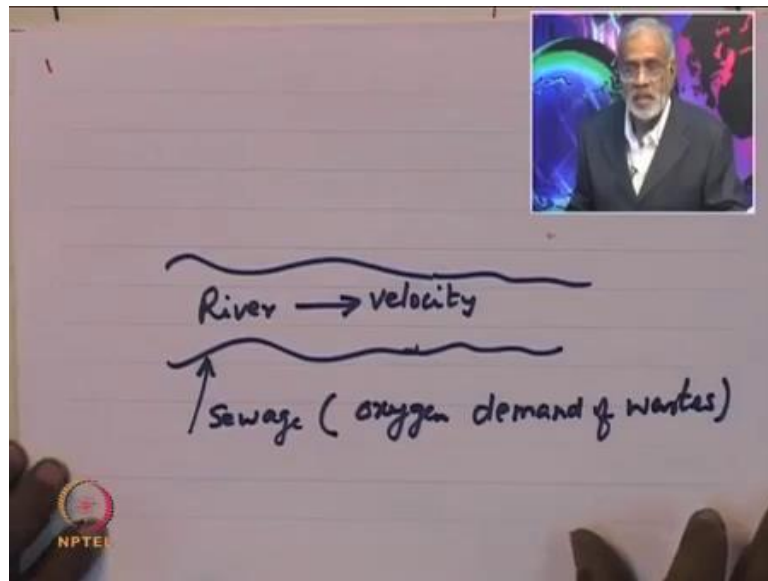
So, what we would like to do today is, to look back at what we have done so far and then apply with for different applications. Specific example I would like to take today is oxygen sag a river systems.

(Refer Slide Time: 00:36)



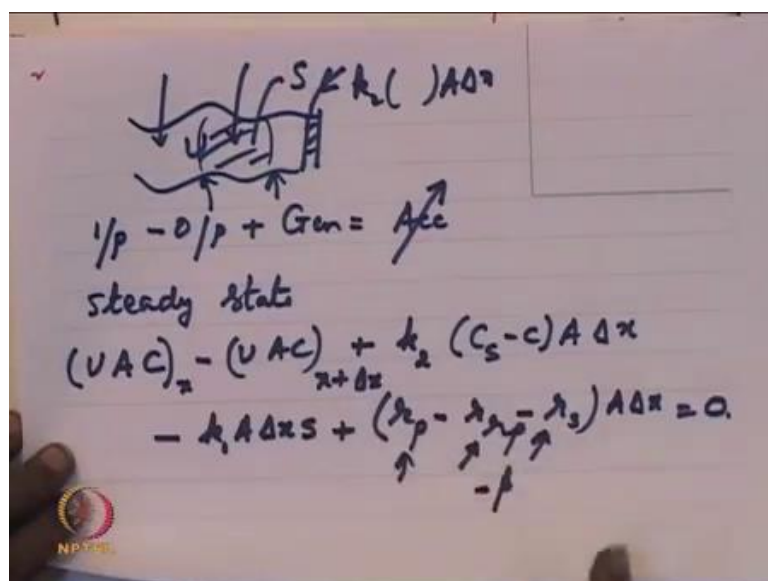
We have spoken about it at an earlier stage, but today we thought we look at in a how various number is come together in trying to understand what happens to oxygen in a river. How the pollution loads that we introduce into the river due to verity of reason either man made or natural. Whatever that is, how it affects the rivers oxygen supply and then how we can restore things like that to do in this exercise. So, let us quickly recall what we have done before.

(Refer Slide Time: 01:16)



So, we have a river system going for say and you have let say Sewage River, now this river may have a certain velocity as is river flows and as this sewage gets mix to this river, therefore the sewage oxygen what is called the oxygen demand of the wastes. So, these oxygen demands of the wastes will start to utilize whatever oxygen that is present in the water whatever that diffuses from the atmosphere into the water. Therefore, you will find the oxygen availability would be different from what it was previously we would like to off course see how this oxygen levels deplete how the oxygen level 0 store itself and so on. So, let us write out to just to recap let us write our equations once again.

(Refer Slide Time: 02:35)



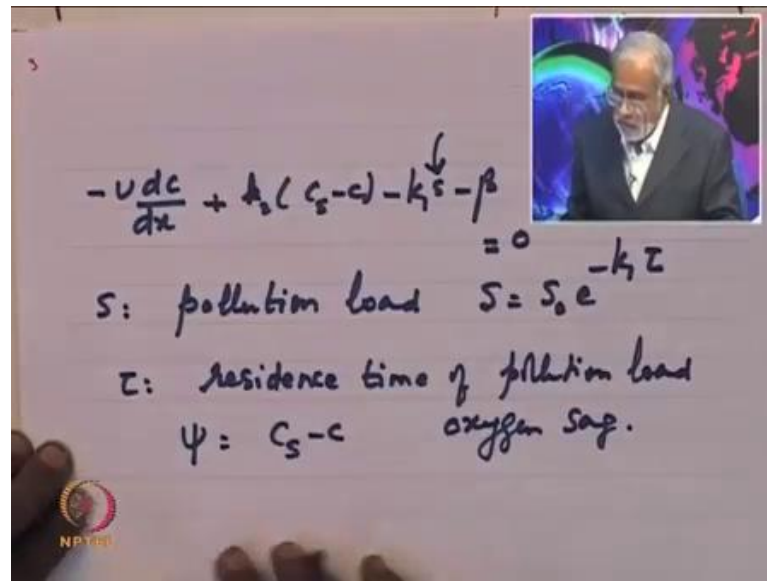
So, we have we have doing writing a balance for oxygen, see input of oxygen minus output of oxygen, plus generation of oxygen equal to accumulation of oxygen. So, let us write it first steady state by steady state we mean that there is no change with respect to time as we go along very were this is not there. So, let us say the velocity of the river velocity of the river is U , so u and cross section area v is a . So, at any position x minus of $U A C$ at any position x , plus Δx , and then you have oxygen supply because of oxygen transfer. So, I will call that $S m k^2 \text{ times } C s \text{ minus of } C a \Delta x$. This is the amount of oxygen that is coming in from the atmosphere oxygen.

You have the oxygen present in the river, which is consumed by the pollution, let us say due to our sewage input the swages what our in this region. So, let us call the swage load is, so this is the oxygen consumed by the sewage plus you have off course what is called as $r r p \text{ minus of } r s$. So, these are this is photosynthetic oxygen production, this is photos this is the spirit oxygen demand and sidemen oxygen demand time a Δx equal to 0. Now, this is something we have done am just doing it once again just to recap what we have done before. So, we have the material flowing into this control volume let us say this is control volume, let us say this is a control volume.

So, this is coming in so much of oxygen coming in so much of oxygen is going out and so much of oxygen is supplied during in this. This is oxygen supplied $k^2 \text{ times}$ multiplied by $a \Delta x$. Then, some under oxygen that is used up, because of the pollution load, which is this quantity. Then, naturally you have photosynthetic oxygen production; you have oxygen consumption, because of respiration there oxygen consumption in the sediments. All that is taken to account and we call this as minus beta we call this minus beta, so let us simplify this, and please simplify this will help me.

So, we can put all these numbers, so what I am going to do now is divide throughout by a Δx and take the limit as extends to 0 and so on. So, when you do that you get minus $U dc dx$, so the first term you can see here please see here b divided throughout $b \Delta x$. So, it becomes minus $U a$ cancels of $dc dx$ o, k plus $k^2 \text{ times } C s \text{ minus } c \text{ minus of } k^1 s \text{ minus of } \beta$ equal to 0 please mixture that our numbers our equations are proper, you can see here. So, let me glance through this is once again, so this gives you minus u times $dc dx$, so this becomes no change in sign this is a minus sign, this is of a minus sign nothing has been nothing new has been done.

(Refer Slide Time: 06:01)



The image shows a whiteboard with handwritten mathematical equations and definitions. At the top right, there is a small video inset of a man with a beard and glasses, wearing a suit, speaking. The main text on the whiteboard is as follows:

$$-v \frac{dc}{dx} + k_2(c_s - c) - k_1 c - \beta = 0$$

S: pollution load $S = S_0 e^{-k_1 z}$

z: residence time of pollution load

$\psi = c_s - c$ oxygen sag.

Now, we have to solve these equations and we have done this before we recognize here this S is pollution load. This pollution load we assume that it decays. Therefore, S at any time is some S naught it is as per of minus k 1 times tau while tau equal to residence time of pollution load. Now, what we are trying to say is that time at if k 1 is the rate constant for the removal of the pollution that we put into the water and tau is the residence time. Therefore, any instant at any position after at time of residence in fine, that the value of s would be this.

So, that means s not minus the S would have been the consumption of the oxygen of the river. Now, we have done all these things if you would recall we have defined atom called oxy, this is what is called oxygen sag, and this is called oxygen sag, why it is called oxygen sag? You would generally expect the river to be at near saturation, but other moment it is a different from saturation. So, it is this difference, which is called oxygen sag what is important for us is to recognize, that the oxygen sag should be as small as possible, showing that the oxygen level in the river was very close to saturation. So, that aquatic life is able to performance natural functions.

So, we have solved this problem for the situation of our interest I just write down this solution, because we have done this before. So, we will not do this again, because it is already well known to us, so we will not do this again, I will just write down the solution that we know.

(Refer Slide Time: 08:11)

$$\psi = \frac{k_1 s_0}{(k_2 - k_1)} (e^{-k_1 \tau} - e^{-k_2 \tau}) + \frac{\beta}{k_2} (1 - e^{-k_2 \tau}) + \psi_i e^{-k_2 \tau}$$

k_1 : pollution Removal Rate Constant
 k_2 : Re aeration Rate constant
 $\psi_i = (C_s - C_i)$: initial deficit

So, what we have shown before ψ equal to $k_1 s_0$ divided by k_2 minus k_1 , $e^{-k_1 \tau}$ minus $e^{-k_2 \tau}$ plus β divided by k_2 $(1 - e^{-k_2 \tau})$ plus $\psi_i e^{-k_2 \tau}$. So, this is the solution we have already gotten, here k_1 is pollution removal rate constant and k_2 is re aeration rate constant ψ_i is initial sag, what is initial sag? It is C_s minus whatever is the oxygen at suppose we have the river like this, so you have pollution coming in.

So, as soon as we mix the two, whatever is the initial level of oxygen in the river as soon as it is gets mixed up with the pollution, that it is called initial sag, called initial often it is called as initial deficit, now the problem that we would like to solve. Let me just quickly displaying the problem that we want to solve, see the problem we want to solve is of following.

(Refer Slide Time: 10:14)

Handwritten notes on a whiteboard:

- $C_s = 6 \text{ mg/L}; 90\% \text{ Sat}$
- $0.1 \text{ km/hr} \rightarrow 1000 \text{ m}^3/\text{d}$
- $C_i = 5.4 \text{ mg/L}$
- $100 \text{ m}^3/\text{d}$
- $\text{COD} = 500 \text{ mg/L}$
- $\text{DO} = 0.0$

NPTEL logo is visible in the bottom left corner.

You have a river, and then you have its coming in at thousand cubic meters per day, you are putting in 100 cubic meters per day of pollution containing COD equal to 500 milligrams per liter. It does not seem have much B O D in the sense that, this is an industrial pollution, which is C O D, B O D has been removed in the treatment, what is left behind only C O D, so it is going into the river. Now, it is coming with door oxygen as in nil, that means there is no oxygen in this water, while the river the oxygen is coming in at it is oxygen solubility is given as 6 milligrams per liter, its 90 percent saturation.

So, it is here is c equal to 5.4 milligrams per liter, so river velocity 0.1 is not flowing velocity is low per river 0.1. Now, this numbers are chosen just two sweet of swam come as closes possibility some other problems that we main countering tropically is in such is ours because of particularly the Deccan rivers. For example, there is in much waterfall much in the year except in during monsoons. Whatever river flows, it would be largely due to a dam, which holds water, and then release the water as per the requirements of the downstream population. Therefore, the river flows may not be large, so keeping that in mind the velocity is there mentioned here are very low.

So, the question is let me just take the questions, so we want to k_1 is given as 0.3 per day in k_2 , which is the re aeration rate constant, it is given as 0.7 per day k_2 equal to 0.7 per day, so this is what is given.

So, we have to find out max sag number 1, number 2 where max sag occurs and 3 where downstream reaches 90 percent saturation. Then, where downstream and four what is the C O D when max sag occurs some usual questions. These are all usual questions that we would like to know about a river, when it is affected by a humoring defense. So, someone these things of course, we have done, so this is not new to us, so let us look at this whole situation one by one. So, our equation is recall here that this is the equation; this is the equation that tells us what the sag at is for different residence times.

Now, what is psi is defined as saturation minus of c I, so first thing to start with we have to find out what is the initial state. So, let us find the initial state and then once you know the initial state we can do all the rest, so what is the initial state? The initial state is now let you say we have 6 into 0.9, so this is 5.4 into 1000 cubic meters per divided by total flow is 1000 plus 100. So, this is the D o in the river, so is it correct, so the multiplied by 100 multiplied by 0, so what is D o in the river. Now, let us calculate what is the D o in the river as soon as the pollution reaches, so we have calculated.

(Refer Slide Time: 14:42)

$C_i = 4.9$ $C_o D = 50$
 DO in River

$$\frac{(0.9)(0.6)1000 + 100(0)}{(1000 + 100)} = 4.9 \text{ mg/L}$$
 COD in River.

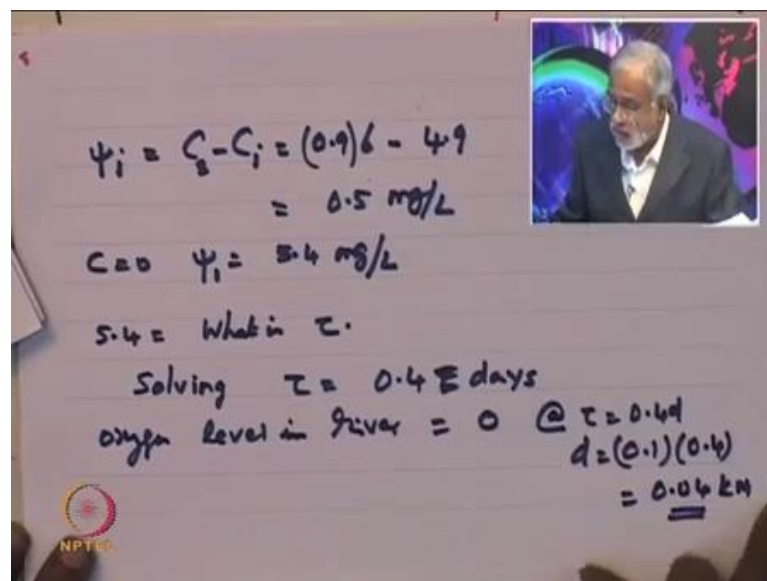
$$\frac{(500)(1000) + (1000)(5 \text{ mg/L})}{(1100)} = 50 \frac{\text{mg}}{\text{L}}$$

So, D o in river equal to 0.9 multiplied by 0.6 multiplied by 1000, this is what is coming in plus from swage there is nothing is coming divided by 1000 plus 100. So, we can do this simple calculation that comes into 4.9 milligrams per liter is the oxygen. Now, similarly, C O D in river C O D in river is how much you are coming up 500 multiplied by 1000 correct? Such C O D 500 multiplied by 100, 100 is the swage, that is coming in.

Then, you have thousand cubic meters into it is coming in at what is mentioned. We correctly check what is coming at 5 milligrams per liter C O D of the incoming water is 5 milligrams per liter. So, divided by 1100 that is it is 1000 plus 100 is 11, so when you do this calculation, which comes out be 50 milligrams per liter what are we saying, now we have a river into which swage has enter. Then, as a result of that we C O D here is 50 milligrams per liter oxygen, which is C i equal to 4.9.

So, there is the state at the point why the swage has entered, this is the state of the river at the time why we enter this. Our intention is what happens as it goes a long how long what is the timing what is the residence time or what is the length of the river, that it takes for it to restore itself. So, what this is the question that you want to answer, but we have solve this kinds of problems before.

(Refer Slide Time: 16:39)



Handwritten work on a whiteboard:

$$\psi_i = C_s - C_i = (0.9)6 - 4.9$$

$$= 0.5 \text{ mg/L}$$

$C=0 \quad \psi_i = 5.4 \text{ mg/L}$

5.4 = What is τ .

Solving $\tau = 0.4 \text{ E days}$

oxygen level in river = 0 @ $\tau = 0.4$

$$d = (0.1)(0.4)$$

$$= \underline{\underline{0.04 \text{ E d}}}$$

So, we can very quickly find out what is psi i by definition it is C s minus of C I, which is 0.9 multiplied by 6 minus of 4.9 that becomes 0.5 milligrams per liter, we start like this, is that clear? So, our equation is here, so we want to know when this how low this can become. Of course, we have done some trial calculations etcetera, let us put psi equal to 0 and find out where this becomes at what residence time and this psi becomes 0. So, when we put that when you put psi when we put C equal to 0 or psi equal to 5.4 milligrams per liter. So, what is meant by that when you put C equal to 0 that means at any position psi equal to C s minus of C correct?

So, if you put C equal to 0 then psi becomes C s correct? That is the highest sag that we can have. So, we can see at what position it this very poor, very bad situation arises. So, when you put s psi equal to that corresponding to 5.4 equal to what is the tau, which means what we will have to put psi equal to 5.4. Here you know k 1, you know k 2, you know s naught is 50, you know that entire beta we generally take beta as a 0, because saying that, basically this oxygen is not available for our purpose. So, we take beta as 0, so only thing in that we have to take this term and this term, so we can solve for given psi equal to 5.4.

We can find what is tau it is just fairly is not a difficult calculation, I am not saying it is simple, we can do it graphically what is tau. So, we get solving I am done this for you solving we get tau equal to when we do this value. I will get of tau about 4 per day 0.4 days, is that clear, what are we saying now? What we are saying is that after putting this up to about after about 0.4 days, our velocity is 1 kilometers per hour.

(Refer Slide Time: 18:51)

$$\psi = \frac{k_1 S_0}{(k_2 - k_1)} (e^{-k_1 \tau} - e^{-k_2 \tau}) + \frac{\beta}{k_2} (1 - e^{-k_2 \tau}) + \psi_i e^{-k_2 \tau}$$

k_1 : pollution Removal Rate Constant
 k_2 : de-aeration Rate Constant
 $\psi_i = (C_s - C_i)$: initial deficit

So, this means that after a distance of about 0.4 kilometers, we find that the oxygen has become 0, oxygen of the river has becomes 0. That is the point we have trying to say, which means that oxygen level in river becomes equal to 0 at tau equal to 0.4 days or distances d equal to 0.4 multiplied by 0.1 multiplied by 0.4. That is equal to 0.04 kilometers at this distance, it is become very quickly it cause to very small value, so this

is the first question the second question is, I mean it becomes 0. What happens to C O D and this point what are the values C O D at this point, if I ask you what will you say?

(Refer Slide Time: 20:00)

$$S = S_0 e^{-k\tau}$$

$$(50) \exp\{-0.3 \cdot 0.04\} = 44 \text{ mg/L}$$

$$\tau? \text{ when } S = 5 \text{ mg/L}$$

$$(S) = (S_0) e^{k\tau} (-0.3 \tau)$$

$$\tau = 7.67 \text{ days} \Rightarrow \underline{18 \text{ km}}$$

So, we can calculate that also, what is this C O D at this point C O D at this point, it is equal to $S_0 e^{-k\tau}$. So, S_0 is 50, so $e^{-k\tau}$ is 0.3 with a minus sign multiplied by τ , which is 0.4, so these terms have to be that is about 44 milligrams per liter. Now, the point of interest to ours is I mean what distance after, which you know the systems start to improve it. In other words, I mean we want this value of 44 to become 5, because that is what the river was when it time when it time this entry, in other words what is τ when S equal to 5 milligrams per liter.

So, how do you calculate that we put S as 5 and S_0 as 50, so we can calculate, therefore we can do that calculation, so 5 equal to 50 exponential of minus of $k\tau$ is of 0.3 times τ . So, we can do this calculation 5 by 50 is 0.1 $e^{k\tau}$ is 2.303, so we come 2.303 minus divided by 0.3. This is 2.303 divided by 0.3 equal to 7.6 exact τ equal to 7.6 seven days or implying velocity is velocity is fairly low is 0.1 kilometers per hour. So, 7.67 multiplied by 0.1 multiplied by 0.24 equal to 18 kilometers C O D level reaches the initial, earlier point at 18 kilometers this small error here just correct this for you to multiply this by 24 I just forgot.

(Refer Slide Time: 22:11)

$$\psi_i = C_s - C_i = (0.9)6 - 4.9$$

$$= 0.5 \text{ mg/L}$$

$$C=0 \quad \psi_i = 5.4 \text{ mg/L}$$

$$5.4 = \text{what is } \tau.$$

$$\text{Solving } \tau = 0.4 \text{ days}$$

$$\text{oxygen level in river} = 0 \quad @ \tau = 0.4 \text{ d}$$

$$d = (0.1)(0.4) \times 24$$

$$= 0.96 \text{ km}$$

So, when I multiplied this by 24 it becomes 24 is about 0.96 kilometers, so let us take a long story short that, so if I ask you what is the level of oxygen level, let us write this here because it is its saving to much space.

(Refer Slide Time: 22:33)

$$S = S_0 e^{-k_1 \tau}$$

$$(50) \exp\{-0.3 \cdot 0.04\}$$

$$= 44 \text{ mg/L}$$

$$\tau? \quad \text{when } S = 5 \text{ mg/L}$$

$$(5) = (50) \exp(-0.3 \tau)$$

$$\tau = 7.67 \text{ days} \Rightarrow \underline{18 \text{ km}}$$

what DO at
 $\tau = 7.67 \text{ days}$

So, if I ask you what is what is that D o at tau equal to 7.67 days, which means at after 18 kilometers, so we can calculate that once again our equations are all with us.

(Refer Slide Time: 22:51)

$$\psi = C_s - C$$

$$\psi = \frac{k_1 S_0}{(k_2 - k_1)} (e^{-k_1 \tau} - e^{-k_2 \tau}) + \frac{\beta}{k_2} (1 - e^{-k_2 \tau}) + \psi_i e^{-k_2 \tau}$$

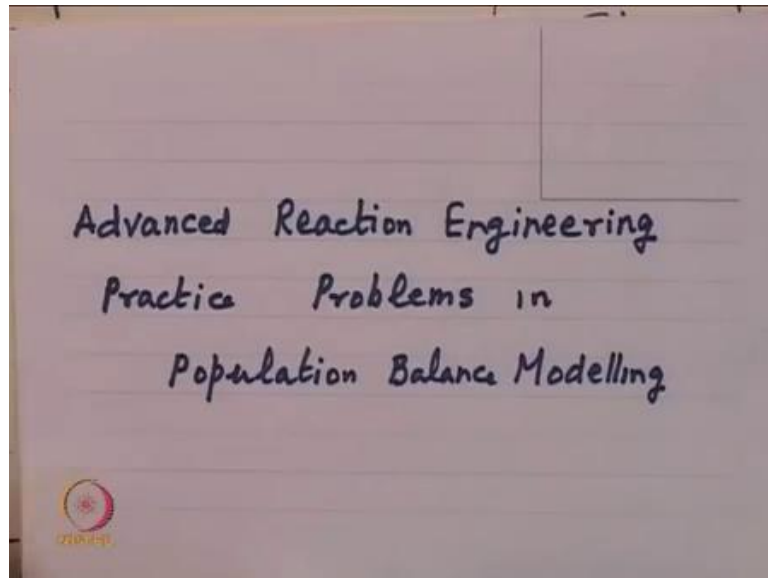
k_1 : pollution Removal Rate Constant
 k_2 : de aeration Rate constant
 $\psi_i = (C_s - C_i)$: initial deficit

We have to once I say that tau is 7.67. We can find out what is the D o at that point, so that means a psi is non means a D o is not, because is simply if you put psi c s is 6 or 5.4. You can find out what is the D o, so what we are trying to say here is that, I mean that point is not forgotten that, if you have an industrial ways coming into water our solutions. You know your require sufficient distance before restore itself very clearly showing that you know our reverse systems, really do not have the oxygen capacity to manage the industrial position. So, we need to look at these problems seriously, see that do not allow our what has to be interfere by you know C O D kind of pollution you must treat them prior to.

So, that what you put into the river is not C O D of 2 and 50 and 305, 100, but some think like the less than 5 less than, so that you know river has the ability to manage the pollutions that is that is entry. So, what in other words you are trying to say is that the natural variation capacity of the river is really meant for the natural biology of the river and not for human genera try pollution. So, human genera try pollution must find its mechanisms of treatment and not be dependent on the river systems, let the point we are trying to put across.

So, this one example is to illustrate how fundamental of reactions nearing comes together to solve or to a to explain how we can manage our system, how we can keep it in good shape and ensure that at our biology of the river is a in good health. So, our dependence on the river system for our health etcetera is ensured.

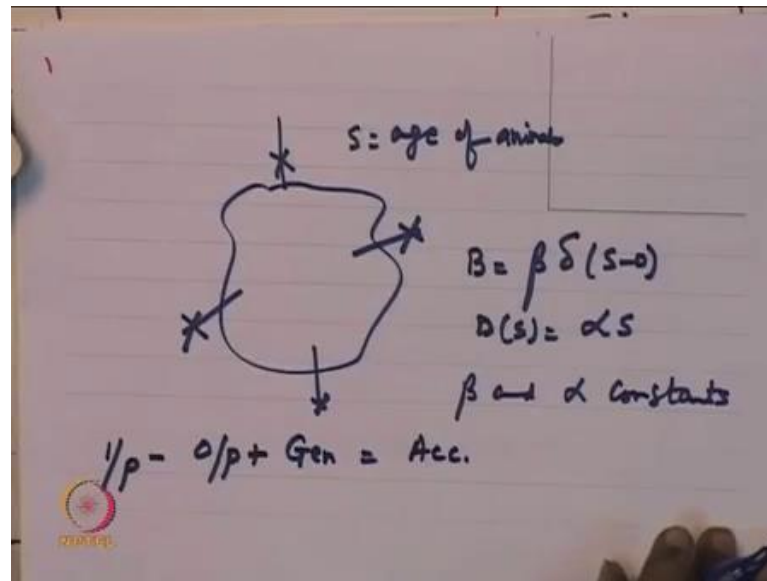
(Refer Slide Time: 24:50)



Let us go to the next example; the next example you would like to look at is something, which all of us have encountered. That is you want to see how we can use population balance modeling to understand a forest. I will still in the general area of environment see our forest are source of water source of a raw materials source of food source of fruit source of timber, name it is forest that really a sustains human populations. Unless we know how to look after our forest we love problems in feature, let us remember forest is not just you no greenery forest as animal, it is this animal and huge indeed by be dense insure the forest is good health.

So, this particular exercise, which you would like do it treat something, this says there are forest, there are animal in this forest it says what is the say animals are bond must be rate S minus of 0.

(Refer Slide Time: 25:42)



So, s is the age of the animals age of animals of bond at rate, which is given, the animals die at the constant rate animals die castrate rate D , which is given alpha times s beta and alpha are constant. So, what happened we have a forest, which is not interference, there is no such movement out of the forest, there is nothing going out what we saying is the close system is a close system in which animals of bond animals grow animals die.

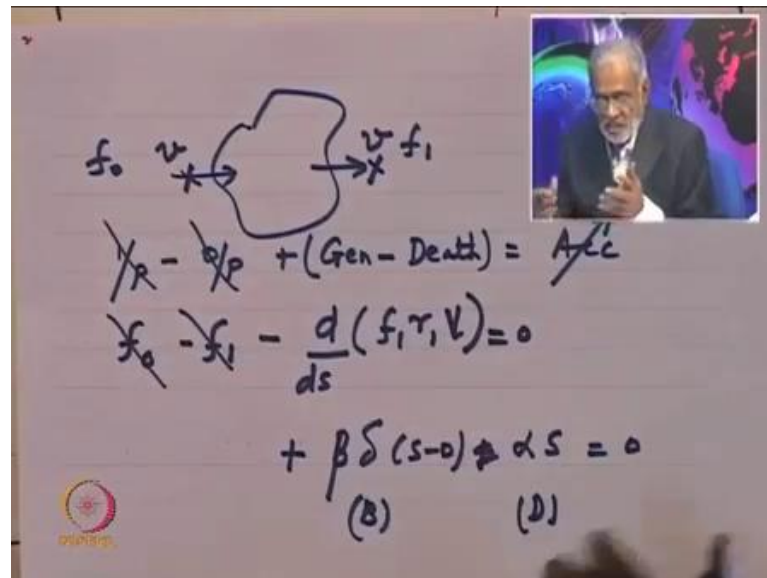
Now, it is reasonable well known is delta exist in the world that a forest left to itself is at study state, by study state what do you mean is at study state by study state what we mean is that if you take example of population. Wherever it may be that population does change it time, in other words the birth death process that is taking going on inside its environment call forest remains reasonably well balanced.

Now, it is when living me if this thing to all of control, in other words of course you are require for depends, we require and to hardest very products on the all these. Perhaps, what is important is that, whatever we interfere are whatever the take of the must be all compare to reason capacity. So, the health forestry essentially and this forest world is this pericardia is a all know forest or forest on the Indian some continent, you name it all over the world its shape and move.

We can understand in the fundamentals of these better it is for less. So, for this particular exercise in just to give your feel for in how things might happen, just a model in does a mean that it is the best way to under understand going on.

It is very simple way understood and in very complex phenomena mind be first, so what is that want to do? What we do now is to write our number balance the about what is happening is forest, so we have let say the balance is input output generation equal to accumulation. This is our balance there writing for long time you must have this are forest, let us adapt this now, so adapt with our forest.

(Refer Slide Time: 28:59)



So, input output plus generation or birth minus death both are there equal to accumulate. Now, we say that it does not change this is the most important point that be want recognized understand and very home and necessary that, forest is essentially studies state if we must understand the fundamental how equipment itself state studies. So, in that if it all the interview we need interview minimally, so this is not disturb to much astral they require the forest for our timber, for the forest for our fruits required for the fire road may be. The required the for managing the carbon dioxide oxygen balance that you know the aerobic problem processes of the environment processes, in all this balances.

So, many issues are connected with healthy forest, so we roundly looking that very small aspects of what going on. So, let us say of the balances if you recall when we wrote our balance certain earliest stage we look like this. So, that is when we wrote for a process is continuous, where it is coming in going out this is f naught this is f 1, so this is the balance that be writing for long time.

If you recall now, we on write this for the case of batch process that what we want write this part. Let us learn to write since there are no in floes and out floes, so these term must be deleted, so this term is deleted we know that, but we must add to this because, i to write this birth and death terms the birth term, which is beta times delta C s minus 0. Then you have other term, which is alpha f. So, this is the birth term, this is the death term, equal to 0, so what of the we have to done what we done is that we have return the material balance, we have derived all there is not going this is once again, we derived all this.

So, we have the population balance in which we f deleted the input term and little the output term where is no input there is no output. The sense out flow of out to the forest we need the terms, so this is the term, which is work we call a change in agent, so this is the birth time and this is the death term.

(Refer Slide Time: 32:12)

The image shows a handwritten slide with the following content:

$$\beta \delta (s) - \alpha s = \frac{d}{ds} (f_1 \tau_1 V)$$

Below the equation, it says: $r_1 =$ Rate of change of property.

Below that, it says: $\tau_1 = \frac{d. (s)}{dt} = 1$

There are some additional handwritten notes: "(age)" and "s=t" with a circled "s" below it.

So, what we have beta delta half S minus 0 not to 10 this alpha f a s equal to d by d s half f 1 r 1 v this is alright what we say now? When we talk about this is what this is rate of change of property what is property this property is a each. So, it is sense it is time, so our this r 1 by definition, this is what I call these property as s, so we have d by d t half our property, which is s, which is age and age is times here s equal to t. Therefore, it is equal to one something we have to said when we talk about resistance time distribution from we recall.

We will say this earliest stage we talk about resistance distribution term, there, we said that there we said that if you want find out the resistance distribution why am to do this again and again is that, if you learn how to play population balance. We are talking about numbers when we can deal with variety of situation of daily like this madly walk mode human of shares in the market.

(Refer Slide Time: 33:49)

$$\cancel{f_0} - \cancel{f_1} + (\text{Gen} - \text{Death}) = A/c$$

$$\cancel{f_0} - \cancel{f_1} - \frac{d(f_1, r, v)}{ds} = 0$$

$$+ \beta \delta (s-0) \approx \alpha \bar{s} = 0$$

This may be you know how population of a town village extra changes variety of things of we would like to know in daily life. So, it thought it important we try to understand this that it closes system there is no input there is no output. Therefore, remove the thus two terms generation because the birth and there is a death. There is a continuously can happen at all times it is over taken in both terms, therefore this particular term, which talks about what happen inside the forest that effect also take in account. So, birth and death how the affect, how the population changes that effect is shown here. So we have are equation I put r 1 equal 1 here minus equal to 0.

(Refer Slide Time: 35:00)

$$r_1 = 1$$

$$\beta \delta (s-0) - \alpha s = \frac{d}{ds} (i.v. f_1) = 0$$

$$\beta \delta s - \alpha s = \frac{d}{ds} (v f_1) = 0$$

$$\left(\frac{\beta}{v}\right) - \left(\frac{\alpha}{v}\right) \text{ constants} = \frac{\beta'}{v} = \beta$$

$$\frac{\alpha'}{v} = \alpha$$

$$\boxed{\beta \delta s - \alpha s = \frac{d}{ds} f_1}$$

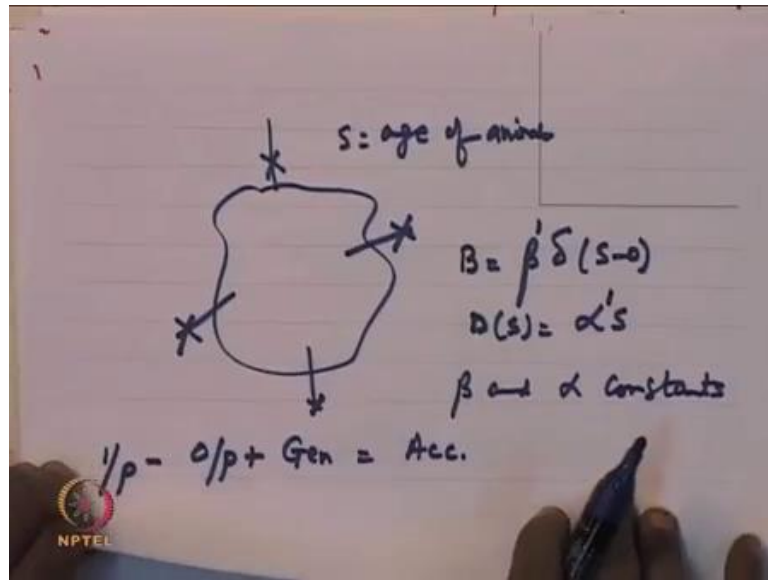
So, our equation looks like this is beta times delta of s minus of 0 minus of alpha f minus d by d s of r 1 equal to 1 shown just now multiplied by v distribution find f 1, is it clear? So, we have r equation beta of s minus of alpha s equal to d by d s of b times plus 1 beta by v and alpha by v. There are constant beta by v alpha by v are constant. So, that now we have I mean for the movement denote beta by v as beta alpha by as alpha. For the case of this is beta when I, we call this beta dash, so that this becomes beta this becomes alpha s equal to d by d s half s 1, so minus or minus equal to 0 equal to 0 now, so delta of s delta of s, so our equation, so what is that we have now?

(Refer Slide Time: 36:42)

$$\beta \delta (s-0) - \alpha s - \frac{d}{ds} (f_1) = 0$$

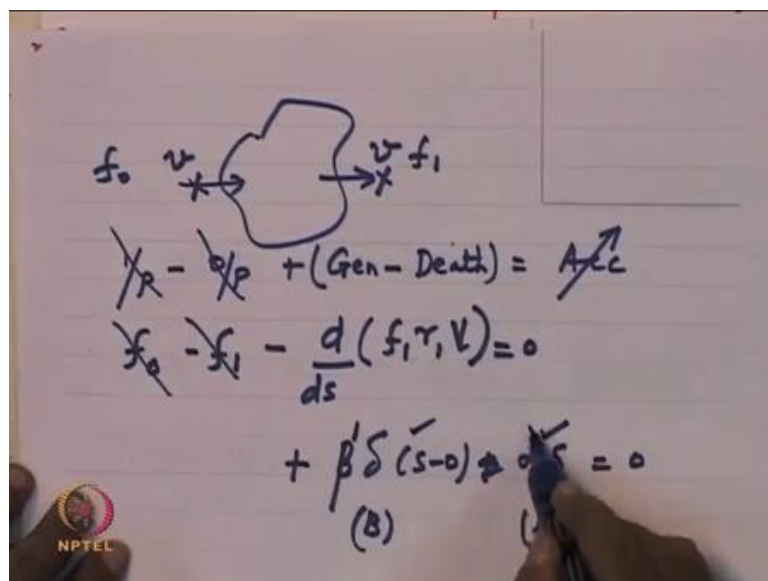
What we have now is beta time delta of s minus 0 minus of alpha times s minus of d by d s half f 1 equal to 0. This is the equation that we have to solve, now this one to spend few minutes here and go through the whole thing once again, please bear with me all put in dash here.

(Refer Slide Time: 37:10)



So, we have a forest where the birth rate given is a beta delta s minus 0 death rate is given is alpha s minus half 0, this is r equation that is kind the numbers density.

(Refer Slide Time: 37:21)



Now, we said that no input and output, how the start with population balance by there is no input, there is no output. That is how this equation looks like this birth and death through on.

(Refer Slide Time: 37:34)

$$\beta \delta(s) - \alpha s = \frac{d}{ds}(f_1, r_1, V) = 0$$

$$r_1 = \text{Rate of change of property.}$$

$$r_1 = \frac{d}{dt}(s) = 1$$
 (age) $s=t$

So, then we said that we can write this dash beta dash, then so that input this rate of changes property are r_1 as 1, because we said s refers to age f refers to time delay, therefore, d/dt that how b is substitute all this minus 1, that is how we said r_1 is 1. Then beta dash b we said beta therefore, we called this equation.

(Refer Slide Time: 38:00)

$$\beta \delta(s-0) - \alpha s - \frac{d}{ds}(f_1) = 0$$

$$f_1(s) \text{ age distribution in front}$$
 I) Solve homogeneous eqn.

$$-\frac{d}{ds} f_1 - \alpha s = 0 \Rightarrow f_1(s) = -\frac{\alpha s^2}{2} + C$$

Therefore, we have this equation which full tell us this is the equation, which we tell us how what is the f_1 it is the age distribution age distribution in forest, how the age distribution forestry, how this age distribution is a fest it by beta and alpha. In other word how birth and death process adjust and self to keep the age distribution and change in the forest this is what we times now solve this equation. We said that we will remove the unbounded term and solve the humanism equation.

Then, get the consort the integration generator boundary chemical, therefore, solve first is solve homogeneous equation what is the homogeneous equation $d f_1$ by $d s$ minus half s equal to 0, what is the solution? We can say f half s equal to minus alpha s square by 2 plus c , this is the elementary theme this nothing to thing much here I called f want solution is f_1 is s alpha s square by 2 plus c these the solution. So, how do find constant the integration how the find constant integration?

(Refer Slide Time: 39:28)

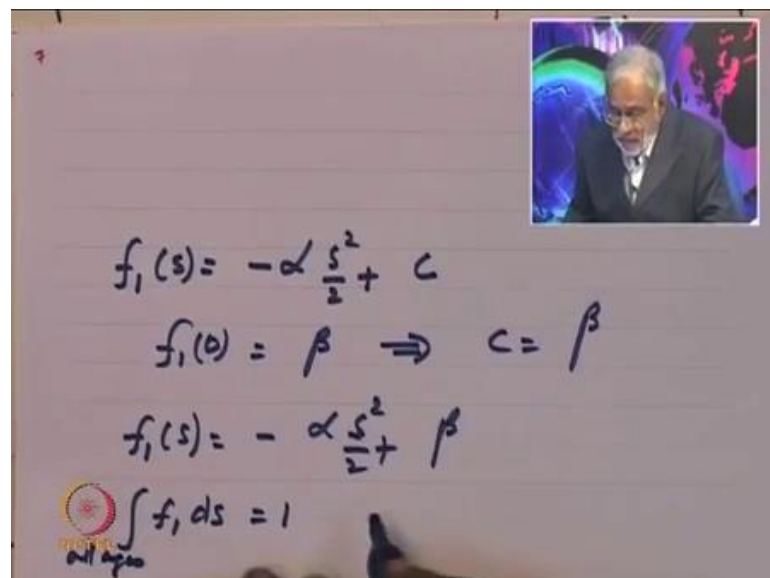
To find constant of integration
 Take Balance $[(s-ds), s]$ $s \rightarrow 0$ $ds \rightarrow 0$
 $\frac{d}{ds} \left(\frac{f_1}{v} \right) - \left(\frac{f_1}{v} \right) + \beta \delta(s-0) ds v = 0$
 $f_1(0) = \beta$

So, find constant the integration to find constant of integration for that what we do is that, we take balance, which is the s minus $d s$ and s ash s tense to 0 $d s$ tense to 0. So, write the all inputs all the outputs all the generations in generations, and then birth death and so on equal to 0. So, there is no input, there is no output, so what is the generation we said f_1 r_1 v and s minus $d s$ f_1 r_1 v at s . Then you have generation, which is β δ s minus 0 s minus half 0 $d s$ into v minus α s $d s$ v equal to 0. This is alright to everybody what we are said we are a writing balance between s minus $d s$.

Therefore, this is material generated it as contributing to s minus s minus d s to s . Then s to outside therefore, this is the difference that accumulates in the interval this is due to birth. This is due to death and there is no generation is this clears what we saying? Now, we have to do in limit at s times to 0 d s times to 0 d s times to 0, this is goes away there is no element that belongs to less then 0 goes away. Then β Δ is d s this whole thing becomes, therefore, this from here we get f_1 r 1 is 1 v cancels of we can see here f_1 at 0 equal to β is it clear what we saying?

How beautifully the material balances come together, so what we have said here I mean this is common sense, this is also common sense, so what are we got we have got here please not this.

(Refer Slide Time: 42:13)



$$f_1(s) = -\alpha \frac{s^2}{2} + C$$

$$f_1(0) = \beta \Rightarrow C = \beta$$

$$f_1(s) = -\alpha \frac{s^2}{2} + \beta$$

$$\int f_1 ds = 1$$

So, our solutions our f_1 of s we said, this minus of α s squared by 2 plus constant that constant of integration and we are said f of 0 equal to β . So, we put β here this gives you c equal to, so our solution is s equal to minus of r squared by 2 plus β . Now, we should have integral f_1 d s equal to 1 that means of all ages all ages in we integrate our all ages this equal tone by definition.

(Refer Slide Time: 42:56)

$$f_1(s) = -\alpha \frac{s^2}{2} + \beta.$$

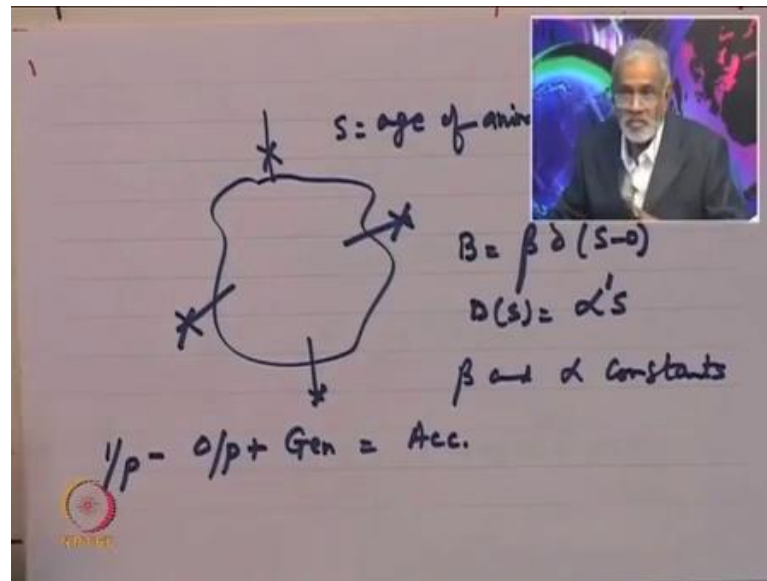
Integrating over all ages assuming $s(0,1)$ normalized

$$1 = \left[-\alpha \frac{s^3}{6} + \beta s \right]_0^1$$
$$1 = \beta - \frac{\alpha}{6} \quad \text{or} \quad \boxed{\beta = \frac{6 - \alpha}{6}}$$

Therefore, that implies do this once again f_1 of s equal to minus of αs square by 2 plus β . Then, integral fit x integrating, please help me integrating over all ages assuming s 0 to 1 normal as all I call it normalized, when I say normalized age. So, this becomes 1 equal to minus of s cube by 6 plus β , this is all right β , shall we say β is a correct β s going from 0 to 1. So, this is become 1 equal to β minus of α by 6 or we get β equal to 6 minus of α by 6, this is the kind of relationship that exist between.

So, this is the kind of relationship that exists between α and β . Let just quickly review this we have said what we have said is that, if we have a forest, if we have a forest, then there is birth and death process in the forest.

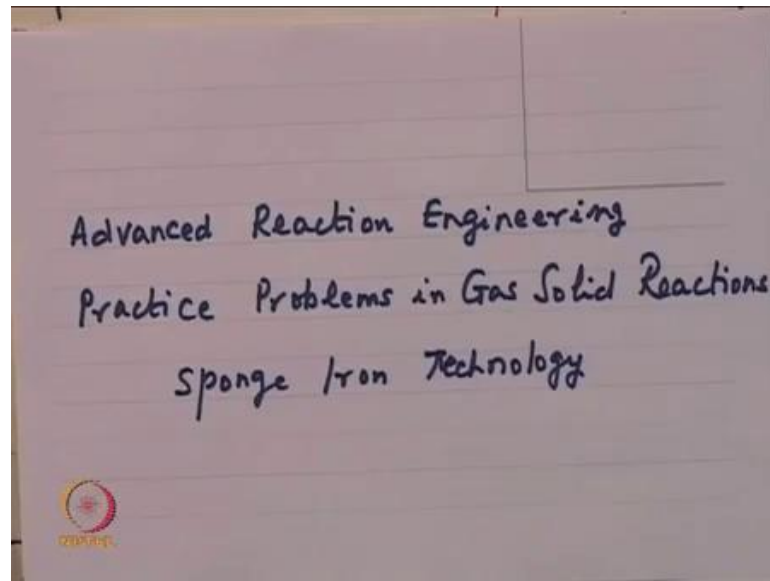
(Refer Slide Time: 44:28)



Then, you will I mean in order that study state, this established we must have that the birth and death process had just themselves otherwise the forest will not be a steady state. So, if there is no external interference if there is interference clearly these problems are almost serious, so the point of this exercise is that that we must appreciate. We must design our inroads in to forest within appreciation with the fact of alpha and beta is related.

We must not disturb those relationships that are the fundamental message that this kind of problems that bring it to us, therefore, to our daily life. So, let us go to the third exercise a relatively simpler kind of exercise, which we want to look at know, which is what we call as the third exercise.

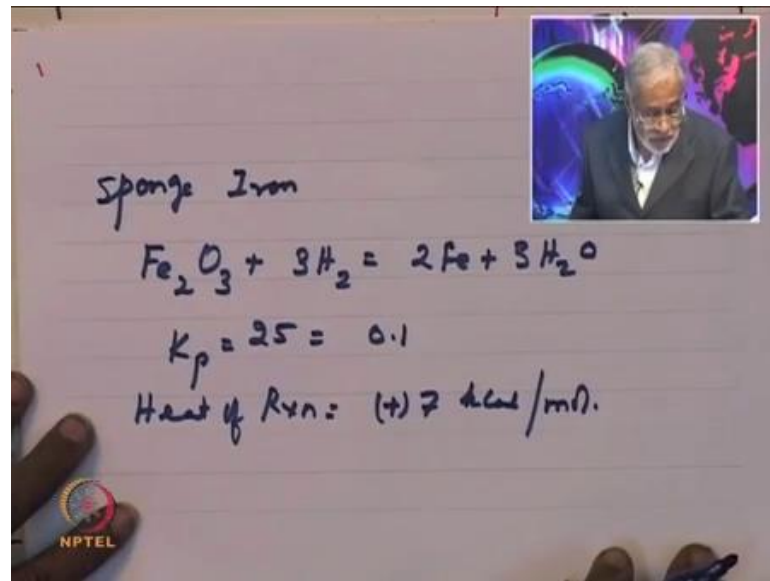
(Refer Slide Time: 45:25)



We want to look at of course it is a very big industry steel making is a huge industry in this world. Steel making comes from blast furnace as you all know. Then, you now start asking in a why is that sponge iron, which is a more recent kind of technology and gaining more and more popularity. The reasons are many now the reasons are that blast furnace technology is suitable for very large scale productions now, and therefore huge investments. Then huge interference they environment so on, but sponge iron is a more expensive technology, but it suitable much smaller scale of operation.

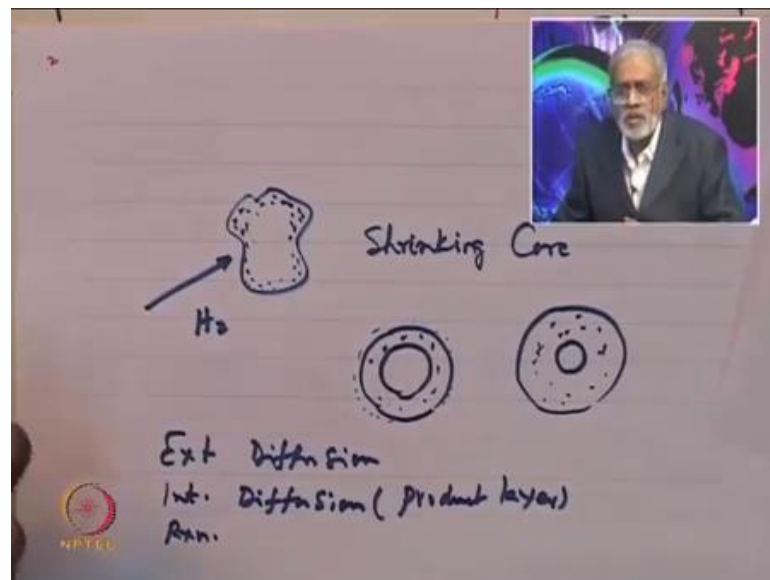
We see that the most important part of technology and I thought we should spent some time and more importantly that some chemical reaction that are so central to sponge iron technology. So, that we appreciate how we can also make sponge iron for various purposes, because once more sponge iron, it is can use making and this is the next part of the process.

(Refer Slide Time: 46:40)



So, we are looking a sponge one what is the technology, F e 2, let us F e 2 O 3 plus 3 H 2 gives you twice F e plus 3 h 2 on notice this K p for this reaction at 25 c is very small. The heat of reaction heat of reaction turns at b 7 kilo cal per mole, it is endowing thermal you need to supply heat energy to be a out of conduct this reaction.

(Refer Slide Time: 47:34)



Now, lets us look at, so you have solid, this is you have hydrogen diffusion as hydrogen goes inside goes inside, then it starts to react and then we say that in our the core keeps on drifting inside, so it is a shrinking core we have done that shrinking core.

So, if you look at a model assuming that it is a spherical particle, so as reaction proceeds we would expect that this is the reacted layer. This is the unreacted layer some other time, if you look at another time, so maybe it is like this, this is the reacted layer. So, what we have external diffusion external diffusion of hydrogen, what is meant by external diffusion that means there is a layer through, which the diffusion of hydrogen occurs.

So, there could be a resistance to a external diffusion, then there is a internal diffusion this there could be a resistance because of internal diffusion, which is the product layer. Then, there could be what is called resistance due to chemical reaction, that means the chemical reaction also of there is at a mode of resistance. So, essentially in any of these reactions, you should expect that external diffusion internal diffusion and chemical reaction. All three of them could be important on other words, when this unable to calculate what the resistance due to each of these controlling steps, let us quickly calculate these controlling steps.

So, we have an idea of the resistances, let us do a quickly calculation write quick calculation for you one minute in write these equations down. Then, know we have if you recall that we have shown we have shown if you have a single pellet see we have done this we have done this before.

(Refer Slide Time: 49:44)

3

$$A(g) + bB(s) \rightarrow \text{Products}$$

○

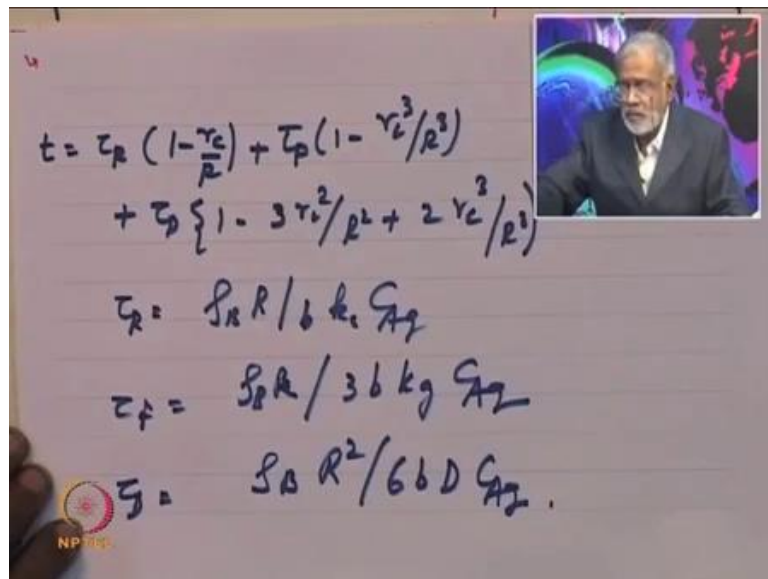
$$t = \tau_R \left(1 - \frac{r_c}{R}\right) + \tau_P \left(1 - \frac{r_c^2}{R^2}\right) + \tau_D \left[1 - 3\left(\frac{r_c}{R}\right)^2 + 2\left(\frac{r_c}{R}\right)^3\right]$$

$$\left(\frac{r_c}{R}\right) = (1 - X_B)^{1/3}$$

We have taken clerical pallets, then we have shown by the times for certain extent of reaction is given by R_c by R . Let us explain this in give a minute, then I will be mat with you r_c cube by R cube plus 1 minus thrice r_c by R whole squared twice r_c by R whole cubed, where we know that r_c by R equal to r_c by R equal to one minus of x b to the power of 1 by 3. So, I am just say a gas plus let us say b b solid equal to products, so if you have this kind of reaction, we have the time.

That is required for the reaction suppose r_c by R can be put in terms of x , that is why let us say we want 50 percent conversion that is why we replace r_c by R . Like this we can calculate what is the time required for 50 percent conversion 60 percent conversion 80 percent conversion. In other words, what we are trying to say here is that we can calculate the time for given extent of reaction is all these numbers τ_r . Let me just put them down because this is let me put them in the next page, because not enough space here bad with me, so this is bad with me I will just write here.

(Refer Slide Time: 51:18)



$$t = \tau_r \left(1 - \frac{r_c}{R}\right) + \tau_p \left(1 - \frac{r_c^3}{R^3}\right) + \tau_d \left\{1 - 3\frac{r_c^2}{R^2} + 2\frac{r_c^3}{R^3}\right\}$$

$$\tau_r = \frac{S_R R}{b k_s C_{A_2}}$$

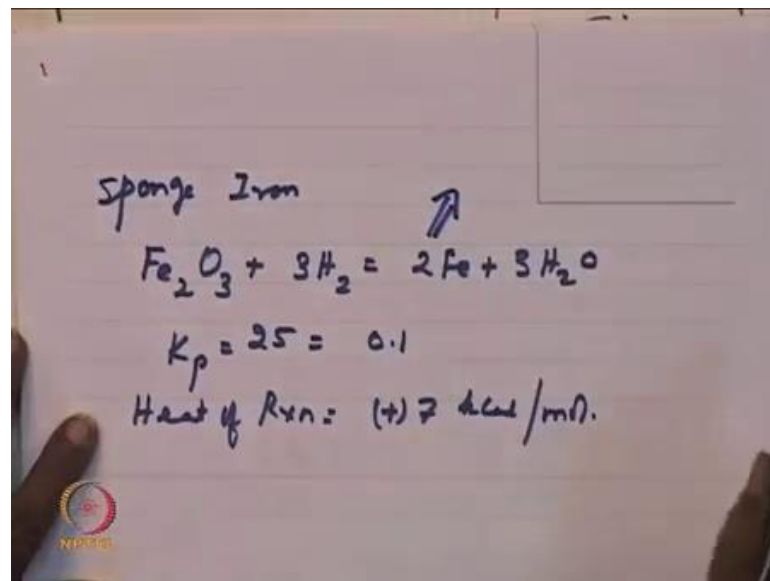
$$\tau_f = \frac{S_R R}{3 b k_g C_{A_2}}$$

$$\tau_d = \frac{S_B R^2}{6 b D C_{A_2}}$$

So, you know t equal to τ_r 1 minus of r_c by R τ_f 1 minus of r_c cube by R cube plus τ_d 1 minus of thrice r_c squared by R square plus twice r_c cube by R cube. Now, τ_r you know this row b r divided by b times k_s times C_{A_2} row f , we may see all these things from our previous exercises thrice b k_g times C_{A_2} and τ_d is equal to row b r squared divided by $6 b D C_{A_2}$.

What we have trying to say here, now is that since these are all main quantities because we know the concentration we know the rate Constant we will tell you how to calculate all this for in this particular example. So, let we are trying to say is if you have sponge iron to be manufacture, then if you know all these number for reaction, which we do will shortly. Then, we can calculate time that required the reaction go to different extent, so we will look at these in detail in next time.

(Refer Slide Time: 52:53)



So, what I am trying to put across is that sponge iron technology essentially is example of reaction between ferric oxide hematite with hydrogen; this is was go out to steel making. Thank you very much.