Advanced Chemical Reaction Engineering
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Lecture - 34
Illustrative Examples: Population Balance Models

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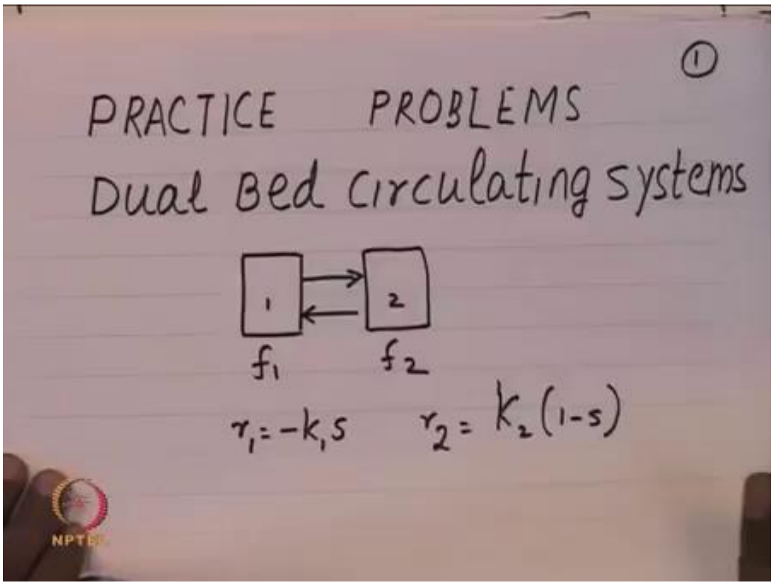

See, we talked about reactor regenerated systems. Just quickly run through what we have already said. For this type of rate functions, we have formulated the population valence equations sometime back. And we got those results, we will not do it again I will just set out what we have already done.

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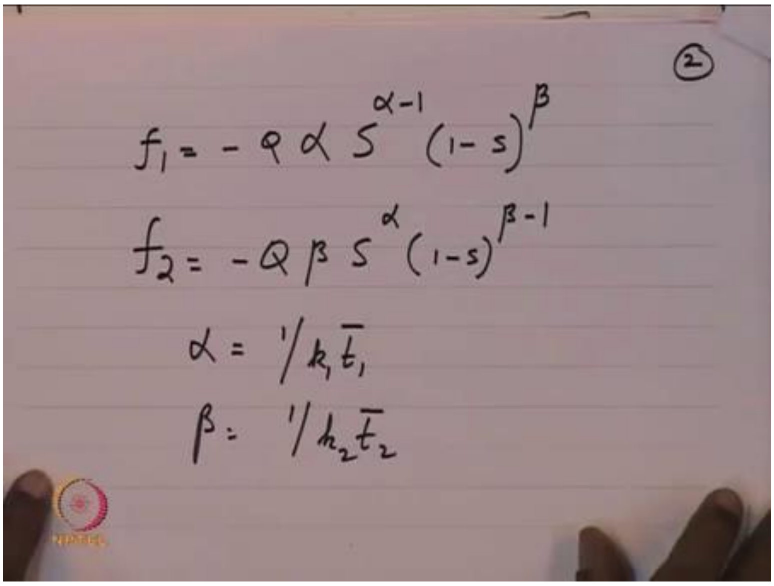

Where we are shown that f 1 is minus of Q times alpha S of alpha minus of 1, 1 minus of s to the power of beta. F 2 is Q beta s alpha 1 minus of S beta minus of 1; where alpha is 1 by k 1 t 1 bar, beta is 1 by k 2 t 2 bar. This is something we have done. Now, for some cases we have taken an example and show out, what are the mean values and so on. What I want to do now, is use this result we look at some, some works that was done, in this department some time ago.

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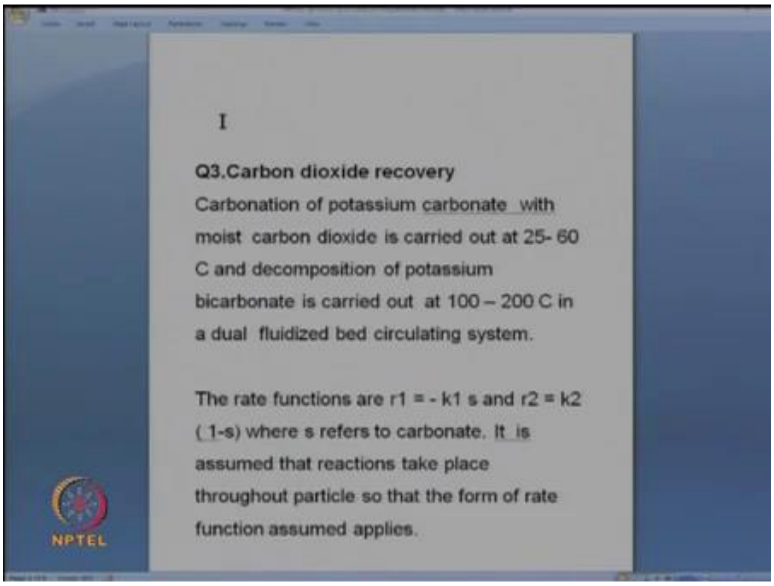

That is use of reactions for recovery a carbon dioxide, I will just set on the equations.

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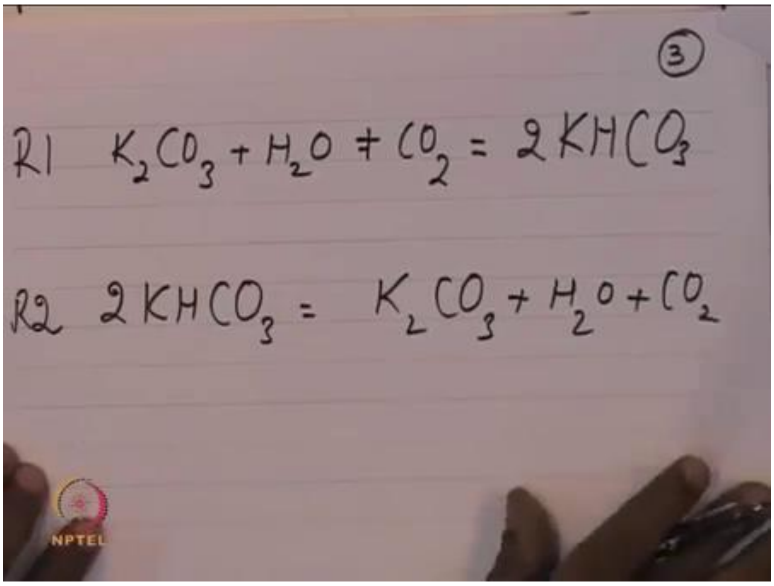

K 2 C O 3 plus H 2 O plus C O 2 equals to twice K H C O 3. And then twice K H C O 3 giving you K 2 C O 3 plus H 2 O plus C O 2. Now, if you in this model reaction, what we having here is carbon dioxide is picked up by potassium carbonate, to give you potassium bicarbonate. And potassium bicarbonate decomposes to give you potassium carbonate. So, that the two equipments, if I call this as a R 1 reactor. If I call this is R to regenerator. If we can appropriately bring them together, you can continuously remove carbon dioxide for some purposes. So, what you have let me just put it out in the context, you have a reactor.

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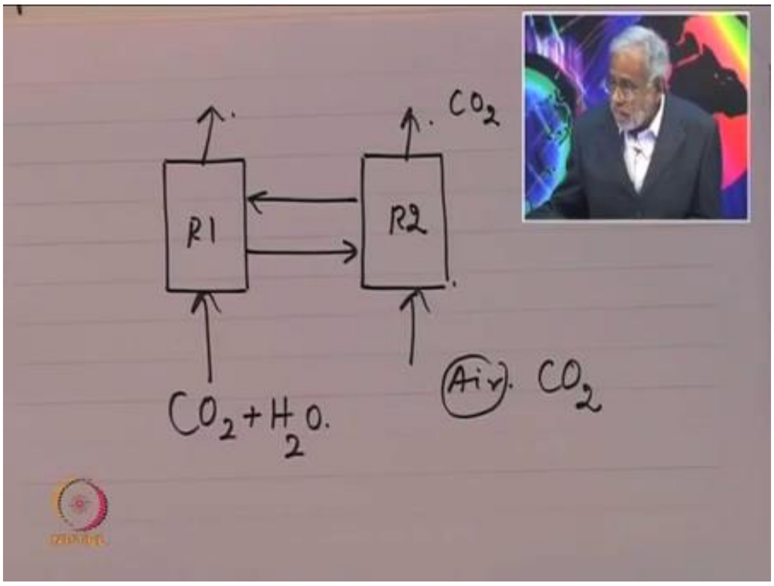

You have another regenerator, solids circulating between the two. You have carbon dioxide plus moisture, goes in and comes out. Then you have air comes out. So that, carbonate circulates between the two environments, is it clear.

Student: ((Refer Time: 03:45))

Now, this is R 1, this is R 2. If now instead using air, if we use C O 2 itself. Then this carbon dioxide you can recover for some uses. So, if you really want recover carbon dioxide. Then you can use C O 2 for the fluidizing gas. And then, recover the carbon dioxide, is that clear. Now, what we have done in the previous sec, is that we have assume certain form of the rate function in R 1 and R 2.

And we determine the populations the activity distribution and so on. So, we want use that result to understand how we can explain this particular data.

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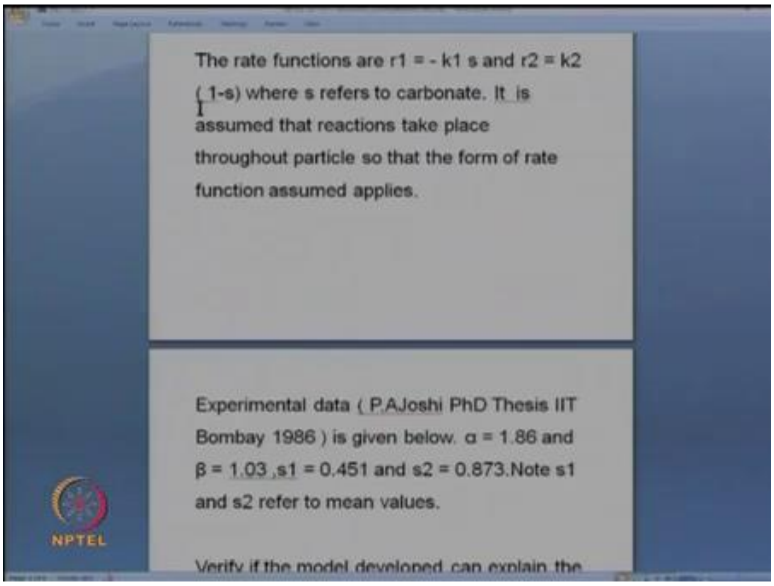

The question is there is some experimental data that has been found. And so the data is given here.

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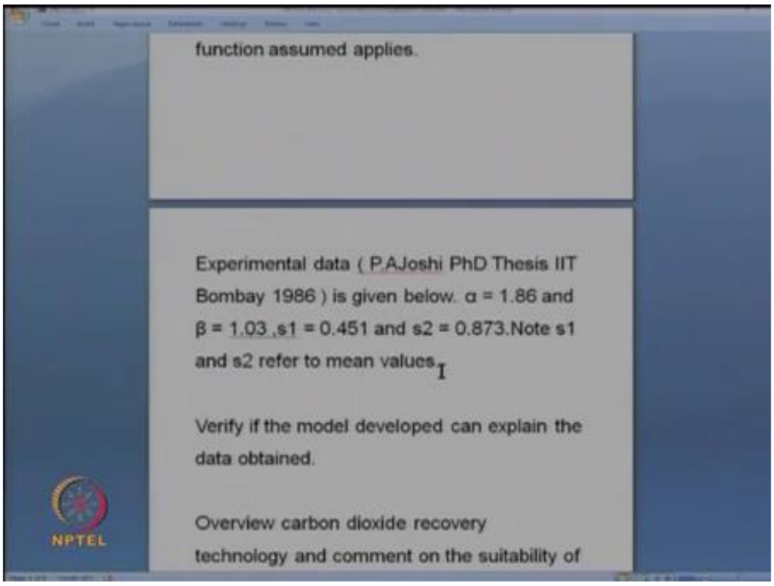

So, value of alpha 1.86, value of beta 1.03. The mean value, this s 1 and s 2 there mean values here. I am not put a bar on the top, s 1 is given as 0.45, s 2 is given as 0.873. So, you would verify, whether this model that we have developed, this is the model. The model is this model says, ((Refer Time: 04:54)) f 1 is this, f 2 is this, and given the values of alpha and beta as 1.86 and 1.03.

What is the mean value is right we should expect from the model, and what are mean value is given in this experiment can we tell how good is the model, is that clear. This is the problem that we want to solve. How do we do this? The procedure is that we first find the value of Q given.

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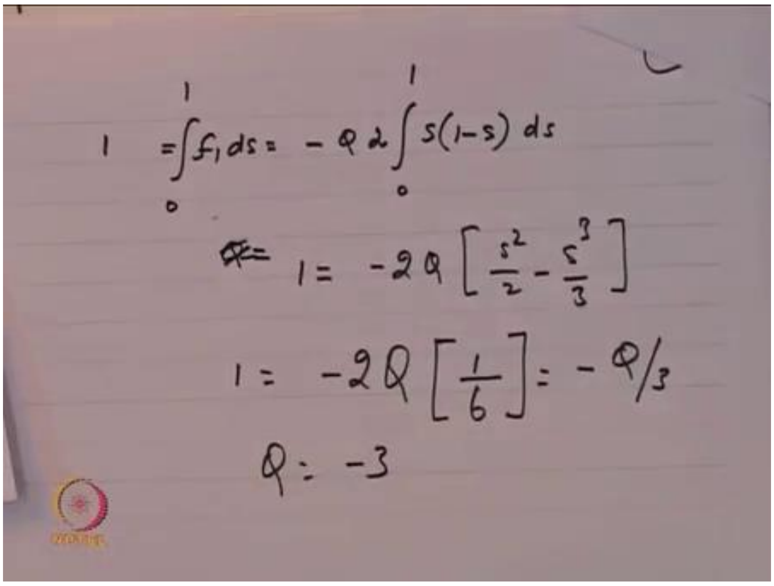

So, to find the value of Q we do integral f 1 d s. What is f 1? f 1 is minus of Q. What is value alpha? Value alpha is 1.86 just to make the integration easier am just putting it as 2, when you go home and do this you can give 1.86 integral s to the power of alpha minus 1. So, s 2 minus 1 is 1. 1 minus beta I take it as 1 d s. Is it alright? So, this is equal to 1. This how we calculate or estimate the value of Q.

So, what is value of Q. Q equal to let us integrate 1 equal to minus of 2 times q within bracket s square by 2 minus of s cube by 3 1 equal to minus of 2 Q, so it is 1 by 6, so it is minus of or Q equal to minus of 3. Now, let us see, there is consistency whether, we get this same value of Q s minus 3 from the other from f 2.

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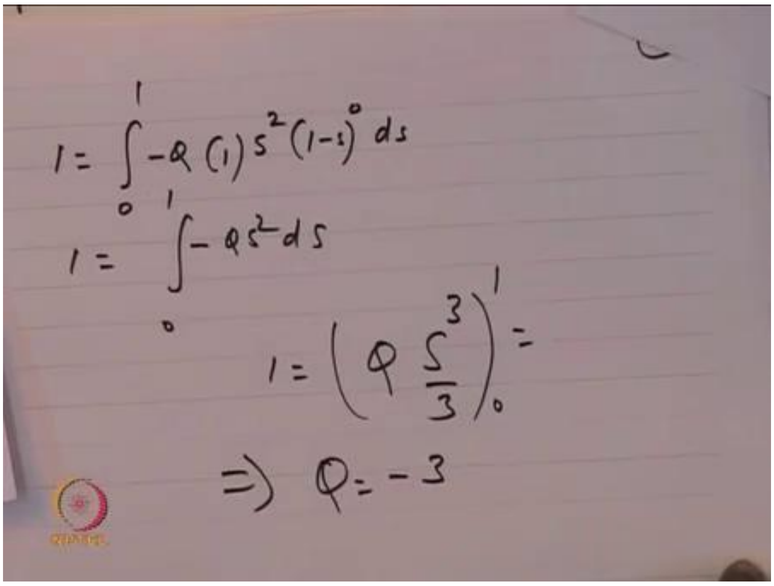

So, integral 1 equal to 0 to 1 f 2, Why does the f 2? f 2 minus of Q beta, beta is 1. s alpha s 2, 1 minus of s beta minus of 1 is 0. So, that is minus of Q s square d s is 0 to 1. That gives you 1 equal to Q times s cube by 3, 0 to 1. Or it also give you Q equal to implies Q equal to minus 3. So, it is consistence, f 1 and f 2 are consistence.

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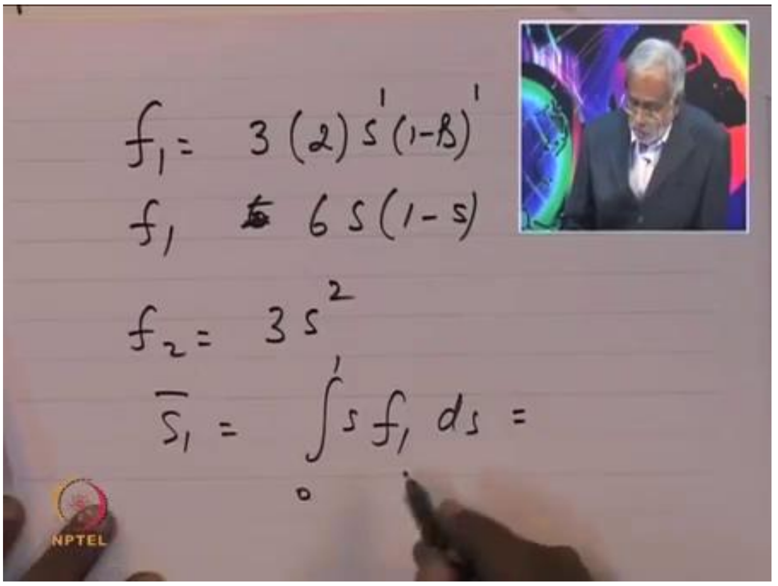

So, we know what is f 1, so f 1 from our this one gives you 3, alpha is 2, s the power of 2 minus of 1 that is 1, 1 minus of beta is 1. So, there is 6 s into 1 minus of s, that is a f 1. f 2 equal to f 2 is 3 beta is 1 3 s square. So, what is s 1 bar mean value of s 1 integral s f 1 d s 0 to 1. The mean value of s and what is s, what is the meaning of s in this problem carbonate, the carbonate fraction in the reactor. Meaning of s in the region rater also is the same carbonate fraction.

So, you would expect the carbonate fraction in the reacted do we less, because it getting consumed. Carbonate fraction in the regenerated to be more, because it is getting regenerated. So, what is this equal to, so let me substitute for f 1. So, it is let me do it in the next page.

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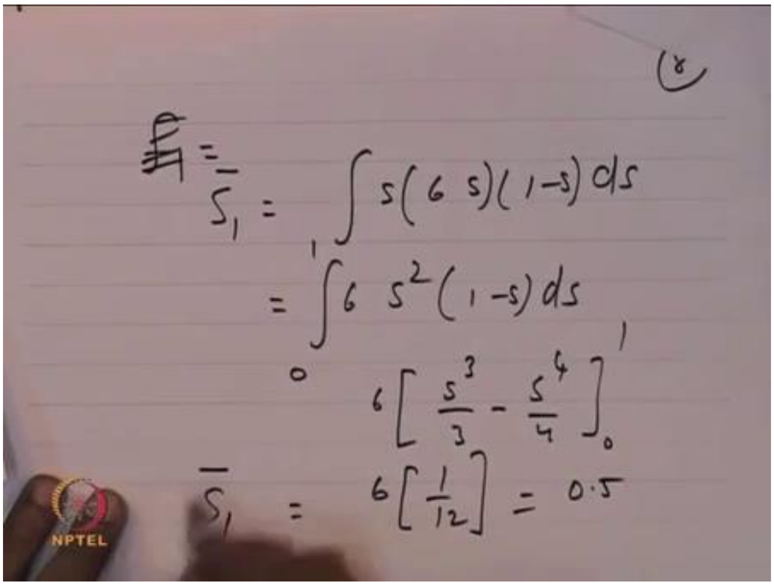

S 1 bar equal to integral s 6 s times 1 minus of s d s. That comes to 6 times square 1 minus of s d s integral 0 to 1. So, that is 6 times s cube by 3 s power of 4 by 4 is 0 to 1, that comes to where's 6 1 by 12 is both 0.5. So, s 1 bar terms out as 0.5.

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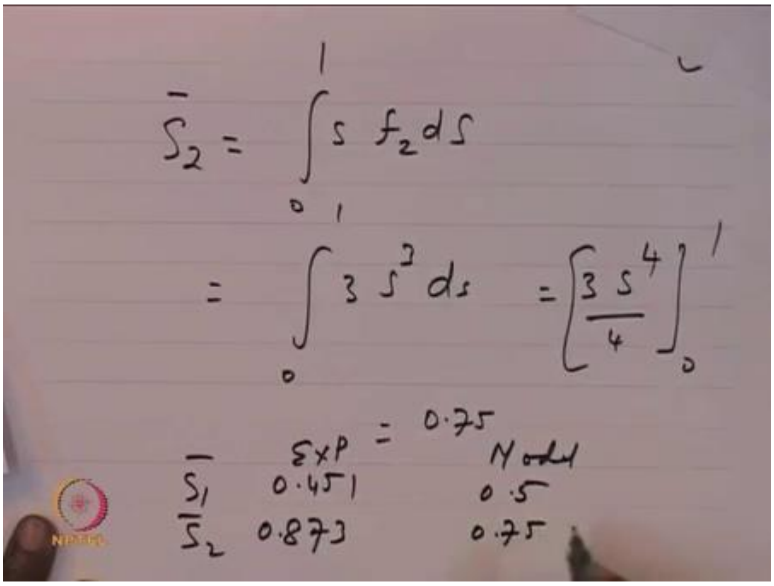

What is s 2 bar, s 2 bar is integral s times f 2 d s is 0 to 1, integral 0 to 1, 3 s cube d s.

That is equal to 3 s the power of 4 by 4, 0 to 1. So, that is about 0.75. Now, the results says it is 0.45, so we just compare s 1 bar and s 2 bar. So, experimentally, it is 0.45 if and then 0.873 and then this is experiment. Model gives you 0.5 and 0.75. Now the context to this a exercise is whenever, we find your model does something well different from the data.

So, you should look at the model, model is not satisfactory. You have to look at what is be in the assumption you have made. And what you can do it to the improve the model, that is the important thing. And in this particular case, I want you appreciate this problem.

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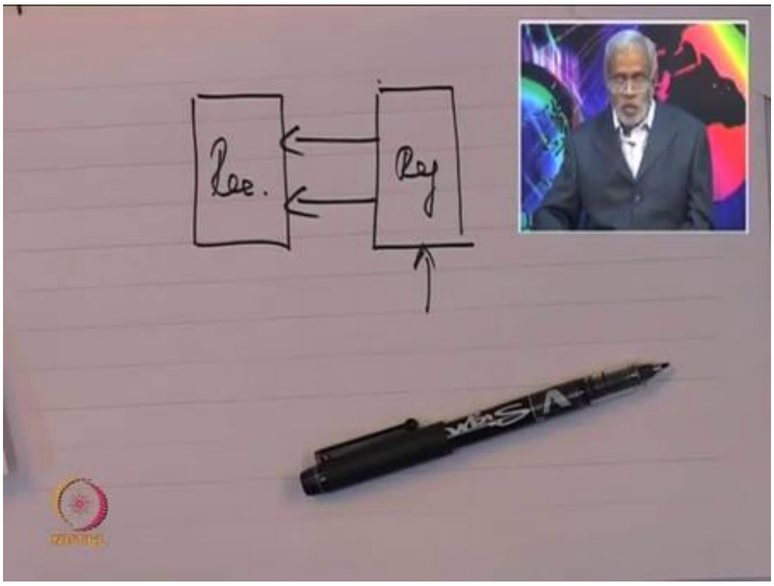

The problem is let me just do it once again. Material coming in an going out. Now, if this is a regenerator, this is the reactor. So, what happens in the regenerator, carbonate is getting generated. So, you would expect that the way the model has been formulated, if look back at the rate function, the rate function says ((Refer Time: 10:46)), k 1 time says. That means, it reacts throughout the partial. As per the model the reaction takes place throughout the partial.

But, in the shrinking code as we have said with gas solid reaction there is a an interface between reactions and the solid. There is an interface, where is the reaction occurs? Or other words, we have model the system using an assumption of uniform reaction throughout the particle. Well, while in shrinking core, you find that the particle actually reacts, the gas only interface. So, this is the replaced the assumptions. Now, when would this assumption the very good. This would be very good, if it is a highly porous particle.

So, that the react is react to able to the take place everywhere. The fact that this data does not fit so well, implies that assumption that this rate function may not be as good. That is the meaning of this. So, the important point I want it appreciate here is that. Every time you postulate a model does not quiet describe the data. We look at the model and make refinement to the model. So, that is the point of this exercise.

Student: ((Refer Time: 11:58))

Yeah.

Student: ((Refer Time: 11:59))

How do you say? Is formulate, because it is a k 1 times s, which means that there is a when we say reaction rate is k 1 times. See, which means that everywhere in the equipment it is reacting at that rate. But, in the shrinking core the particle and the gas wherever is the interface only it is reacting, it is not reacting everywhere. So, that assumption that we have made in this formulation is not what is happening in the reaction.

So, they two are not consistence, that why we are seeing a difference. Is this point cleared all of you? That is why we are not finding a such a good agreement. Let us go to in equally important exercise, which is of course those of you who have look at the cat cracking. What is in the cat cracking process?

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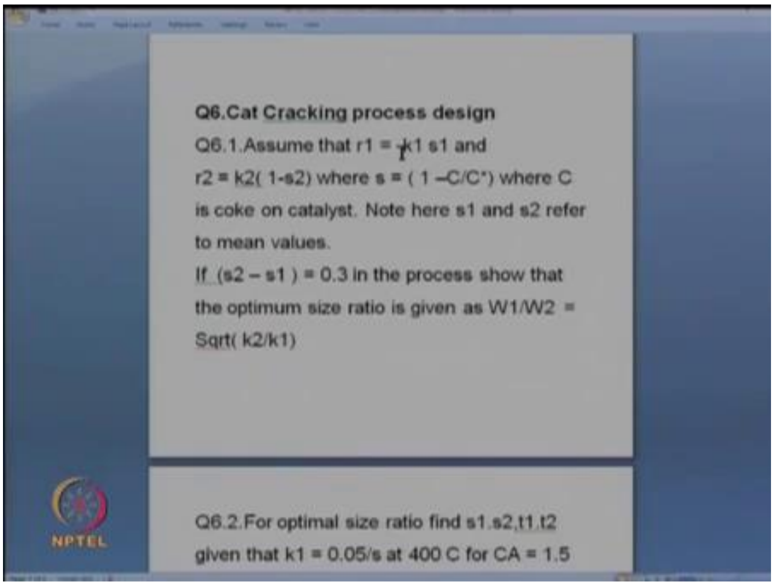

We have a cat list and this cat list is pores. And as the reaction proceeds the cat list has a deposition of cook. So, let me this is problem number 6.

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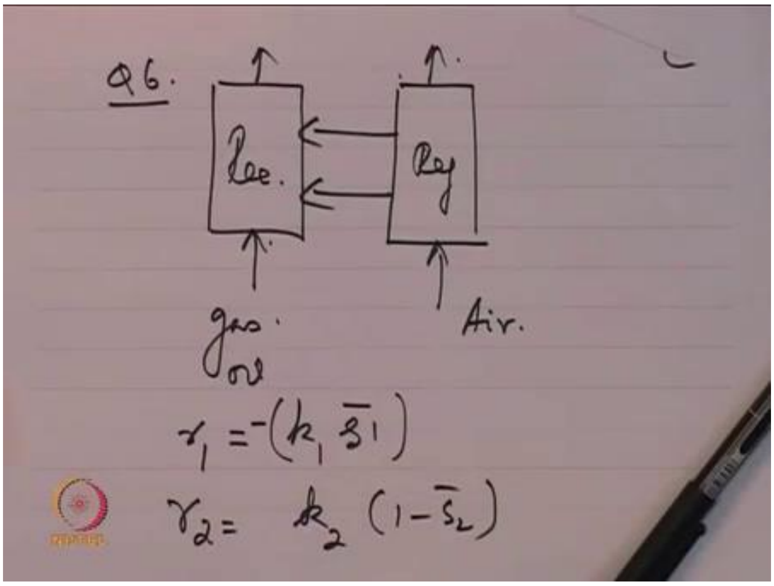

So, this is the cracking reactor you this cracking reactor, where gas oil is coming in it is gas oil say. And it is regenerated with air. So, as this reaction proceeds, as this reaction proceeds the cat list you just present in the react a, there is cock deposition. And because of cock deposition, which is uniform throughout the particle. And this deactivates a cat list. So, the first part of the exercise is that what is the extent to which this catalyst is getting deactivate that is the first part.

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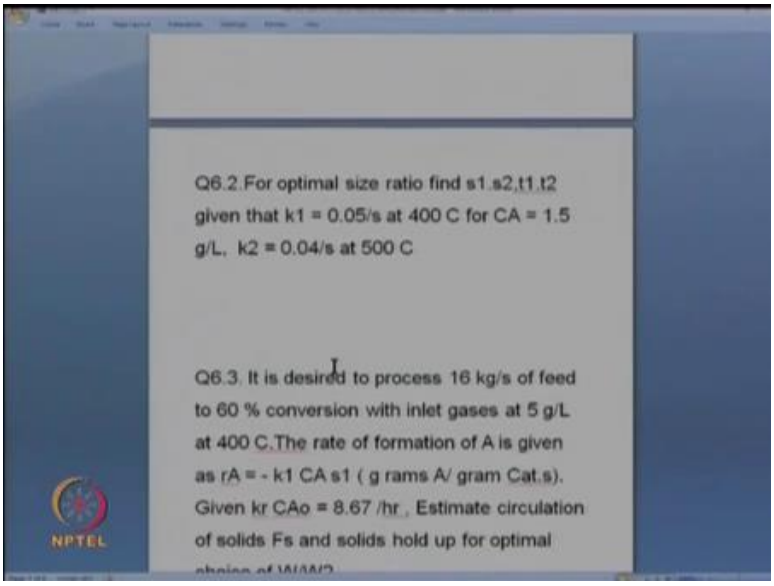

The second part is how do we run this process second part. Third part is that, what kind of in conversion we should expect in our process that three aspects to do it one by one. First part ((Refer Time: 13:55)) says the reactor gets deactivated by this rate function k 1 time s 1, where s 1 now refers to main values. What do you mean, previously we talked about distributions. Now, we are not talking about distribute. We are saying ((Refer Time: 14:11)) R 1 is simply k 1 times s 1 bar with a minus sign.

Indicating that the rate at which this, how we have formulating the problem. Because, we already found out how to determine as 1 bar, we have done that problem already. For the case, we know what is a value of the main value of the catalyst activity. Because, you have done this problem just now. Now, what we are saying is let us now look at this reactor deactivating by this function k 1 time as 1 bar, s 1 bar is known.

Similarly, r 2 which is the regenerator, where it is getting regenerated by this kind of function r 2 equal k 2 time s 2 bar. This is given to you k 2 and k 1 value is a given to you. Now, what this problem now says is if s 2 bar minus s 1 bar ((Refer Time: 15:01)) the mean values s 1, s 2 are mean values. This is let us say it 0.3. What is being said this is a mea value of activity s 1 bar, mean value activity s 2 bar is about 0.3. This how the process is run, you can run the process at different value.

It is depending upon, what you feel is the appropriate to you. In this particular problem, we are saying it is s 2 bar minus of s 1 bar is 0.3 it is given. The question is show that the optimum size ratio of w, that means, optimum size ratio of the reactor and regenerated. That is given by this square route of k 2 by k 1. This is the first part of the exercise. So, let us try to do that problem.

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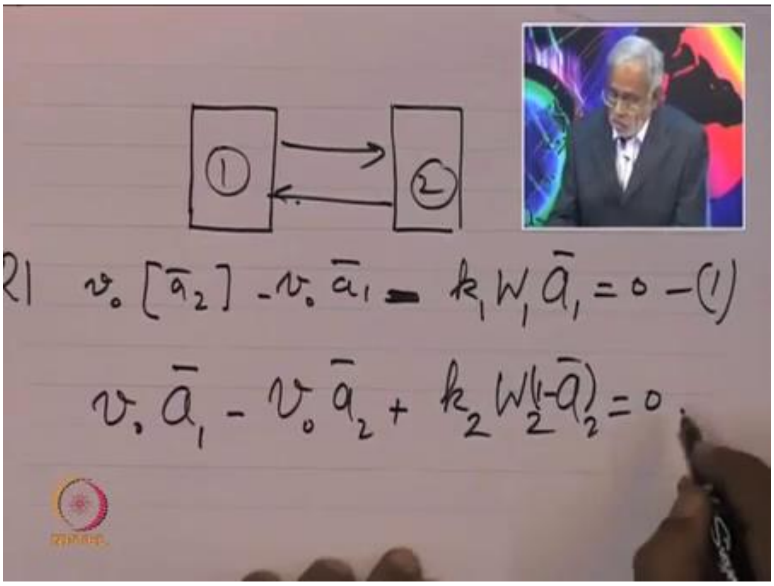

So, I will do it once again. You have react a, we have regenerate a solid just circulating between this two. Now, we are writing a material balance for cock deposition. This is what is responsible for the loss of activity. Or we can also write material balance for activity itself. Because, we understand the activity is because of loss of activity, because coke formation. So, v naught times a 2 bar or s 2 bar I may have written a 2 bar here v naught times a 1 bar minus k 1 w 1 a 1 bar equal to 0.

What we are saying the denote time this is reacted to this is react 1. Material is from related to material is coming at an activity of a 2 bar is flowing at v 0. Similarly, it is going out at v 0 at a 1 bar. And the rated which the activities is lost is given by k 1 bar a 1. So, this is equation 1, this further reactor. Similarly, we can do for v naught a 1 bar minus v naught a 2 bar plus k 2 w 1 a 2 bar equal to 0. But, 1 minus or 1, yes or no alright. we want to solve these two, let us quickly solve. So, what I have written please tell me is it ok.

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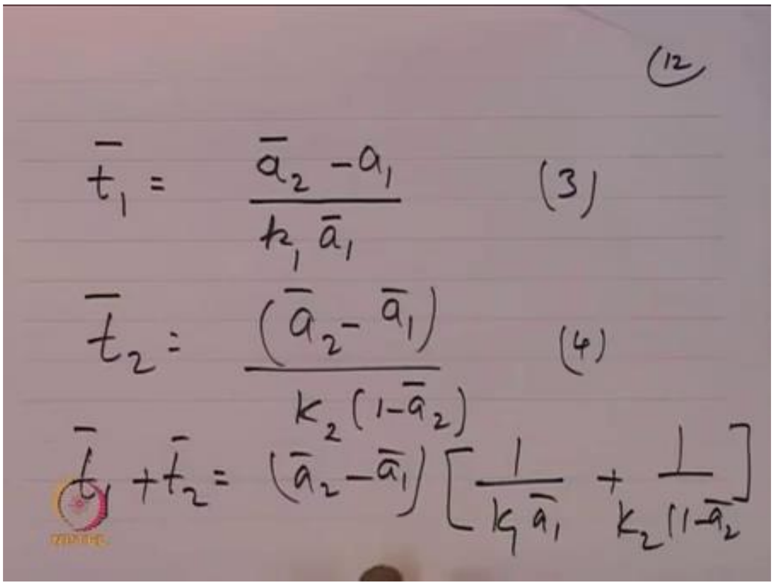

T 1 bar equal to a 2 bar minus of a 1 bar divided by k 1 a 1 bar, t 2 bar equal to a 2 bar minus of a 1 bar divided by k 2 times 1 minus a 2 bar. Look at this. So, I just call this is equation 3 and equation 4. Do all agree with this, yes or no?

Student: ((Refer Time: 17:49))

No. a 2 bar minus a 1 bar is actual a process design decision. You will decide how you will run your process. That means a 2 bar, which is a average activity in the react re generator a 1 in that difference is a process decision. You will find various values in this case it is given as 0.3. On other words, t 1 bar plus t 2 bar if I add this two a 2 minus of a 1 bar within brackets 1 by k 1 a 1 bar plus 1 by k 2 times 1 minus of a 2 bar. The question we ask now is we have made a decision of what is a 2 bar minus a 1 bar.

Let us say, that is fixed for a process. Therefore, if you want to find the best choice or the minimum size of this equipment w 1 and w 2, you have to minimize this quantity with respect to a 1 bar or a 2 bar. But, the difference is fixed.

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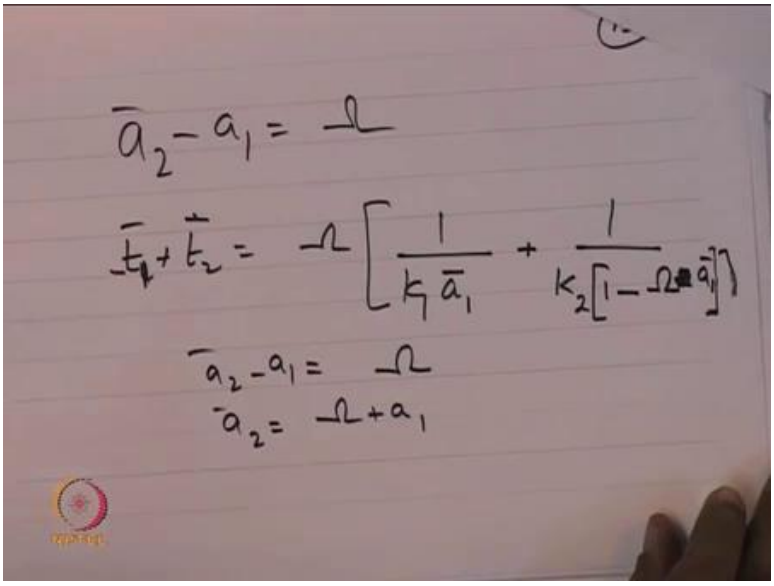

That means, what we are saying is that a 2 bar minus of a 1 bar is fixed, it is omega. And then what is the best choice that you can make for a 1 and a 2. How do we solve that problem? You solve this problem by recognizing that t 1 bar plus t 2 bar t 1 bar sorry. So, this is omega 1 divide by k 1 a 1 bar, I can say plus 1 by k 2 times what is this, a 2 is

Student: ((Refer Time: 19:34))

Are you say, omega plus a is it all right. So, let me say a 2 minus of a 1 bar is omega. So, a 2 bar equal to

Student: ((Refer Time: 19:49))

Correct, you are right, you are right. minus alright. Now, you can differentiate this and find the best values. Please do that differentiate this and find the best values. So, you differentiate, which aspect to a 1 set it equal to 0. I have done this here.

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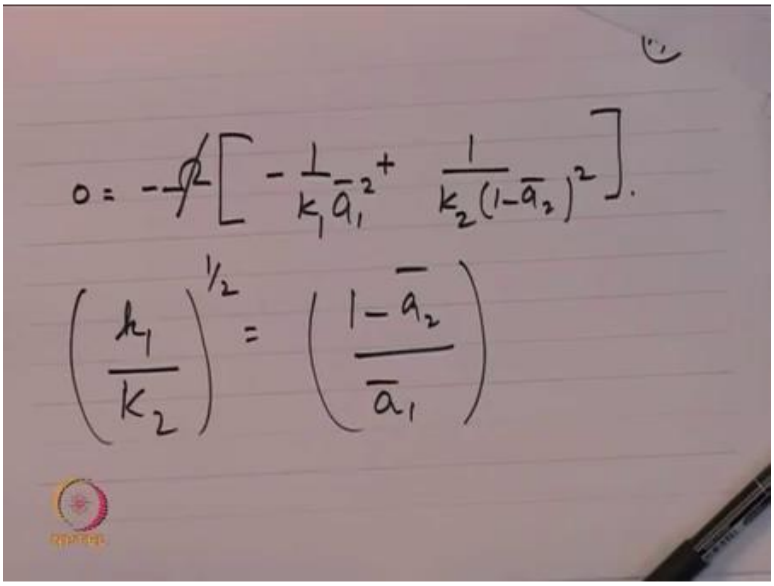

So, 0 equal to minus omega, this what I get please tell me whether, I got it write a 1 square plus 1 by k 2. I have removed the a 2 bar I have removed the omega from here. Please tell me, I have differentiated this and then replace omega and terms of a 1 a 2. yes or no? Please differentiate with respect a 1. And in the result replace omega in terms of a 1 a 2. These a result I get, yes or no please 1 or 2 is not enough. So, what do we get now?

So, we get k 1 divided by k 2 to the power of half equal to 1 minus of a 2 bar divided by a 1 bar. Yes or no or so shall we go forward. Let us, look at or first equation t 1 bar is given t 2 bar is given by this. So, from here k 1 t 1 bar I can write. So, I will write, I will write like this.

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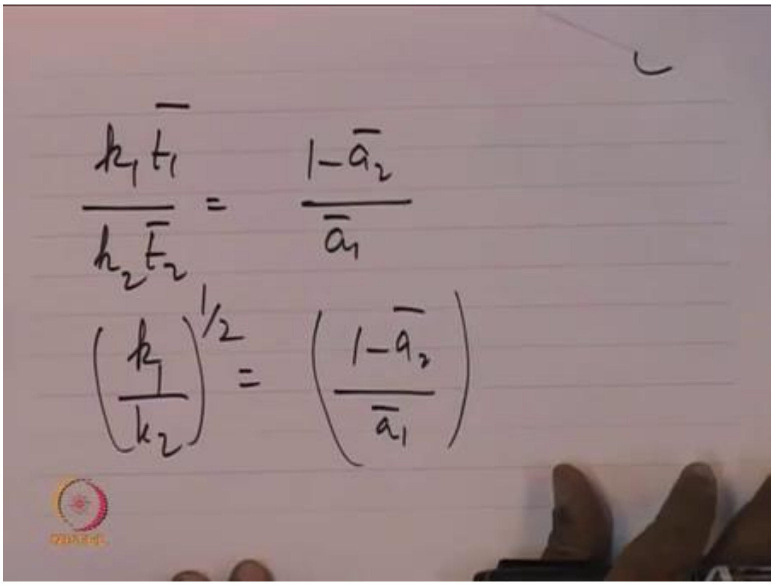

Please tell me if it is k 1 t 1 bar divided by k 2 t 2 bar equal to 1 minus a 2 bar divided by a 1 bar. That comes in equation, that we written earlier. Now, you also have this. Let me, put this in here k 1 divide by k 2 to the power of half equal to 1 minus of a 2 bar divided by a 1 bar. So, we this what kind of simplification can we make regarding by t 1 by t 2. So, I mean, it follows directly from here t 1 by t 2 bar is simply k 2 by k 1.

This what is ask we look at here ((Refer Time: 22:07)). See, w 1 by w 2 is simply t 1 by t 2, so k 2 by k 1 that is what we get. So, what do we get here.

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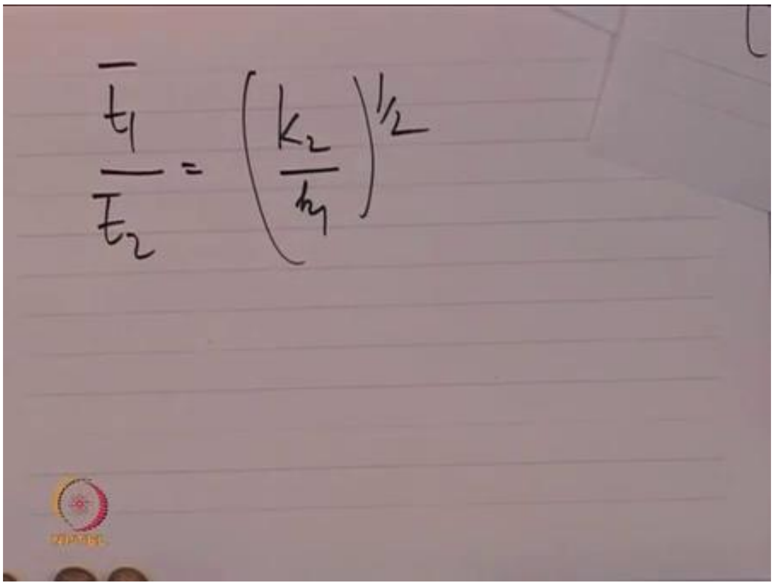

So, it is simplifies as t 1 bar dived by t 2 bar equal to k 2 divided k 1 to the power of half. That is same as what you have to show. You have to show is w 1 by w 2 ((Refer Time: 22:26)) is same as t 1 bar by t 2 bar. So, what we are saying is that the size ratio, the point that is being put across here this ratio for this we circulating the systems, the any depends upon the rate Constance. So, essentially your cattle’s design see, if you have a rate Constance.

So, essentially depends upon your cattle’s how good how bad it is. On that basis only the size ratio will be determined. I mean nothing this unusual I mean the something that we all know, the rate constants are high then you can have smaller equipments so on. That is what it say. Now, the next thing is for the optimal size ratio we found out these numbers s1, s 2, t 1, t 2 and so on. For optimal size ratio, so if the optimal size ratio is given what is a 1 bar and a 2 bar.

That is our next question. That means, the size ratio is optimal is given first we have to find out what is s 1 bar, what is s 2 bar or a 1 bar a 2 bar, what are the t 1 bar t 2 bar this is what we have to. So, next let us do that now.

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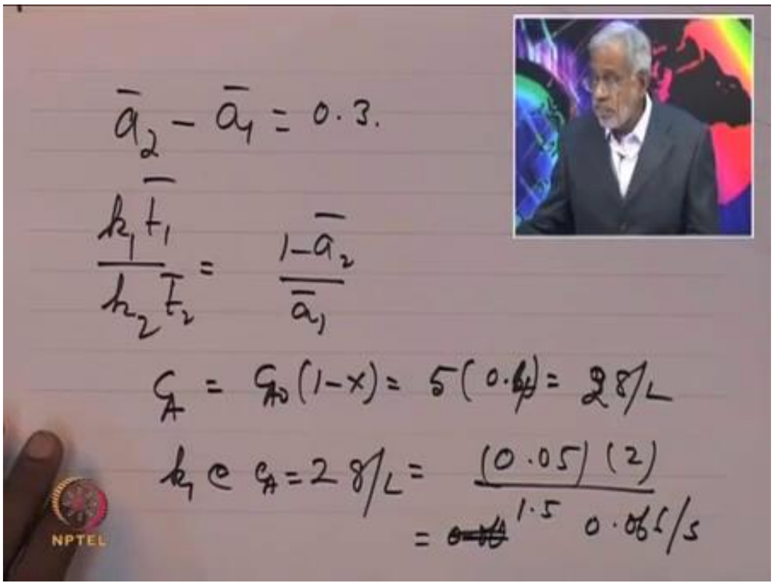

What is given is a 2 bar minus of a 1 bar is 0.3. We know that k 1 t 1 bar by k 2 t 2 bar. If derive just now is 1 minus of a 2 bar by a 1 bar, we have derived just now. Now, what is k 1? From our exercises, what is k 1 value ((Refer Time: 23:51)) 0.05 per second at 400 degree C for CA equal to 1.5 gram moles per liter. That means, what happens please appreciate in the cat cracking system the rate, at which coke deposits really depends upon the concentrational gas in contact with solid.

So, if the gas concentration is high to that extend the coke deposition is high. Is the gas concentrational low, although issues are there with the cat cracking system. So, here it says, if CA is 1.5 it is point 0.05 per second. Actually, ((Refer Time: 24:26)) what we are given a little later. In fact, this to related, that we are operating 5 grams per liter that is the concentration. And we wants to operate at 60 percent conversion.

So, what is value CA for this for the problem we want to solve? 5 grams per liter, if the is the extent reaction is 60 percent, what is this a value in contact with the solid in the reactor. CA corresponding to 60 percent is conversion is what, CA equal to CA 0 times 1 minus of x, which is 5 multiply by 0.6, which is 3 grams per liter.

Student: ((Refer Time: 25:03))

What we are saying 2 grams per liter. So, if k 1 is 05 at CA equal to 1.5, what is k 1 value corresponding to 2 grams per liter. These something common sense says, they little be propositionally higher. So, k 1 value, k 1 at CA equal to 2 grams per liter equal to 0.05 multiplied by 2 divide by 1.5. You all agree, is it alright, How much is it?

Student: ((Refer Time: 25:44))

0.066 shall we say per second. So, k 1, so for us to proceed further the value of k 1 is 0.0 double 6 and k 2 is 0.04. No, k 1 is 0.0 double 6, that is what we calculated just now.

Student: ((Refer Time: 26:08))

1.5 or value is 2. Just now, we said

Student: ((Refer Time: 26:16))

What am saying is see we are said a little later that it is actually going to be operating at 2 grams per liter. It said a little later, it said been earlier unfortunately it said later. So, we calculate value of CA corresponding and therefore, calculate k 1 at 2 grams per liter. So, there is 0 double 6 k 1 is 0 double 6 k 2 is 0 4. So, let us find out what is k 1 t 1 bar.

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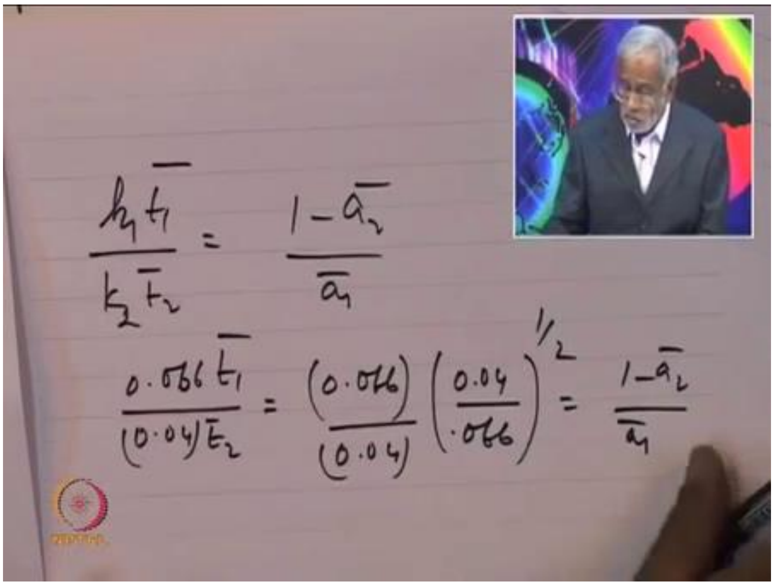

Let me, said once again. k 1 t 1 bar divided by k 2 t 2 bar equal to 1 minus a 2 bar divided by a 1 bar. k 1 is 0.0 double 6 t 1 bar. And what is k 2? What is k 2, 0.04 t 2 bar, what is t 1 bar 2 bar we also done that. What is t 1 bar by t 2 bar? So, let substitute that also 0 double 6 t 1 bar by t 2 bar is k 2 by k 1. k 2 by k 1 to the power of half. So, that become 0 4 by 0 double 6 to the power of half equal to 1 minus a 2 bar divide by a 1 bar. Yes or no. So, what is the value know please tell me or in the numbers.

Student: ((Refer Time: 27:39))

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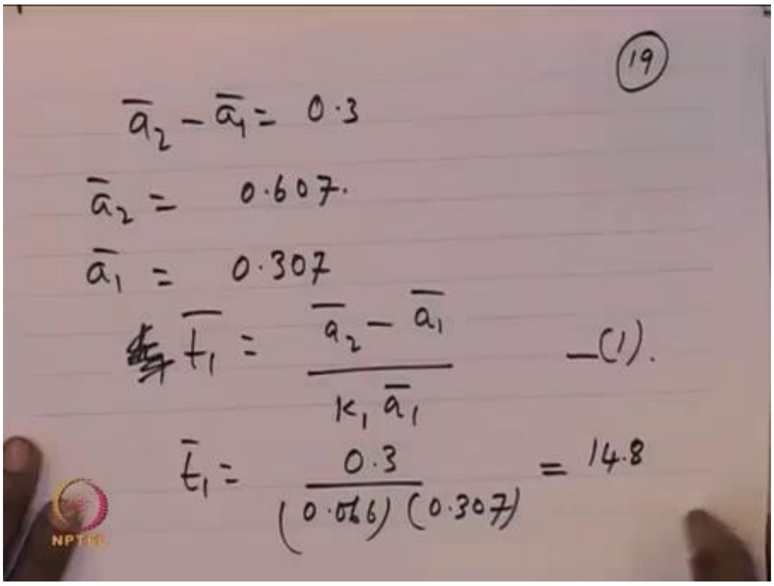


So, we know that a 2 bar minus of a 1 bar is 0.3 is given. So, this gives us a 2 bar equal to a 1 bar equal 2 what, tell me is this clear ((Refer Time: 27:51)). What we are saying a 2 bar, this a 1 bar a 2 bar minus a 1 bar is given therefore, you can do all the manipulations in give me the results. What is a 1 bar and what is a 2 bar? What is the answer? I get a value of a 1 bar as 0.307. A 1 bar is 0.307 I got and a 2 bar is 607. Anybody else, anybody else please is it alright.

Student: ((Refer Time: 28:24))

Yeah. Regenerate c it is says, just a minute how do you say that?

Student: ((Refer Time: 28:36))

6.2 Yeah. See, ((Refer Time: 28:42)) the temperatures have regenerates is always higher than the reactor, because you have to burn of the cock that data is given.

Student: ((Refer Time: 28:50))

Yeah. Why k 1 is yeah, yeah, yeah, good, good, good. See, what am saying is this 6.2 is related to 6.3. ((Refer Time: 29:02)) They should have been said earlier. In the 6.3 what they saying in the this gas oil cracking that goes on the petroleum industry with ((Refer Time: 29:11)) gas are coming in the 5 grams per liter. And 60 percent conversion is typical. Therefore, the gas solids in contact with gases gas concentration is about 2 grams per liter.

Because, ((Refer Time: 29:25)) it is 2 grams per liter and that data k 1 is at 0.5 corresponding CA 1.5. And our CA values are more like 2. That is why we have upgraded this number and said that it is this is of this 05 multiplied by 2 by 1.5. That means, higher the concentration higher is coke formation that is what is being said. Is it ok now? Now we have a 1 bar is 0.607, a 2 bar is 307. Then the next we have to find out is what are the find s 1 s 2 we have done t 1 bar t 2 bar.

How do you find t 1 bar t 2 bar? We go to our equation number 1, k 1 t 1 bar equal to a 2 minus of a 1 bar divided by k 1 a 1. This our equation 1, this something that we have written earlier.

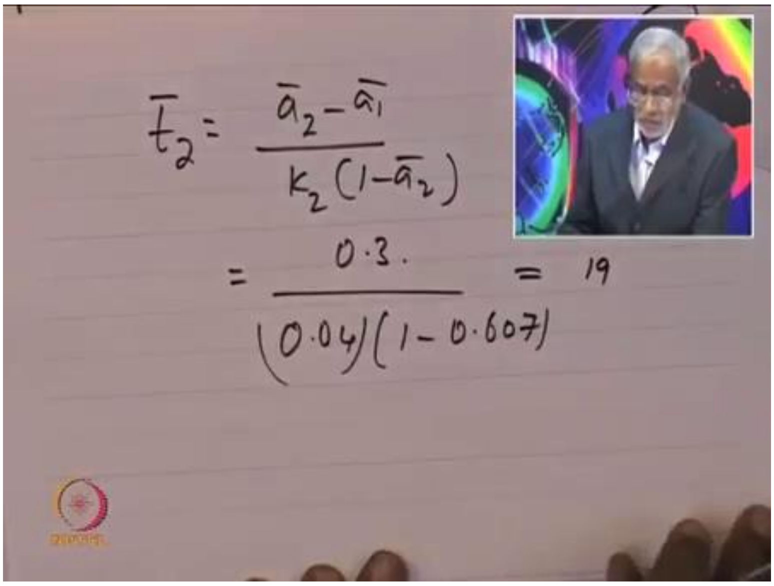
Student: ((Refer Time: 30:26))

Sorry, thanks. Now can we find out t 1 bar?

Student: ((Refer Time: 30:34))

So, t 1 bar equal to 0.3, k 1 is 0.0 double 6, a 1 bar is 0.307. Is it correct? 14.3 correct 14.8.

(Refer Slide Time: 30:54)

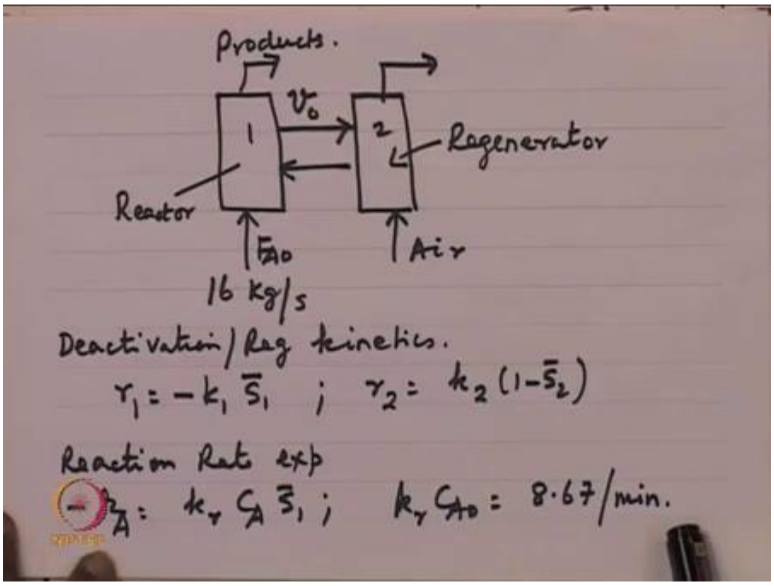


Now, what is t 2 bar, t 2 bar equal to we can do the same thing a 2 bar minus of a 1 bar divide by k 2 times 1 minus of a 2 bar. So, what is t 2 bar equal to 0.3 divided by 0.04 multiplied by 1 minus of 0.607. How much is it?

Student: ((Refer Time: 31:17))

19. Everybody 19. Now, we have set up the equations for the dual bet circulating systems.

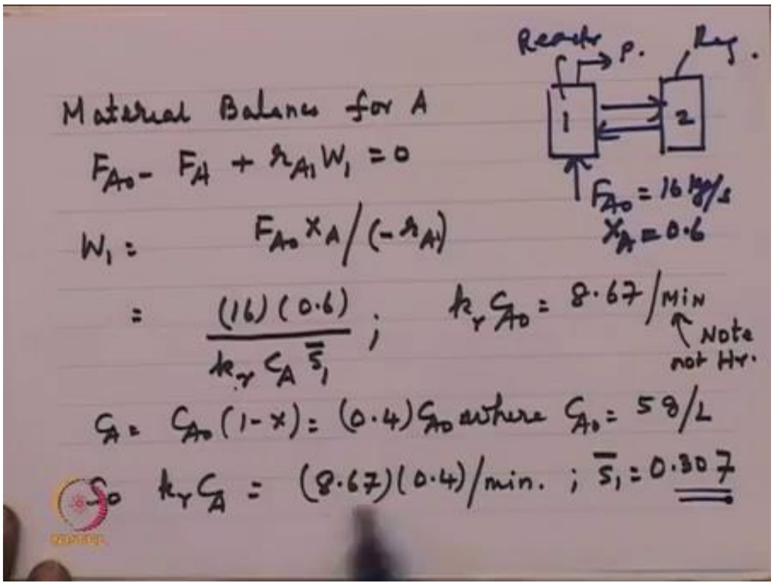
(Refer Slide Time: 31:37)



And we are said, that we have this reactor and regenerator and material going between become at we not and in the reactor that the reaction takes place. And in the regenerator the coke deposited gets regenerated and so on. Now, our equation is r 1 is minus of k 1 as bar, r 2 is k 2 times 1 minus of s 2 bar. I sometimes use a here instead of s. So, you know both r in to changeability used. So, you recognize that, our reaction rate expression said is minus of r A k r C A times s 1 bar k r C A 8.67 per minutes.

We would like to find out, what is the size of w 1, what is the size of w 2. I think that is the point that we want answer now, because we have done at the modeling and it found out what is t 1 bar and what is t 2 bar, that we already done. Let see, how to do this.

(Refer Slide Time: 32:32)

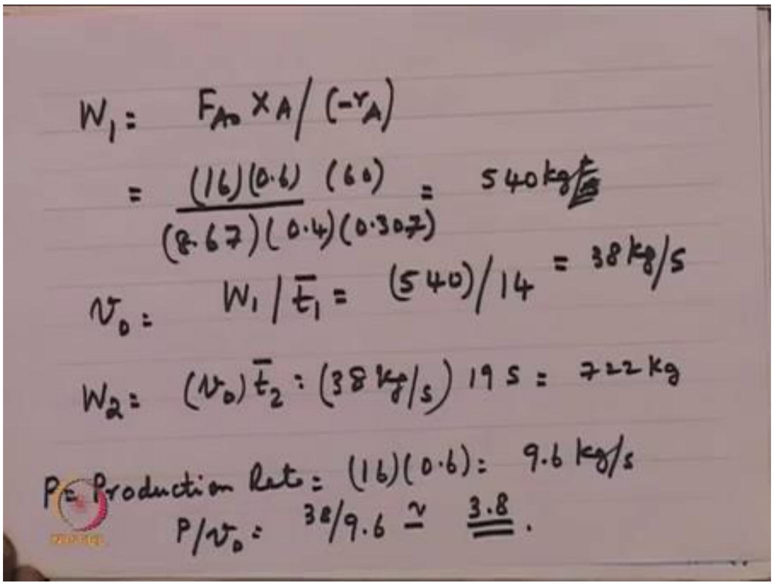


Now, what is our a material balance for component A in the reactor. What is the material balance for component we have this reactor. Here, we have this reactor here. We have this regenerator here. So, this is reactor, this is regenerator. Now, we have this products coming out problems. This is the F A 0. So, what is their material balance? Input, output generation equal to 0. What is the steady state? Therefore, size of the equipment w 1 is F A 0 X A minus r a 1 here. r a 1, this r a 1 alright.

So, what is F A 0 we are saying F A 0 is 16 k g per second. We already said that 16 k g per second. And the reaction extend is X A is 0.6, that is also given. And what is k r C A s 1 bar. What is k r C A we have given k r C A 0 is 8.67 per minute. So, what is C A then, C A is C A 0 time 1 minus of x. Therefore, C A is that number is C A 0 s 5. And 1 minus of x 0.4, because of x is 0.6. Therefore, we find that k r C A is 8.67 multiplied by 0.4, which is k r C A. And s 1 bar is already we have derived it 0.307, this already done.

Therefore, how do you find out r a 1 in k r C A times s 1 bar, k r C A is given here, s 1 bar is given here. Therefore, we can substitute here and find out what is the size of w 1. Let us do that now.

(Refer Slide Time: 34:12)



We need do that what do we find. F A 0 is 16 this is 0.6. Now, 8.67 by 0.4 this we are multiplied by sixty to convert this is per minute. That is what we are because our numbers are given in minute. Therefore, 8.67 by this 8.67 by this ((Refer Time: 34:36)) 8.67 per minute. So, 8.67 is per minute not per hour. Therefore, what have done is that, 8.67 per minute I will divided by 60 to get per hour. So, that we get, v naught equal to w 1 t 1 bar this is 540 k g.

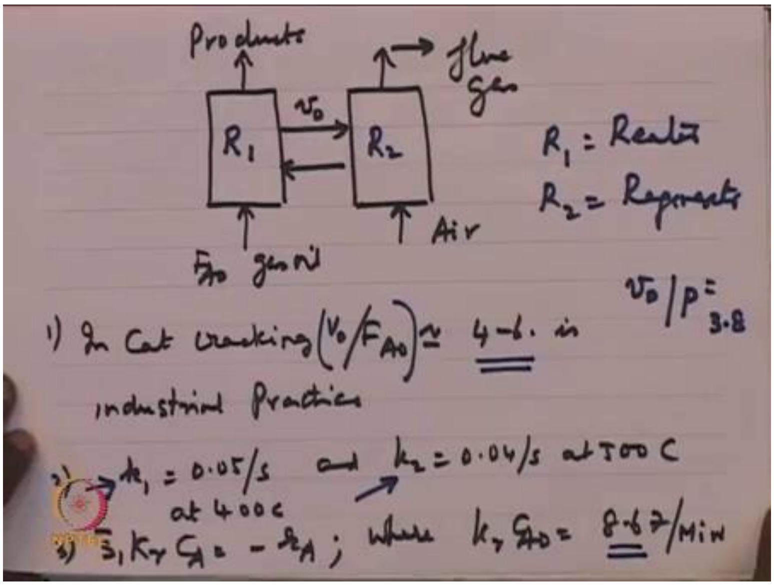
Therefore, divided by 14 we get 38 kg per second. Let us go through this once again, what I we done. What we have done is w 1 is F A 0 X A minus r A oneself. We have only at substitute all the numbers to get our answers. This is 16 kg per second, while this 8.67 is given per minute I have this converted to second. That is why I gotted the 60. That is why we get size of this equipment w 1 540 kg. What is v naught, v naught is circulation. So, we not is simply w 1 divided by t 1 bar.

We already found t 1 bar to the 14 second, we already done this. Therefore, we get v naught to be 38 kg per second, which means we must circulate this simulates between the two reaction equipment. Let me, just run through this ((Refer Time: 35:46)) once again. What is saying is the this is v naught that means, we are saying the this v naught must be circulated at 38 kg per second. That is that is what we say. So, because t 1 bar is already calculate that 14 seconds. We find v naught is 38 kg per second.

Now, what is v 2, v 2 by definition is v naught multiplied by t 2 bar, we also hold up t 2 bar within 19 seconds. Therefore, we find w 2 is equal to 722 kg. So, what we have done now, we have done this design for a reactor regenerated system, where we have done in optimal choice of t 1 bar and t 2 bar based on that optimal choice of t 1 bar and t 2 bar. We determine w 1, because we knew the reaction kinetics. And based on that we found out v naught and hence we found out w 2.

Therefore, what is the production rate of the products from reactor. Because, that is what we want a convert grass files some useful product. But, for a production rate is 16 multiplied by 0.6, 9.6 kg of products were coming out of reactor. And therefore, production rate divided by v naught, what is this ratio 9.3 divided by v naught, How much is that? V naught reverse v naught by P equal. So, the circulation divided by production that 38 by 9.6 this is 3.8. So, the important point that I want put across to you here is the following.

(Refer Slide Time: 37:26)



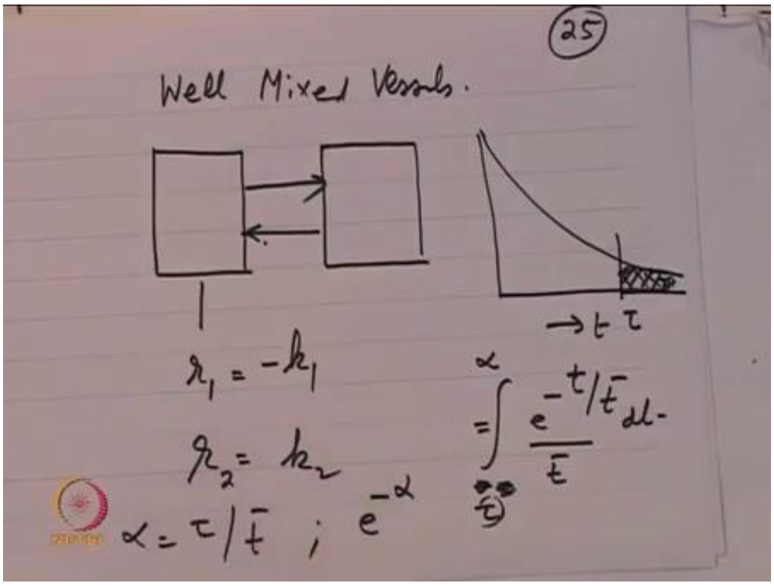
That is, if you have a reactor, this is reactor I will call it r 1, this is regenerator. So, reactor is reactor and r 2 is regenerator. So, what we are saying is that, in reactor regenerated system we not divided by F A naught this typically is supposed to be 4 to 6. It supposed to be 4 to 6. In our case, we are found v naught divided by production rate is about 3.8 or v naught divided F A 0 is likely less.

What we are trying to say is that, we industry norm is in the range of 4 to 6, which means we must circulate this solids with at about 4 to 6 times at the rate, which you want to process ((Refer Time: 38:19)), this is the industrial ((Refer Time: 38:22)) this is for the case of k 1 is so much. And k 2 is so much. k 2 is k 1 is 05 per second and k 2 is 04 per second at 500 degree centigrade, this is 400 degree centigrade. Now, ((Refer Time: 38:37)) improving and so on.

And we can do that and then movement better catalyst of course, 0 degree design a systems to give you the appropriate optimal choices. And another point to be mind is that, our reaction rate this in the reactor the gas oil is coming in, if s 1 bar k r C A. In the rated with reaction takes place in the equipment. We this k r C A 0 is experimentally found to be 8.67 per minute. On other words, we can improve the catalysts to improve this rate reaction rate.

So, that we get higher productions. That is important things. That means, for a given circulation, if you want higher production we have to improve the catalyst. Of course, these are all important features ((Refer Time: 39:22)) understand this. So, ((Refer Time: 39:24)) we are trying to say that in reactive regenerated system. We must circulate this solids at roughly 4 time the rated, which we process the gasses the industry practice. And that, something that we you do design. So, that we get a results of out choice. All right, let us go to the next one.

(Refer Slide Time: 39:47)



Our next, we have a reactor please talk to me. We have a reactor regenerated system, in which our rate function r 1 and the rate function r 2 are k 1 minus k 1 plus k 2. He started this exercise sometime back, where he said that whenever we have a rate function which are 0 order, what is 0 order mean, 0 order means that the time required for complete consumption of the particle is finite. And these are all well leads to essence. That means, you are looking at well mixed vessels.

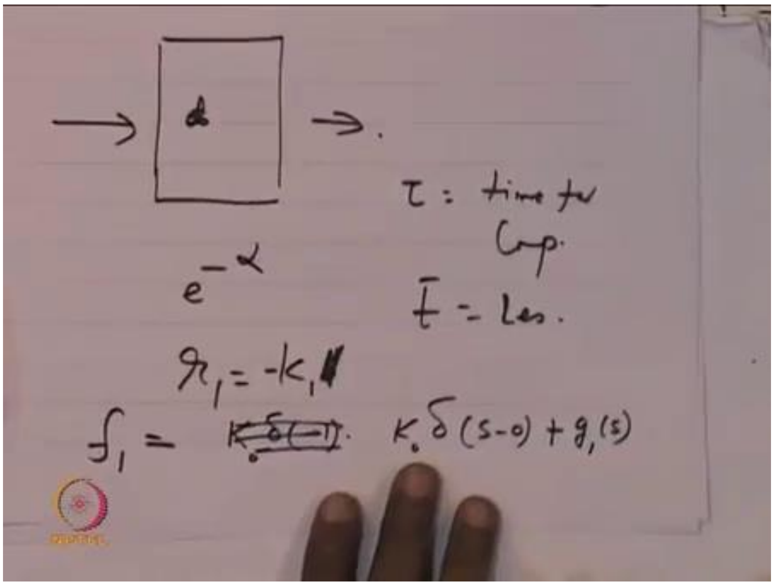
Well mixed to vessels means what, that the R T D is the exponential R T D. We have done that the R T D is exponentials R T D and then if tau is the time if complete consumption, there is a finite fraction of material, which is in the equipment which is completely consumed. And what is that fraction, we said the fraction which is find completely consume is 0 to infinity sorry tau to infinity e ((Refer Time: 41:13)) minus of t by t bar d t divided by t bar. We have done that.

This is the fraction, which is completely consumed. Because, any material which is staying time for a more than tau will be completely consumed. Therefore, this what is this fraction may be integrated this. And if he call alpha we could tau by t bar, we said that fraction is e ((Refer Time: 41:41)) minus of alpha. What we are saying is that, minister vessel there is finite material which has a time up residence greater than tau.

And therefore, that material will be completely consumed.

Because, it is confused consumed it will you can be seen as a either completely consumed or completely regenerate, which ever may be the case depending upon what is their situation. Therefore, the fraction that is completely consumed is e to the power of minus of alpha in a ((Refer Time: 42:14)). On other words, if you have a single vessel, let me state this once again, because is important.

(Refer Slide Time: 42:21)

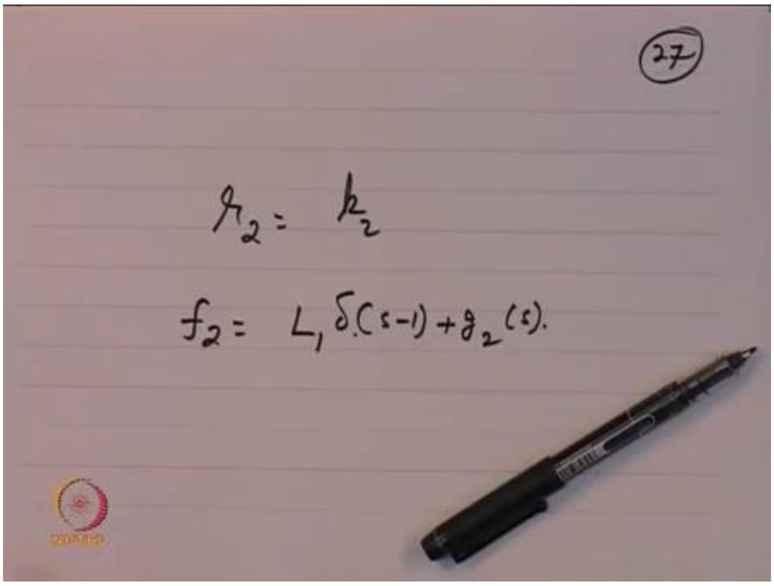


If you have a single equipment, material coming in material going out e to the power of minus of alpha tau is time for complete consumption. And then, t bar is then residence time. Then, this is the fraction which is completely consumed or completely regenerated. Therefore, in the context of population balance, if you do the instance of a reactor in which there is a only deactivation that means, the rate function of use of this form. Then distribution function will have a discontinuity. The discontinuity will be at is equal to 0.

Because, this completely consumed, completely consumed means what, the property while uses is equal to 0. And what is that fraction we said that fraction is e to the power of minus of alpha. This is cleared to all of you. If you have a fluidize bed for example. If you have solid coming in and solids going out. And the solids are reacting as per this rate function. Then there is a finite fractional material, which is completely consumed. And that fraction is given in by ((Refer Time: 43:46)) e to the power of minus of alpha.

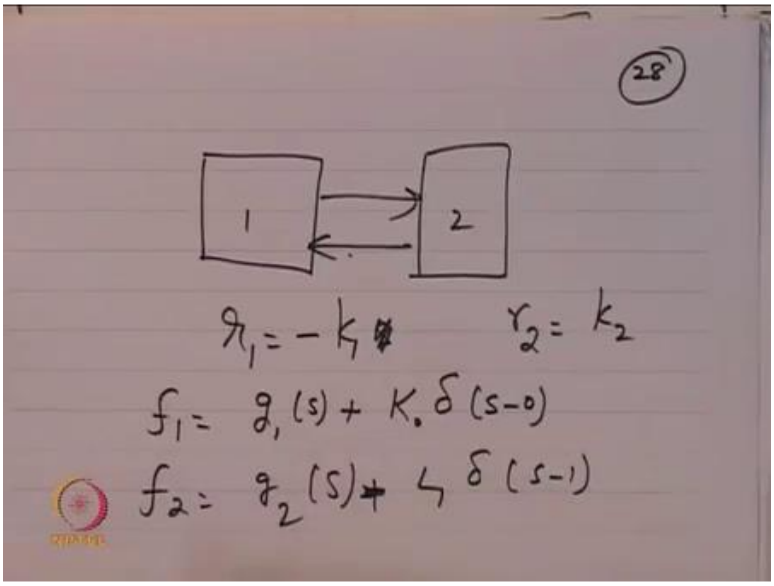
And therefore, the distribution function f 1 will have a discontinuity at s equal to 0. Therefore, we say that, if that quantity is so delta, ((Refer Time: 44:07)) let me write it in k 0 times delta of s minus of 0, plus a continuous part this is clear. Physically, what it means that, there is a finite fraction, which belongs to delta s minus 0 that means, it is a property value is s equal to 0.

(Refer Slide Time: 44:35)



If this rate function is of the form r 2 equal to k 2 is this form. Then the fraction that is completely consumed. We will be let say it is f 2 equal to and denoting this at as s and l1. I will give a reason why I will denoted as a l1. The fraction that is completely consumed, if is of this form it is regenerated. So, the discontinuity is that s equal to 1.

(Refer Slide Time: 45:12)

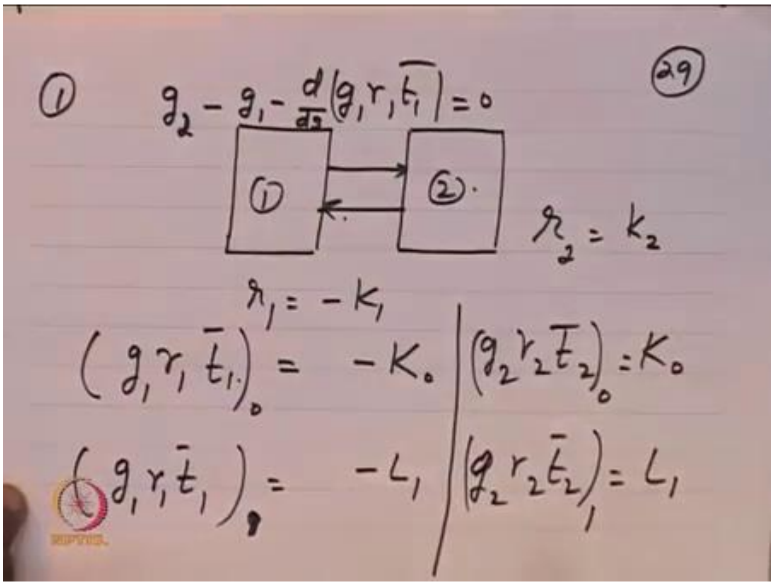


Now, the context, the context is this problem context is this or context is this. This is reactor, this is regenerated r 1 is minus of k 1, r 2 plus k 2. Therefore, f 1 is some g 1 at s plus k 0 delta of s minus 0. f 2 is g 2 of s plus l 1 delta of s minus 1. I draw you attention to this once again. That I said, please just give minute I said when there is a single ((Refer Time: 45:52)) fluid bed, this fraction k 0 is e to the power of minus of alpha, when there is a single. When there are two of them.

Then this fraction is determined by the interaction between both. Therefore, it is not e to the power of minus k 0 alpha. Is not this something else derived at now? What is important to recognize here is that, function f 1 and f 2 have a discontinuous part and a continuous part. And then, now we should wave the derive what is k 0, what is l 1, what is g 1, what is g 2 this is what we want to do now. Now, this general approach here is that whenever we have a function, which unbounded we try to get rid of it, is it.

We tried to get rid of it and the look at the homogenous equation, generate boundary conditions and appropriately account for the discontinuity the same procedure. We have done path of this before, but I will run through this once again.

(Refer Slide Time: 46:52)

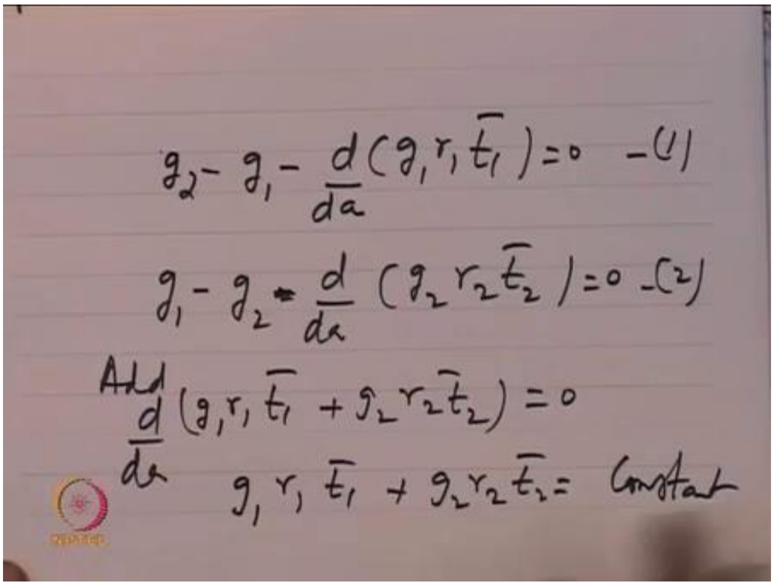


We have already derived, I will write it down, but in the view of done this when we met last time. So, I thought, I will save some time. We did this last time.

Student: ((Refer Time: 47:37))

Sorry, thank you. This is something that we have, how did we derive this. The setting up what happens at the boundaries. We can do it again, but we would not do it now. Because, will lose some time. Now, what is our population balance? If you just write it here. Just for g 1 sorry this is for reactor, reactor g 2 minus of g 1 minus of d by I would sometime variety as sometimes g 1 r 1 t 1 bar, he could similarly I write it again. I will write it again.

(Refer Slide Time: 48:35)



For reactor is g 2 minus of g 1 minus of d by d, I write a and s. I write to a here. So, that is continued that, g 1 r 1 t 1 bar equal to 0. Similarly, g 1 minus of g 2 minus of d by d a of g 2 r 2 t 2 bar equal to same. Now, what is the procedure that we have adopted first solving this, the simply add this two. And then, so let us add 1 and 2. What is the question number I have given here by call this is 1, call this is 2 suppose you add 1 and 2. What do we get? So, we get g 1 r 1 t 1 bar plus g 2 r 2 t 2 bar d by d a equal to 0.

So, that gives as g 1 r 1 t 1 bar plus g 2 r 2 t 2 bar equal to constant. We agree now, to find the constant of integration, we have got the all the boundary conditions here. We can see how it looks ((Refer Time: 49:57)) g 1 r 1 t 1 x 0, g 1 r 2 t 2, 1 is k 0, 1 is minus k 0. Therefore, constant integration is 0. Because, we have see g 1 r 1 t 1 bar at 0. And this minus k 0, this is plus k 0. Therefore, constant integration is 0.

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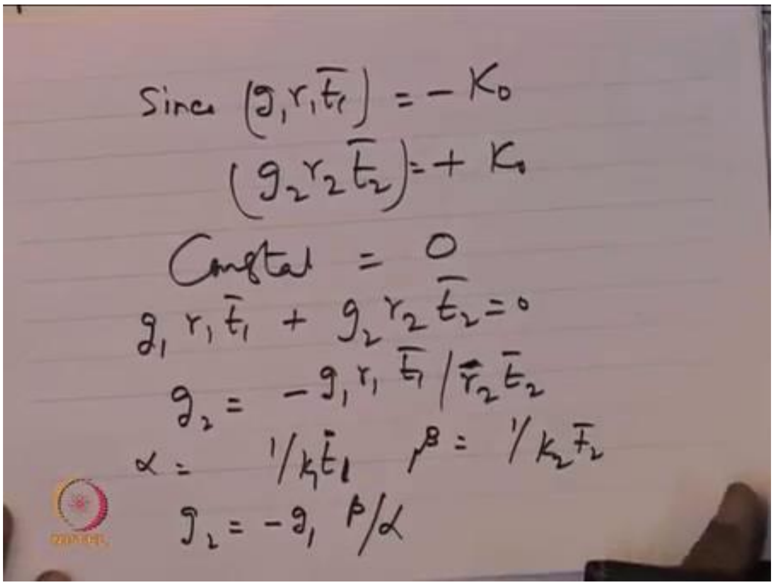

So, since g 1 r 1 t 1 bar equal to minus k 0, g 2 r 2 t 2 bar plus k 0. We have constant equal to 0. Therefore, this if our equation becomes g 1 r 1 t 1 bar plus g 2 r 2 t 2 bar equal to 0. So, what I have done, I said g 2 equal to g 1 r 1 t 1 bar divided by r 2.t 2 bar.

Student: ((Refer Time: 51:01))

Minus. Thank you. Now, I put alpha equal to 1 by k 1 t 1 bar. And beta equal to 1 by k 2 t 2 bar with these I get g 2 equal to g 1 beta by alpha. Please tell me, g 2 equal to g 1 beta by alpha, since alright. Now, we can substitute for g 2 in our equation 1. We can substitute for g 2 here. And then simplify and so on. I have done that, I written correctly here I give it wrongly here. Let us, go for them now please.

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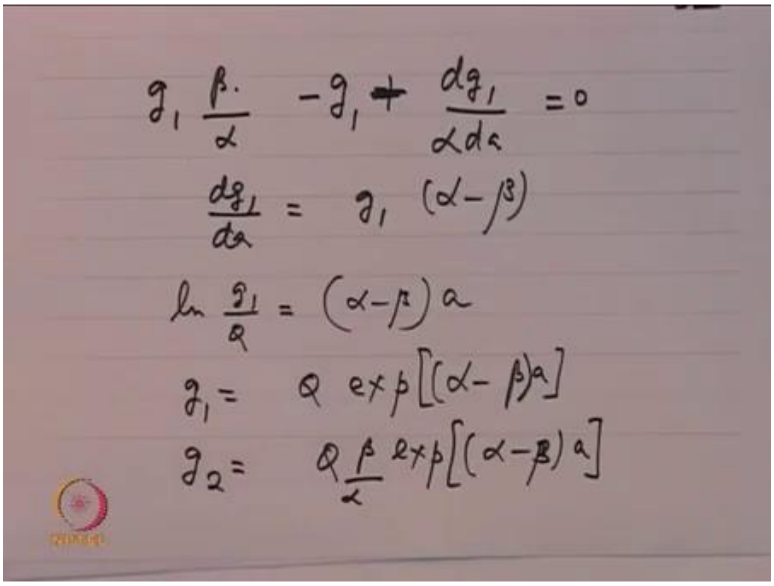


Substitute for this in our first equation I get please see, if it is all right. g 1 by alpha beta minus of g 1 plus d g 1 by alpha d a. So, this how it looks our equation 1. Please see, if it ok. So, substituting for g 2 dissolve ((Refer Time: 52:15)) it is substituting the g 2 in a equation 1. And equation 1 is, this is ((Refer Time: 52:19)) equation 1 and it simplifies like this. Please tell me if it is ok. So, finally, it simplifies as d g 1 by d a, g 1 times alpha minus of beta by this representation is correct please tell me, listen ((Refer Time: 52:39)) you should not make any mistake here. I have written g 2 as g 1 beta by alpha, g 1 is alright, minus d by d a g 1 r 1. So, r 1 is negative, therefore, this become a plus. So, this statement is correct. So, this equation is correct. Now, please help me to simplify this.

Student: ((Refer Time: 52:58))

Can we solve this now?

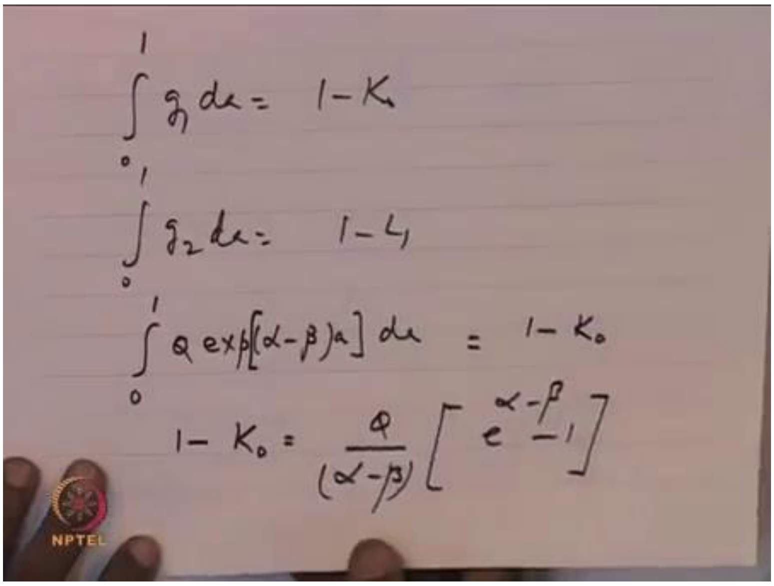
Student: ((Refer Time: 53:00))

What is the solution?

Student: ((Refer Time: 53:04))

So, you get l n g 1 by Qq equal to alpha minus of beta times a. Can we write? Therefore, g 1 equal to q times exponential of alpha minus of beta times a. So, this is the solution. What is g 2? I written as Q times beta by alpha exponential of alpha minus of beta times a. Our solution is complete, if you find the values of k naught and l 1. In terms of system parameters, which is alpha and beta are the system parameters. So, let us try to do that.

(Refer Slide Time: 54:14)

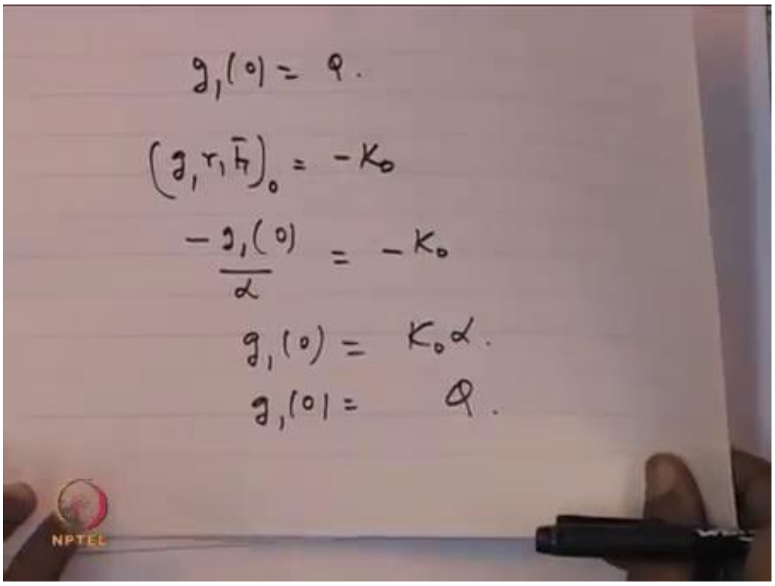


For that, what I have done integral 0 to 1 g 1 d a is 1 minus of k 0. Integral 0 to 1 g 2 d a is 1 minus of l 1. It is my definition. Please our definitions are here. This is a definition ((Refer Time: 54:31)). So, integral f 1 d a is 1. Therefore, g 1 d a is 1 minus of k 0, g 2 d a will be 1 minus of l 1. So, let us, go farther now. So, what is plus, it has to be ((Refer Time: 54:53)) plus. See, please do not forget our physics is what, the function f 1 and function f 2 has a continuous part and a discontinuous part. So, it is plus.

Now, what I have done, is that integral g 1 d a. Can we integrate g 1 d a integrals 0 to 1. What is g 1 Q times exponential of alpha minus of beta times a d a. That is equal to 1 minus of k 0. I have integrated this and it is see whether what done it right or not. Q minus of alpha minus of beta within brackets of this what I get. We will keep this there some manipulations are required I have done the manipulations.

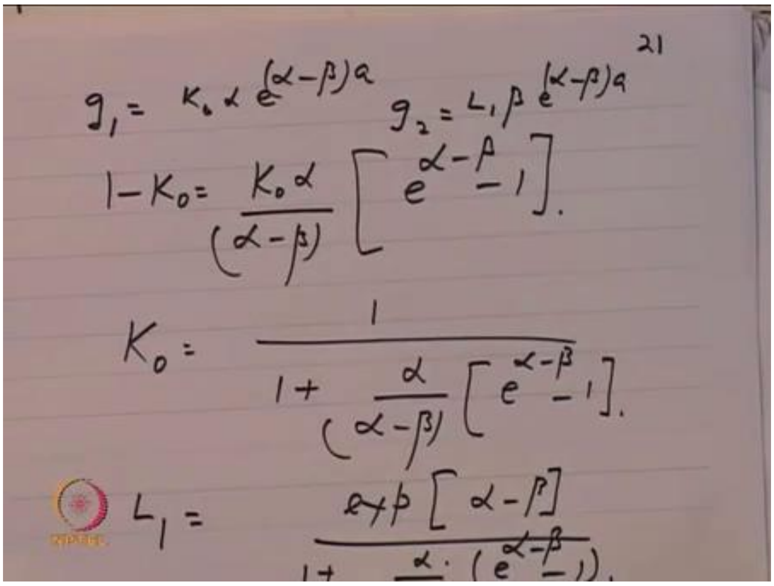
Now, let us look at g 1, where is g 1 look ((Refer Time: 56:01)). You look any way this is g 1, what is g 1 at 0 g 1 0, g 1 0 is cube I will write.

(Refer Slide Time: 56:09)



g 1 0 is cube now ((Refer Time: 56:16)) what else do we have I have got something else I have done. Every time g 1 0 by alpha from ((Refer Time: 56:26)) what is ((Refer Time: 56:27)) g 1 t 1 r 0 equal to minus k. So, can Ii write this as minus of g 1 at 0 by alpha equal to minus k 0. Can I write this? So, that tells me that g 1 0 is k 0 alpha. And g 1 0 is also equal to Q. Therefore, our solution looks like this. So, what I am saying is that ((Refer Time: 57:03)) this Q I will replace it as k 0 alpha. So, help me now.

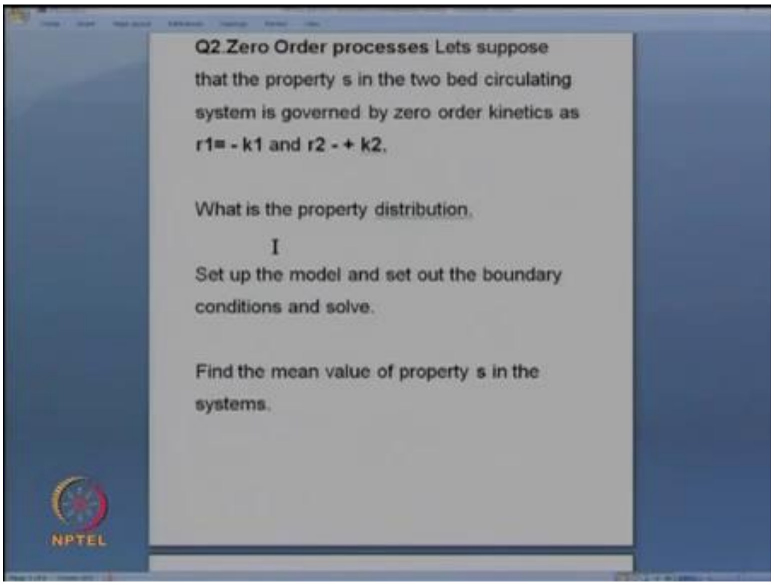
(Refer Slide Time: 57:13)



So, I am saying 1 minus of k 0 equal to k 0 alpha divided by alpha minus of beta e to the par of minus of 1. So, this simplifies and gives you k 0 as 1 divided by 1 plus alpha minus of beta e to the power of alpha minus beta minus of 1. Now, look at this long story short and we can do some more manipulations I will not do this. I will simply write this you can do it yourself not so difficult. I get this 1 plus alpha by alpha minus of beta e to the power of alpha minus of beta minus of we can do this yourself.

So, what we have to try to do is. We have found out g 1, we have found out g 2, we have found out k 0, we have found out l 1. on other words, is your completely specified the distributions in terms of system properties alpha and beta. The important point is to recognize that k 0 and l 1 depends on choice of the operation alpha and beta. That is the point that, is that comes through this exercise.

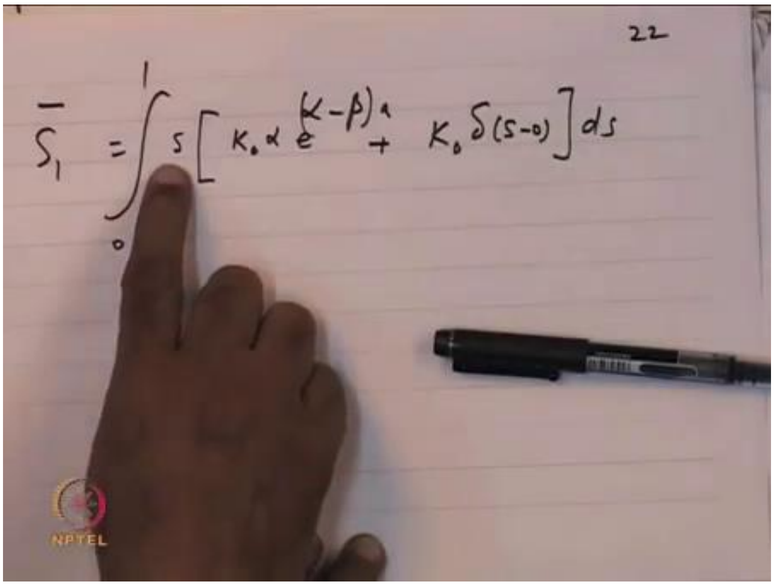
(Refer Slide Time: 58:27)



The property distributions have done it is the property distribution I will just the all the number it will write down. Alpha is known, k 0 is known, l 1 is known, g 1 is known, g 2 is known. Let me ((Refer Time: 58:40)) just write here g 1 you have done this g 1 is k 0 alpha e to the power of alpha minus of beta times a. And g 2 is l 1 beta e rays to power of minus of alpha this is involve this. So, we know the complete solution is this clear. The complete solution is in front of you.

You can see a g 1, g 2, k 0, l 1. The what is tells us you that, if you want the main values. If you want the main values now, you have to take the first movement of the distribution. So, if you one mean value of in reactor, so yes times. So, I just want you to the main, if you just do one of them.

(Refer Slide Time: 59:32)



S times you have k 0 alpha minus of beta times a, plus this is k 0 times delta of s minus 0 d s. This is the integral, this gives you s 1. What is this integral, s times k 0 times delta s minus 0 d s.

Student: ((Refer Time: 59:57))

What is the meaning of the delta function, s equal to 0. This s multiplied by k 0 delta s minus 0 d s that integrates what?

Student: ((Refer Time: 1:00:09))

It is 0. Do you understand s times k naught times delta s minus 0 d s is 0. That is the definition of delta function, delta function k is the value. See, when multiply s delta s time delta s minus 0 is 0. Do not make the mistake. ((Refer Time: 1:00:36)) this is integration this term actually, disappear. This is clear, this is state all of you.

Student: ((Refer Time: 1:00:46))

See, what is important is that you must appropriately use the property of the delta function. So, that is the point at 10 point ((Refer Time: 1:00:56)). So, what is says is that you done the ((Refer Time: 1:01:03)) property distribution. And then boundary condition is all that done the mean value of I have not done. You can do it yourself and ((Refer Time: 1:01:10)). What are the thing you are done. You are done this we have done this 0 order done, carbon dioxide you have done, mixed order see this 4 and 5 they are exercise for you, because we are set up for various cases. So, 4 and 5 are two exercise that you should do. So, that you know how to apply population balance. How to right balances properly take things appropriately in to account. So, this is the very good exercise you know. Unless you right properly do not get the result. And all this are analytical solution. You get analytical solution, you get there is no. So, will be good exercise from both all of you.

Student: ((Refer Time: 1:02:03))

Accept, it is plus k 2, which means that I mean plus I did not every time. r 2 equal to k 2.

Student: ((Refer Time: 1:02:12))

No, no, no this is wrongly written, which should be r 1 is only correct is wrongly written your right I will make the change. It is only minus sorry now it ok. r 1 is minus k 1 s, r 2 is plus k 2. Please make the change. Question no 5, this is correct 5 is ok. So, this of course, you done all this things you do it once again shows it most importantly 4 and 5 you must submit. Because, that I am not done here. So, whatever I am not done here, you should definitely do. And not done fully also you should submit. I am not done fully I think this particular question I am not done fully. This part I am not done.

Student: ((Refer Time: 1:02:59))

We will stop there, we will take it next time.