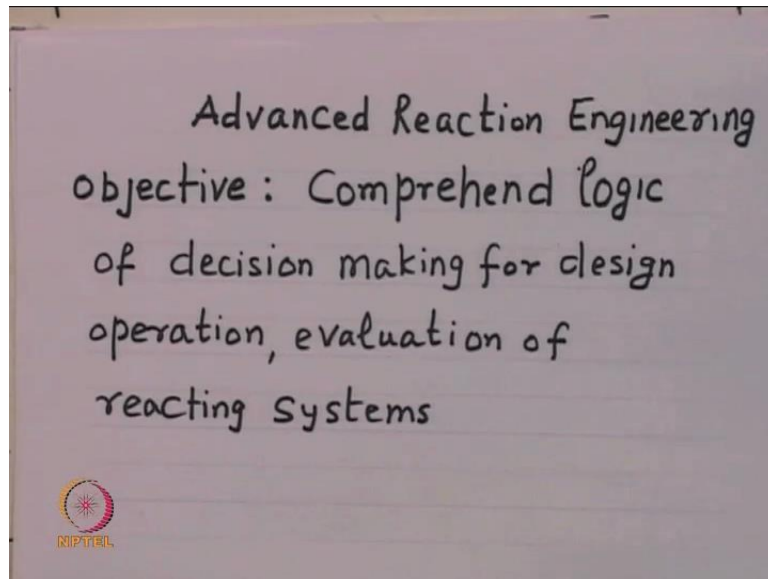


Advanced Chemical Reaction Engineering
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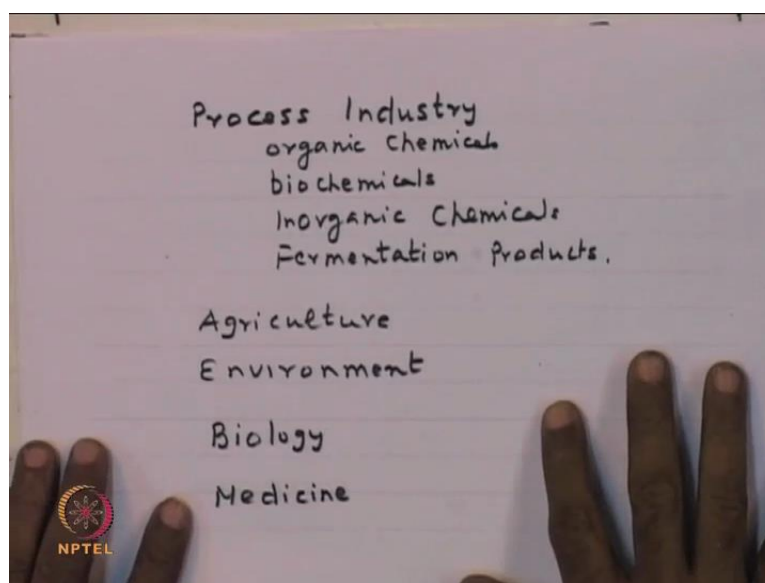
Lecture - 03
Design Equations-1

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We are looking at advanced reaction engineering; the objective here is to comprehend logic of decision making for design, operation evaluation of reacting systems. So, it is a large subject in which we look at reactions has a central way of understanding what is happening? Let us look at some examples, where we might find this subject of much value.

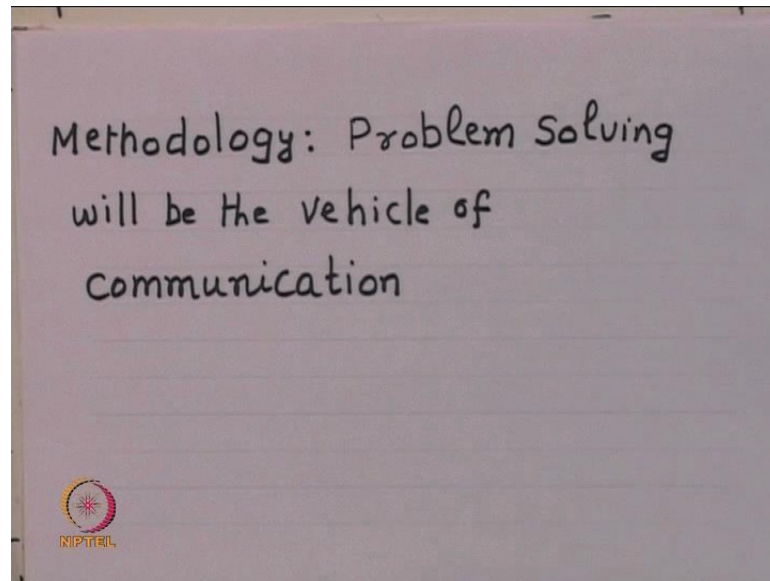
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Process industry, for example, we make organic chemicals we make biochemical various kinds inorganic chemicals, fermentation products and then these are industries were they start with certain raw material to produce certain products. Our raw materials could come from agriculture; it comes from forestry; it come from mining; it could come from oceans in the form of various ocean products, all of which we used to produce various chemicals. Now, we could also be another way by which we look at reaction engineering is we want to understand agriculture in a larger way.

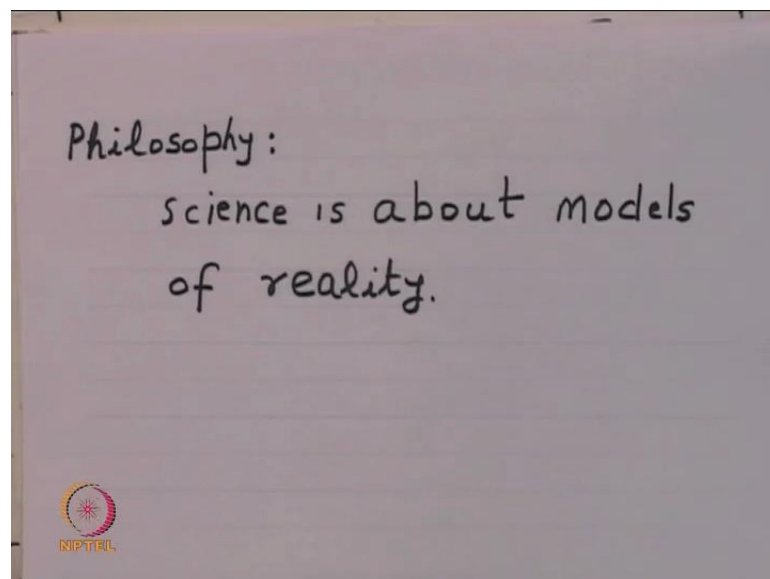
We want to understand what is happening in your environment or maybe we are trying to look at how medicine is performing on a certain organism may be human may be animal. Similarly, how certain medicine can interact with environment and so on. So, these are all situations where reactions are so central to what is happening in the system. So, to say that reaction engineering gives you some fundamental principle to be able to deal with small systems or large systems. What is our methodology?

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Our methodology in this course would be by enlarge problem solving; that means, we try to convey principles to you through solving small but carefully selected problems so that whether it is a process industry problem are from agriculture or environment or biology or medicine. Each of these situations, we will be able to comprehend by looking at these small but carefully selected problems.

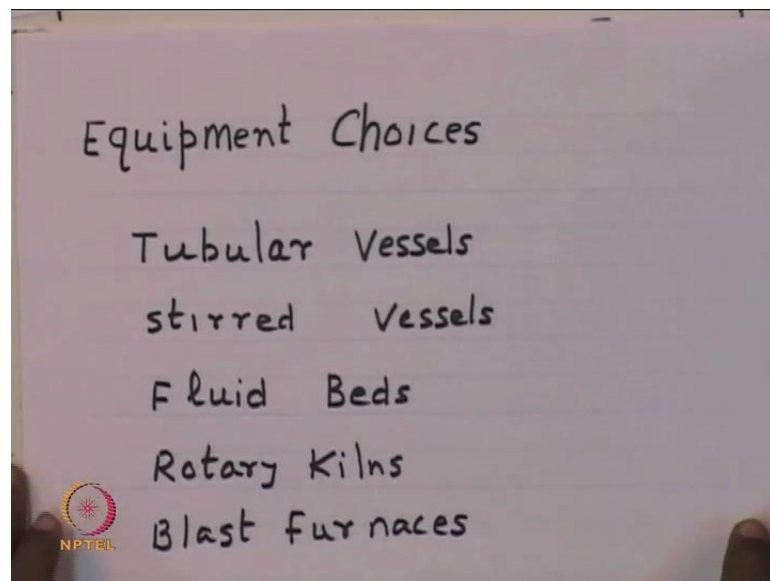
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I mean, we have as you go along this course we will be formulating various models. Models, as we would always remember models have to be seen carefully so, we always

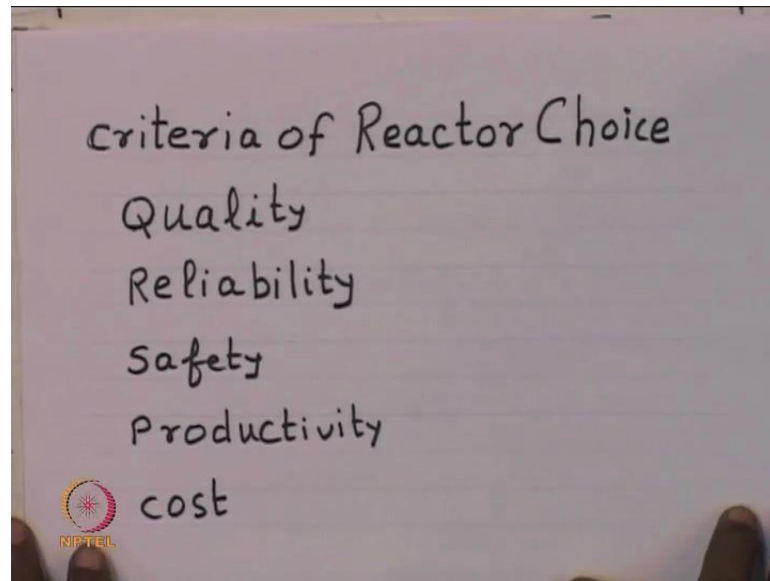
remember that whatever we do sciences are only about models of reality. We like to understand reality we will models and we see how these models describe reality and if you describes well we except and as we understand more and more; we find these models are not so useful; so better and better models are required. So, the base point is the science are only about models of reality and to extent the model to describe reality we will use them when they not describe reality well we look for better models. So, this is philosophy with which we will look at this course. Chemical industry has been around for last more two hundred years now may be much longer.

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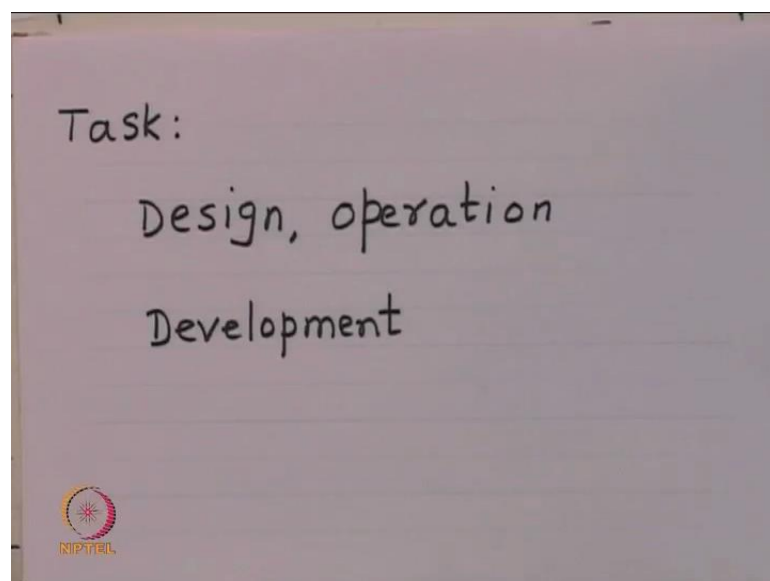
So, kinds of equipment that we will see in the chemical industry is tubular vessels, tubular reactors that is common in the petroleum industry; stirred vessels are very common in chemical industry; when we have fluid beds that you see in cracking industry; rotary kilns are very common in lime industry and of course, steel making and known for very long time blast furnaces or such equipment. On other words what we try to do in this course is to see how we can use or understand of the chemical reactions to build and design equipment and see how they operate. How they can be operated better? How best we can optimize its operations so that we produce our products safely, productively, economically so that we deliver useful value to society. So, whether it is tubular vessels, stirred vessels, fluid beds, rotary kilns or blast furnaces each of these choices are based on understanding of the reactions that we are trying to conduct.

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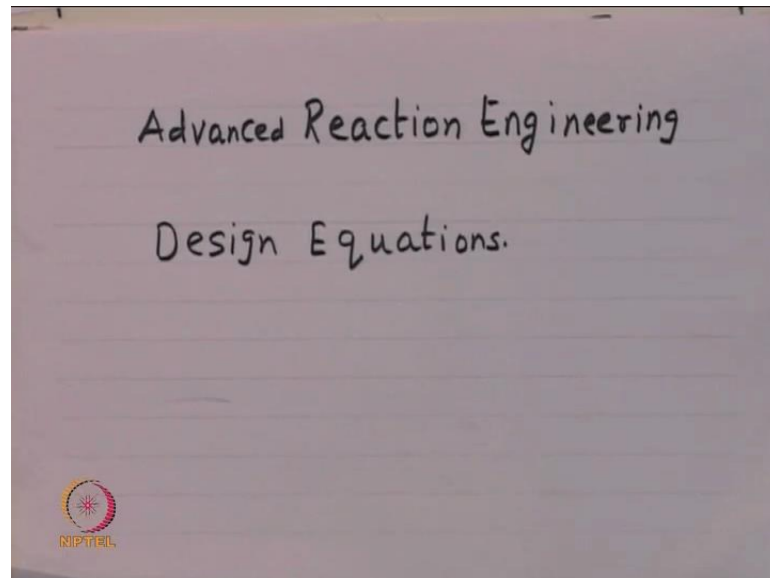
Now, the criteria that we will use the criteria of choice of these reaction equipment of course, depends upon on the quality of the product that we want to produce; the reliability with which you on make this equipment work; the safety feature there are associated with it; the productivity we desire and the cost that we would like. On other words, our choice will be dependent on the importance that we attach on to these five qualities. Of course, we say I mean this not that be compromising these qualities the fact is that all of them are equally important. Therefore, the criteria should be such that every one of the qualities is criteria are satisfied to the extent that is required for the process.

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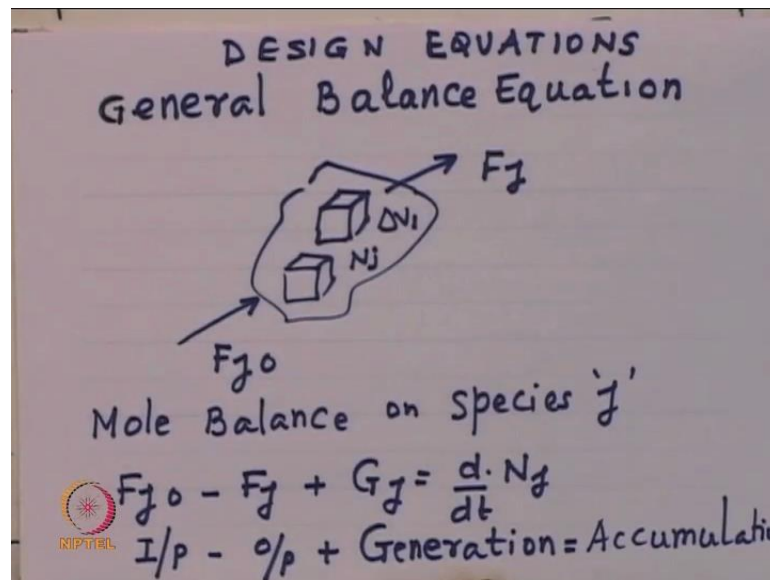
The task in front of us can be design; the task can be operation; the task can be development of a reactor design for a new application. Our principles must apply to all of these at all times so we need to develop principles and methods; so that we are able to address any of these situations that we would like to handle. So, to put this in the perspective this course advanced reaction engineering.

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Essentially, we want to look at the design equations that are required to understand the equipment that we are going to design, to operate the equipment to performance function that we are looking for. So, let us get on with this task of reaction engineering design equations.

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Now, this is a conceptual representation of what might be reaction equipment. Please do not think like this is how the reaction equipment looks like that is not the idea. So, we have an enclosure of an arbitrary shape deliberately chosen, in which a certain fluid F_j at 0 is entry and it is leaving here; fluid enters and fluid leaves in the equipment. Now we can write a material balance; we call it is as a mole balance any other components that enter the system. F_{j0} is the moles per time that entering the system and F_j is the moles per time that leave the system, while G_j is the moles per unit time which is produces in the system produce by means we can we consume it as a negative value. This difference would be the rated which the material is accumulated in the system.

On other words, input minus of output plus generation equal to accumulation is a fundamental statement of material balance; whatever comes in; whatever goes out whatever get generated; this combination that has to accumulate. Generation can be positive in which case this function is positive; it can be negative if it is consumed it is a negative value through general statement of material balance. Now, we recognize that this function G_j this function G_j ; that means, they related which components is reduced or consumed by the point in the equipment this could to vary in the point to point variations. On other words, the rate of generation of this component can be different points in the equipment you must take into an account.

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$$G_j = r_{j1} \Delta V_1 + r_{j2} \Delta V_2 + \dots$$

$$= \int_V r_j dv$$

$$F_{j0} - F_j + \int_V r_j dv = \frac{dN_j}{dt}$$

if r_j is uniform over volume V

We have in a stirred vessel

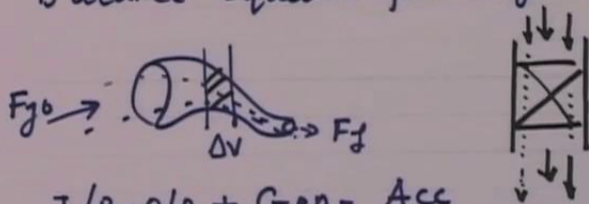
$$F_{j0} - F_j + r_j V = \frac{dN_j}{dt}$$

So, keep in that in mind what you are said in that at different points one two three four five upto n r_{j1} is the rate of generation component j Δv_1 is the volume of the elemental volume. Similarly, r_{j2} is the rate of generation elementary volume Δv_2 upto n . On other words rated which the component j is generated is an integral of r_j multiplied by dv . So, $r_j dv$ is the rate of generation of component j . So, if you substitute this into our general statement here material balance replaces this G_j by r_j times dv . So, that we have more general statement of the composition equation F_{j0} minus of F_j integral $r_j dv$ equal to d by dt of N_j . Now, it could so happen that the equipment that we are dealing with is an equipment where there are is a stirrer so that the composition temperature in other intensive variables are essentially the same and different points in the in which case called stirred vessel.

So, if you have stirred vessel into its continuous input of component the continuous output of component j then, we have we are continuous input of component j continuous output the stirred vessel in which case we can say that r_j is uniform at every point inside the equipment so that we can take r_j outside the integral therefore, this equation can be written as F_{j0} minus of F_j plus r_j times v integral dv is v equal to d by dt N_j . Other words out statement of material balance for the case of stirred vessel can be written as F_{j0} minus of F_j plus r_j times v equal to d by dt N_j . Let us look at this some more detail.

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Balance Equation for Plug Flow Vessels



$$I/P - O/P + \text{Gen} = \text{Acc}$$

$$F_j(v, t) - F_j(v + \Delta v, t) + r_j(v, t) \cdot \Delta v = \frac{\partial (C_j \Delta v)}{\partial t}$$

$$-\frac{\partial F_j}{\partial v} + r_j = \frac{\partial C_j}{\partial t}$$

at steady state

$$\frac{\partial F_j}{\partial v} = r_j$$

Suppose in state of stirred vessel instead of stirred vessel, we have a different type of vessel which is often seen in the process industry which is called as a tubular reactor. What is the tubular reactor? We have a pipe of a chosen diameter, perhaps we have a catalyst inside and fluids come this way; let us say and this is out way a tubular reactor. Now, this is trying to represent a general situation where you have a vessel of an arbitrary shape through which component j is entering component j is leaving; component j is entering and leaving $F_j 0$ is entering F_j is leaving. We want to write a material balance on an elemental volume Δv so, this is the elemental volume Δv are which you want write a material balance.

So, we notice here that F_j at v at any time t F_j at v plus Δv at any time t plus r_j times Δv at any time t equal to this is the total amount of multiply with Δv . So, this is the statement of material balance for a flow through what we call as plug flow vessel. By plug flow what you mean is that every fluid element is that enters here moves through the equipment without recognizing the existence of other fluid elements. On other words as the fluid moves through the equipment it does not mix with other fluid elements. For example, what is mean said here suppose the fluid element comes here it moves through and gets out; another fluid element that enters here it moves through and gets out. On other words this fluid element moves through and this fluid through this and they do not recognize existence of each other.

As a result this is called as the plug flow there is no inter mixing between these two fluids and such situations when we write material balance in the element F_j at v F_j at $v + \Delta v$ rate of generation with the component j this is the difference that accumulates. So, if you take the as Δv tends to 0 and so on. This how the material balance looks like under the unsteady state; when it is steady state where this term disappears the question which like this. On other words what we are trying to say here is there are two types of equipments; ideal equipment that you will see in the process industry.

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$$G_j = r_{j1} \Delta V_1 + r_{j2} \Delta V_2 + \dots$$

$$= \int_V r_j dv$$

if $\left[\begin{array}{c} F_{j0} \rightarrow \\ \downarrow \\ \text{Stirred Vessel} \\ \downarrow \\ F_j \end{array} \right]$ then $\rightarrow F_j$
 r_j is uniform

$$F_{j0} - F_j + \int_V r_j dv = \frac{dN_j}{dt}$$

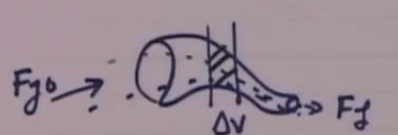
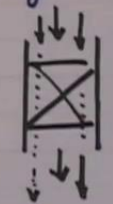
if r_j is uniform over volume V
 We have in a stirred vessel

$$F_{j0} - F_j + r_j V = \frac{dN_j}{dt}$$

Equipment number one, we may pointed out is a stirred vessel. This stirred vessel could be operated such that it could be a batch in the sense there is no continuous input and continuous output, it is still a stirred vessel there is a stirrer; there is a composition is maintained.

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Balance Equation for Plug Flow Vessels

$$I/p - o/p + \text{Gen} = \text{Acc}$$

$$F_j(v, t) - F_j(v + \Delta v, t) + r_j(v, t) \cdot \Delta v = \frac{\partial}{\partial t} (C \Delta v)$$

$$-\frac{\partial F_j}{\partial v} + r_j = \frac{\partial C_j}{\partial t}$$

at steady state

$$\frac{\partial F_j}{\partial v} = r_j$$


The other kind of equipment is what we have here is tubular equipment where materials move through and then exit without mixing with each other that is called as plug flow vessel. The equations that describe if the variation of various properties for a stirred vessel is given by this and for plug flow vessel given by this. For a plug flow vessel at a steady state $\frac{\partial}{\partial v}$ of F_j is given by rate of generation of j and for stirred vessel equation is already shown. Now, these two types of equipment what we call as plug flow this is for a case of plug flow equipment.

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Plug Flow steady state

$$\frac{dF_j}{dv} = r_j$$

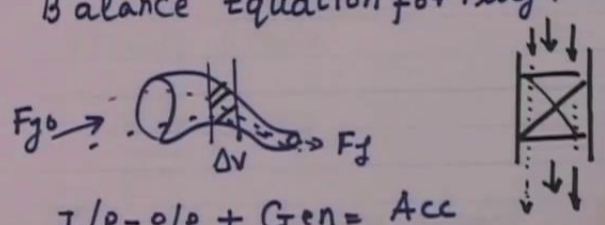
Plug flow means fluid elements do not mix as it moves through equipment



So, you find that plug flow equipment are very common particularly, in the process industry dealing with catalyst. Because this pipe holds the catalyst and the fluids can enter and leave; therefore, the catalyst that is performing work for you can be changed assignment activity goes down. That is the kind of equipment most popularly used in the process industry we will deal with fluids and mode of heat etcetera. Particularly, in smallest scales, you will find stirred vessels in batch and continuous mode. On other words what am trying get across to you is that you have the stirred vessels, which are commonly used for smaller scales operations. Where do you have either in batch operations or continuous operations?

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Balance Equation for Plug Flow Vessels



$$I/P - O/P + Gen = Acc$$

$$F_j(v, t) - F_j(v + \Delta v, t) + r_j \cdot \Delta v = \frac{\partial}{\partial t} (C_j \Delta v)$$

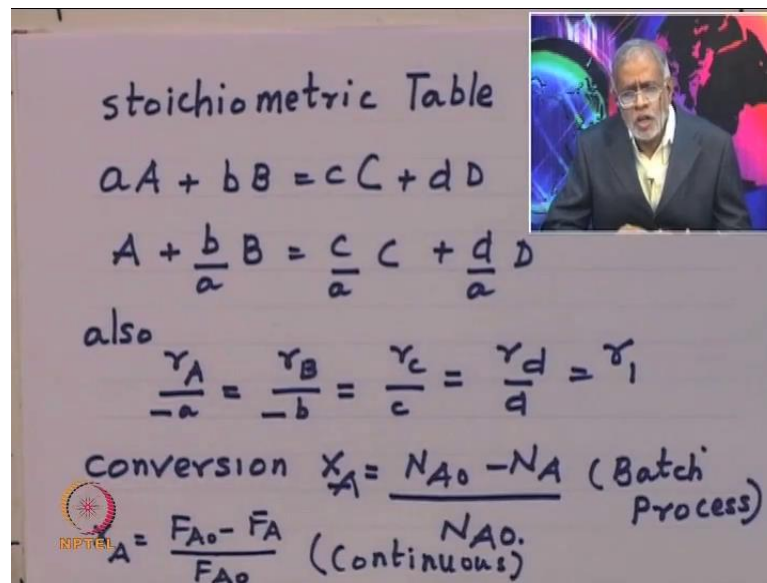
$$-\frac{\partial F_j}{\partial v} + r_j = \frac{\partial C_j}{\partial t}$$

at steady state

$$\frac{\partial F_j}{\partial v} = r_j$$

You have this plug flow vessels particular for catalytic reactions; mostly in very large scales they have a catalyst, which is continuously used for processing a certain material that selecting a final product. So, batch vessels continuous stirred vessels as well as plug flow vessels are the most common thing that you will see in the process industry. We have set up equations both for batch and stirred vessels here and then we are set up equations for steady state and unsteady state for the plug flow vessels. Our job now is to see how reactions perform in these ideal vessels. How do reactions perform and what we can learn from the performance of the reactions?

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Let us just to illustrate let us just assume that the reaction of this form $aA + bB$ giving you $cC + dD$ takes place in our reaction equipment. Now if you choose a as our reference species then we can divide throughout by a so that our reaction now looks like $A + \frac{b}{a}B = \frac{c}{a}C + \frac{d}{a}D$. This is assuming that this is component A is our reference. Now, we know that the reaction takes place at certain rate if I call that rate of chemical reaction as r_A ; if I say that rate of reaction of component A r_A understanding a basic chemistry that r_A divided by minus of a equal to r_B divided by minus of b equal to r_C divided by plus c equal to r_D divided by plus d .

On other words this ratio rate of reaction A divided by stoichiometric coefficient with appropriate sign for a given reaction there are equal and therefore, I turned this r_1 sometimes called as intensive rate of reaction. Now, how do we understand conversion? We understand conversion at steady state conversions are typically we start with so many moles away and end of the reaction so many moles unreacted. So, we express the amount of component A converted with respect to what is started with that is gives to the conversion. In other words, the whole meaning of conversion requires you to recognize certain starting material or reference. So, our reference for defining conversion is N_{A0} .

If you have a batch process you have a continuous process this N_{A0} is moles if the continuous process X_A is defined as with respect to the reference F_{A0} , where F_{A0} has the units of moles per time. So, whether it is batch or continuous we defined

conversion with respect to certain reference. For continuous process, we have taken $F A_0$ reference species and for batch process we have taken $N A_0$ as a reference. Choice of reference is always your choice in the sense you can prefer defined with respect to your reference. Generally, a reference component giving limiting substrate or limiting agent that is in the reaction that is how it is normally reference is ok.

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BATCH SYSTEM $aA + bB = \frac{c}{a}C + \frac{d}{a}D$

Species	Moles (Initial)	Moles Remaining
A	N_{A0}	$N_A = N_{A0} (1 - X_A)$
B	N_{B0}	$N_B = N_{B0} - N_{A0} X_A (b/a)$
C	N_{C0}	$N_C = N_{C0} + N_{A0} X_A (c/a)$
D	N_{D0}	$N_D = N_{D0} + N_{A0} X_A (d/a)$
I	N_{I0}	$N_I = N_{I0}$
	N_{T0}	$N_T = N_{T0} + N_{A0} \left(\frac{d}{a} + \frac{c}{a} - b \right) X_A$

Let us say we have a batch system where the reaction $aA + bB$ just we write it down on the scale our reaction is $aA + bB = cC + dD$ all of you divided throughout by a it will like this. So, for this reaction if you look at species A B C D and I; I is inert typically every system comes into inert. So, you starts with so many moles $N_{A0} N_{B0} N_{C0} N_{D0} N_{I0}$ and so on. Therefore, as per stoichiometry what we have written is if X is a extensive reaction or conversion. So, the unreacted a is given by N_A equal to N_A times one minus of X_A , whose definition and similarly N_B is N_{B0} minus of $N_{A0} X_A$ multiplied by b by a .

So, this term b by a this term comes because of the stoichiometry as we are represented there. So, as per the way we are define conversion and the way we are taken the chemical reaction; we can write what is the unreacted amounts of A B C D and I in the system at any given conversion. This follows directly form stoichiometry you can add up all this. When you add up all these you will find it is number of moles we start with is N_{T0} .

When you add up all these moles this is so many moles to have coming in and so many moles are going out.

When you add up all these you notice here this is N_t N_{t0} and all these terms common $N_{A0} X_A$ is common and X_A ; this term inside d by a plus c by a minus b by a minus 1; this term the effect or the chemical reaction. So, what we get here from stoichiometry number of moles we start with the number of moles we get; then this is the certain amount of reaction X_A .

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$$N_t = N_{t0} + \left(\frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1 \right) N_{A0} X_A$$

$$\frac{N_t}{N_{t0}} = \left(1 + y_{A0} X_A \delta_A \right) \begin{array}{l} \uparrow \\ \text{change in moles} \\ \text{due to rxn w.r.t A} \end{array}$$

$$\delta_A = \left(\frac{d}{a} + \frac{c}{a} - \frac{b}{a} - 1 \right) \Rightarrow \begin{array}{l} \text{change in} \\ \text{moles due to} \\ \text{rxn w.r.t A} \end{array}$$

$$y_{A0} = \frac{N_{A0}}{N_{t0}}$$

Now, we can write number of moles at the end of certain extensive reaction certain conversion X_A is given by N_t N_{t0} plus this term the effect of the chemical reaction as expressed with respect to component A, which is what is called as delta A. Delta A is meaning of delta A is change in moles change in moles due to reaction with respect to the reference component A. On other words delta A tells you what is the number of moles change that comes because of the reaction d by a plus c by a minus b by a minus 1; this is the known quantity for every reaction. So, what we get from stoichiometry is that given a conversion X_A given conversion X_A N_t by N_{t0} is given by this relationship.

y_{A0} is the mole fraction of component A by the start with the N_{t0} is the total number of moles, which we start and delta A is change in moles due to reaction expressed with respect to component A. So, in other words this representation tells us what is the

number of moles at any conversion at which the reactor working. Now, what is important is what we have done here what we have done here with respect to component A; we can do with any other component does not matter. So, just put it in the perspective but I have said here is that.

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When you use B as reference
we get

$$\frac{N_t}{N_{t0}} = 1 + y_{B0} \delta_B X_B \quad X_B = \frac{N_{B0} - N_B}{N_{B0}}$$

where $\delta_B = \left(\frac{d}{b} + \frac{c}{b} - a/b - 1 \right)$

$$y_{B0} = \frac{N_{B0}}{N_{t0}}$$

If you chose B as a reference species then your equation look like this instead of y_{A0} instead of y_{A0} here y_{B0} instead of X_A ; you will get X_B and what is the X_B what is the X_B ; X_B is generic X_B equal to N_{B0} minus of N_B divided by N_{B0} . On other words, X_B is the conversion as measured with respect to the component B. We recognize that inner reaction this could be different in measure conversion with respect to component A and component B there could be different.

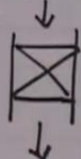

Now if instead of for batch systems so we talked about the stoichiometry all the stoichiometry we have talked about is considering our system is batch system. By batch what we mean is we have equipment, which is a vessel which is got no outlet and no inlet; certain amount of fluid is inside the equipment and then these are the changes that we are going to observe.

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FLOW SYSTEM

For a flow system our equations look similar. Simply replace N by F

$$F_t = F_{t0} + F_{A0} (\delta_A) X_A$$

$$\frac{F_t}{F_{t0}} = 1 + y_{A0} X_A \delta_A$$



But we could also have a system where there is a continuous input there is a continuous input and continuous output like a tubular reactor; there is a continuous input there is a continuous output. So, how do we deal with systems where there is a continuous input and continuous output? So, when we have a flow system our equations now look exactly the same that is not difference or stoichiometric still does not change. I notice here this here is to writing if N_{A0} ; I could written in F_{A0} F_{B0} F_{C0} F_{D0} F_{I0} F_{t0} . On the other words as per the stoichiometric concern it does not matter whether you are talking about plug flow system or a batch system. Therefore, whatever relationship that we get for n_t is the same we will get when it is for f_t because stoichiometric does not get affected because of flow etcetera.

So, we can say now F_t the total number of moles giving the system is F_{t0} , which is total number of moles entering the system multiplied by $F_{A0} X_A$ multiplied by δ_A . Same like here N_t equal to $N_{t0} N_{A0}$ this is δX_A . So, essentially what we are saying that if the flow systems then total moles leaving is related to the system with same kind of equation that we have done for the batch system. So, that F_t divided by F_{t0} is given by $1 + y_{A0} X_A \delta_A$; previously we have N_t divided by N_{t0} given by this. So, whether it is a N_t by N_{t0} or F_t by F_{t0} the right hand side does not change because these are coming from stoichiometric. Now have we set all these things let us go a little bit forward and see how we look at various kinds of design equations for our systems?

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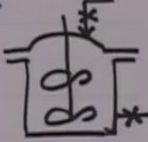

Design Equation(s) Batch System
Well Stirred Batch

I/p - o/p + Gen = Accumulation

$$F_{j0} - F_j + r_j V = \frac{dN_j}{dt}$$

$$aA + bB = cC + dD$$

$$A + \frac{b}{a}B = \frac{c}{a}C + \frac{d}{a}D$$

$$r_A V = \frac{d \cdot N_A}{dt}$$



First system we are taking that is batch systems. Let me just remind you what the batch system is or batch system looks like this. You have a vessel the vessel is closed it is well stirred, which means the composition are same at different points there is an inlet, but this this is closed; there is an outlet which is closed. The idea of this inlet valve and outlet valve so that we can charge and discharge. So, as for the reaction is concerned during the reaction there is no inlet and there is no outlet, whatever is taking inside the equipment that is what brings about the reaction? So, the case of the batch system where there is no input where there is no output there is only generation and accumulation. Therefore, first term and second term disappears therefore, you have r_j times V rate of generation of the component equal to d by dt of N_j .

So, this is the statement of the material balance for a batch system that rated which the reaction takes place equal to the rate of accumulation in the system. Now our reaction is given by $aA + bB = cC + dD$ or in terms of component reference it is looking like $A + \frac{b}{a}B = \frac{c}{a}C + \frac{d}{a}D$. Therefore, now we can say if A is a reference species r_A times V just replacing j by a we have r_A times V equal to d by dt of N_A . So, statement of material balance for a batch system is simply r_A times V equal to d by dt of N_A . Now we can go forward, we will taking about stoichiometric table.

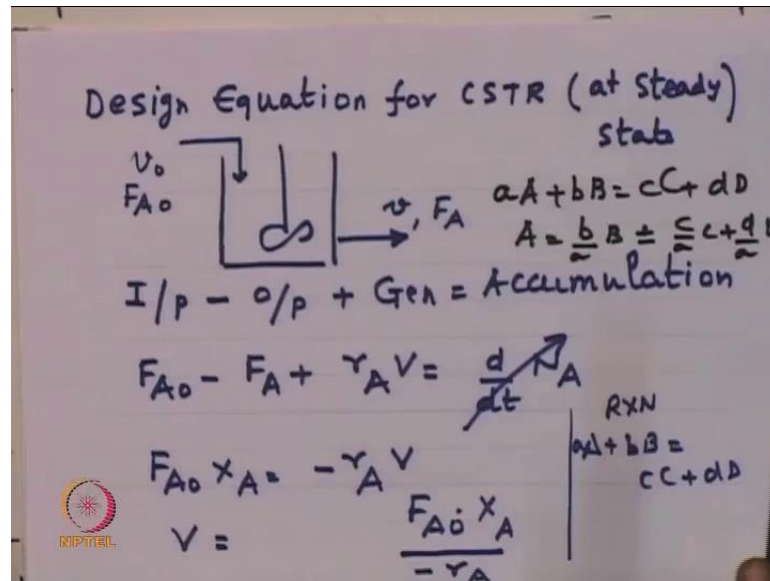
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The image shows handwritten mathematical derivations on lined paper. At the top, the conversion equation is given as $x_A = (N_{A0} - N_A) / N_{A0}$, with an arrow pointing to N_{A0} and the text "Note Reference". Below this, the equation $N_A = N_{A0} (1 - x_A)$ is written, followed by the material balance equation $-N_{A0} \frac{dx_A}{dt} = r_A V$. The next line shows the rearranged equation $dt = \frac{N_{A0} dx_A}{-r_A \cdot V}$. The word "integrating" is written above the final equation, which is $t_r = N_{A0} \int_0^x \frac{dx_A}{-r_A \cdot V}$. An arrow points to the denominator of the integral.

Now, something that we know that conversion is defined as N_{A0} minus of N_A divided by N_{A0} . A point to be noted here is that N_{A0} is a reference; this reference is our choice. This particular case we have taken A as the reference. Now as a result of the this representation know this N_A is given by N_{A0} times 1 minus of x_A . On other words if we look at batch reactor we find minus of $N_{A0} dx_A$ by dt equal to r_A times V ; this is the statement of the material balance. So, that we can see dt equal to $N_{A0} dx_A$ divided by minus of r_A times V , when you integrated this if the reaction time is $N_{A0} dx_A$ divided by minus of r_A times V .

Notice here that volume volume volume effect appears here. On other words if volume of the reactor changes during the process of the reaction, where the effect should have to be accompanied here. If the volume is constant you can take out of the integral so that it becomes N_{A0} ; so that $t_r = N_{A0}$ times dx_A divided by minus of r_A times V familiar representation takes place it is so, why does this is constant important issue we must bear in mind.

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Now, let us take another very common instance that you might see in the process industry what is called as continuous stirred tank reactor. Now, stirred tanks are examples where the fluid comes in continuously; fluid leaves continuously. Batch equipment please recalls batch equipment we said the process works the process works and during the process there is no input. We set this batch equipment is an instance where there is no addition or removal of material as the reaction takes place. That is the batch equipment, which are all the raw materials close all the valves conduct the reaction at the end of the reaction time; you open this valve to discharge the product.

Now, a continuous reactor is of continuous input of fluid continuous removal of fluid, a reaction takes place inside this equipment continuously. Therefore, we are able to continuously withdraw the product at the rate that we are desire. So, continuous reaction what is called continuous stirred tank reactor; one example of process equipment by the you will produce the product continuously. On other words, you do not have to worry about charging time discharging time and so on, which could be very significant in the batch process. So, all the charging and discharging of material is eliminated only on the process time is what is of concerned to you and this way you are able to produce large quantities of materials.

But an advantage of the continuous process is that we do not use time during charging and discharging. Now, let us see how the equation looks like. Our statement of material

balance does not change input minus of output plus generation equal to accumulation these are fundamental statement of conservation of mass. Our input is what if are reaction once again let us recognize let me write down once again a $A + b B \rightarrow c C + d D$ and A as reference it becomes b by a B plus equal to c by a C plus d . So, we are still considering same reaction with A as the reference. So, we have input input is a $F_A 0$ we can see here input is $F_A 0$ output is F_A ; you can see here rate of generation of component is A and d by dt of N_A and their operating at steady state.

We will look at unsteady state of situation later on for the moment let us assume let this process is running at steady state. What is meant by steady state? By steady state we mean if you look at composition of the system at any time you look at composition of the output at any time. We will find composition inside the equipment and composition exit at the same; that means, it does not change with time. Steady state is an instance of a process operation, where the composition that you measure the exit does not change with time. Composition that you measure inside the equipment with time does not change its time; that means, it is steady with respect to time under situation where this is steady if the equation $F_A 0$ minus of F_A plus $r_A V$ equal to d by dt of N_A .

On the right hand side which represents accumulation because it is steady there is no change with respect to time therefore, d by dt of this will go to 0. Therefore, the right hand side is 0. So, steady state representation of material balance gives you $F_A 0$ times X_A equal to minus $r_A V$; because right hand side is 0. Please recognize $F_A 0$ minus of F_A equal to $r_A V$ comes from the fundamental definition of X_A . We define X_A as $F_A 0$ minus of F_A divided by $F_A 0$; that is how X is defined that is why we write this term in this form. So, that this representation gives us volume of continuous stirred tank reactor is given by $F_A 0$ times X_A divided by minus of r_A .

So, a continuous reaction in which you have CSTR bring about this reaction volume of the equipment is given by $F_A 0$ times X_A divided by minus of r_A ; $F_A 0$ is something you know X_A you can measure and r_A is the chemical kinetics at which the reaction occurring. So, right hand side known quantity therefore, given the chemical kinetics given the extent to which you must react given the production that you want all these can be taken together to determine what is the size of the equipment which must design for.

Now, why is it that moved away from batch equipment continuous equipment like CSTR there are many reasons for this batch to continuous is the decision that (()) take depending upon the scale of production. Generally, you find continuous production of preferred if the scales are very large the scales are small of course, the batch productions are preferred. But that is not saying enough lets also recognize the when you have some products which are very expensive, where you require very careful control of the process during the process of reaction the general preference is to batch so that you can address all the issues that that is required to be addressed in the course of reaction.

So, there are certain advantages in doing batch processing and are there are advantages using continuous processing and the decision will be a decision that is taken based on technical economic and commercial issues.

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$$\tau = V/v_0$$
 Since
$$V = \frac{F_{A0} X_A}{-r_A}$$

$$\tau = \frac{C_{A0} X_A}{-r_A}$$

$$\tau = \text{residence time based on inlet flow}$$

Now, what we are now saying is that if our stirred tank continuous CSTR design equation is this recognizing that residents time this actually volume of the equipment divided by the flow at the inlet. Then, we can write our design equation in terms of V; we can also write in times of resident time torque, which looks like this. On other words this is in terms of volume this is in terms of residents time they are stating the same thing the residents and reactor volume are related depending upon the volume flow at which we process the fluids.

So, this is the fundamental statement of design equations for CSTR or this may be another way of saying this is the equation for CSTR. If you know the chemical kinetics if you know the extent, which you want to react and if you know this fluids you can say what is the size of the equipment that is required for your process. What does happen in the process industry that there are situation where you may need there are no one stirred tank there are many stirred tank in sequence.

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Sequence of CSTR's of Equal Volume

$$F_{A0} - F_{A1} + r_{A1} V_1 = 0 \quad \text{Steady state}$$

$$V_1 = \frac{F_{A0} X_1}{-r_{A1}}; \quad \tau_1 = \frac{C_{A0} X_1}{-r_{A1}}$$

Tank 2

$$F_{A1} - F_{A2} + r_{A2} V_2 = 0$$

NIPTEL

There are situations which require many times to be operated in series, but more importantly this sequence of stirred tanks gives us certain inside what happens in the process. Let us say look at what this sequence tells us this sequence tells us that if you look at tank one; input output generation equal to 0 at steady state; we have study all these are steady state; that means, we are writing a material balance at steady state. We will come to unsteady state at later date first now we are looking at steady state.

Therefore our statement of material balance leads us to size of the equipment given by $F_{A0} X_1$ divided by minus of r_{A1} or residents time given by $C_{A0} X_1$ by minus of r_{A1} ; that means, the residents time in tank one is given by this equation $C_{A0} X_1$ by minus of r_{A1} . Now, if you write material balance for tank two which is $F_{A1} - F_{A2} + r_{A2} V_2 = 0$, where this is the rated which the reaction occurs in tank two; r_{A1} is the rated which the reaction occurs in tank one. therefore, this equation is the way of understanding what is the size of V_2 that is required.

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Handwritten mathematical derivation on a whiteboard:

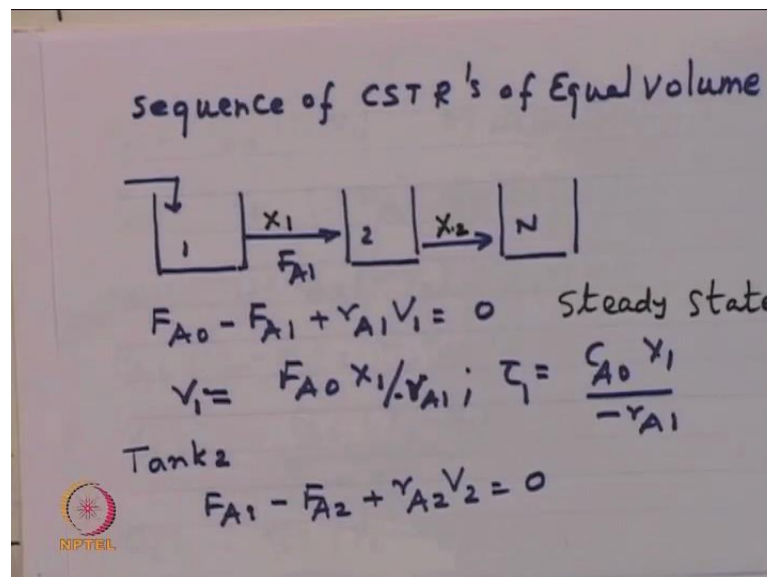
$$F_{A1} = F_{A0} (1 - X_1) \quad \text{by definition}$$
$$F_{A2} = F_{A0} (1 - X_2) \quad \text{by definition}$$
$$F_{A0} (1 - X_1) - F_{A0} (1 - X_2) + r_{A2} V_2 = 0$$
$$F_{A0} (X_2 - X_1) = -r_{A2} V_2$$
$$V_2 = \frac{F_{A0} (X_2 - X_1)}{-r_{A2}}$$

Below the equations is the NIPTEL logo and the equation for residence time τ_2 :

$$\tau_2 = \frac{C_{A0} (X_2 - X_1)}{-r_{A2}}$$

So, we can simplify this write our material balance; material balance this is our material balance for tank two. What is written once again in the next page F_{A0} input output generation equal to 0. We find that the size of reactor two equipment two can be given as F_{A0} times X_2 minus of X_1 X_2 is what is the X_2 ?

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X_1 is conversion at the at the outlet tank here; X_2 is conversion here, what are we saying here?

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Handwritten mathematical derivation on a whiteboard:

$$F_{A1} = F_{A0} (1 - X_1) \quad \text{by definition}$$
$$F_{A2} = F_{A0} (1 - X_2) \quad \text{by definition}$$
$$F_{A0} (1 - X_1) - F_{A0} (1 - X_2) + r_{A2} V_2 = 0$$
$$F_{A0} (X_2 - X_1) = -r_{A2} V_2$$
$$V_2 = \frac{F_{A0} (X_2 - X_1)}{-r_{A2}} \quad \tau_2 = \frac{V_2}{v_0}$$

NPTEL logo and the number 2 are visible in the bottom left corner of the whiteboard image.

What we are saying here is that; the size of tank two depends upon F_{A0} times X_2 minus of X_1 , what is the X_2 minus of X_1 it is the moles of component A undergone chemical reaction in tank two. What is r_{A2} ? r_{A2} is rate of chemical reaction occurs in tank two. So, the total moles converted divided by the rate of conversion using size of the equipment. So, it is stating the obvious it is nothing new I have been said obvious being said but said in a form this is useful for calculations.

So, what do we get, the size of the equipment we told we want to express in terms of resident time our residents time is defined as equal to V_2 divided by v_0 . So, we have size of the equipment two, I mean reactor two is given by C_{A0} the conversion change divided by the rate of change of reaction. Now, we can continue this whole process and then say for tank three tank four tank five and so on.

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The image shows handwritten mathematical equations and a schematic diagram on a piece of paper. The equations are:

$$V_n = F_{A0} (X_n - X_{n-1}) / (-r_{An})$$
$$\tau_n = C_{A0} (X_n - X_{n-1}) / (-r_{An})$$

Below the equations is a schematic diagram of a series of tanks. It consists of three rectangular boxes labeled '1', '2', and 'N' connected by arrows pointing from left to right. An arrow labeled F_{A0} enters box '1' from the top. An arrow labeled X_{N-1} enters box 'N' from the left. An arrow labeled X_N exits box 'N' to the right. Ellipses between boxes '2' and 'N' indicate intermediate tanks. In the bottom left corner of the paper, there is a small circular logo with a star and the text 'NPTEL' below it.

We can say for tank N we are looking at its F_{A0} times X_N minus X_{N-1} divided by minus r_{An} , where what is F_{A0} is recognized, we have N tanks here; tank one and tank two and then tank n. This is tank N, 1, 2. Now inputs this is the F_{A0} this is what is coming out is X_N . So, what we are saying is that size of the tank is V_n this is n if F_{A0} what comes here multiply by what happen this is X_N minus of 1; what is coming here is X_N minus of 1 you see so, X_N minus of 1. This is difference change in conversion this divided by rate of chemical reaction. So, it is stating is obvious, but in forms that is very useful to us for doing calculations. So, what are we said what we are said is that if you have if you have tanks in sequence instead of single tank tanks in sequence.

When we find that we can use a same statement of material balance to find out; what is the size of the equipment one tank one; what is the resident time in tank one. Then we can see what the size of the equipment two is; what the resident time is in tank two. Similarly, we can say what the size of the equipment n two is; what is the resident time in tank n. So, our statement of material balance helps us to calculate all this because it is put in that form. Suppose, as we said when we started this we said we have two types of equipment that you will see in the process industry. One that is what is called stirred vessel that is mechanically stirred; the other equipment you have plug flow vessel, where there is no mechanical stirring inside the equipment.

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PLUG FLOW

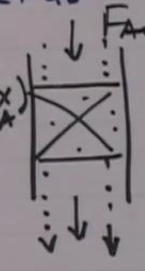
$$\frac{dF_A}{dV} = r_A$$

$$-F_{A0} \frac{dx_A}{dV} = r_A$$

$$V = F_{A0} \int_0^{x_A} \frac{dx_A}{-r_A}$$

$$\tau = V/v_0 = C_{A0} \int_0^{x_A} \frac{dx_A}{-r_A}$$

$aA + bB = cC + dD$

$$F_A = F_{A0}(1 - x_A)$$


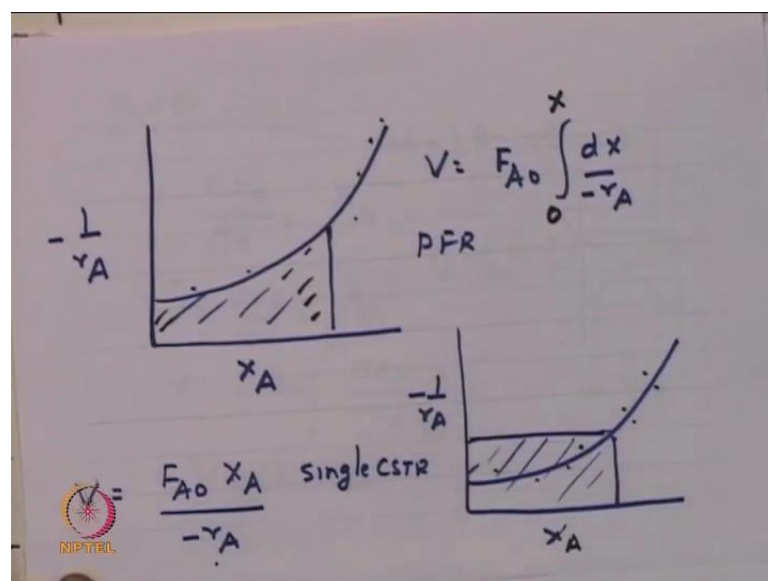
That means the plug flow equipment you have let us say a pipe may be inside the pipe there is a catalyst and the fluid is coming in and going out; there is no mechanical equipment inside here to do stirring. So as a result but expects but there is plug flow plug flow what we mean is that the inside without mixing with each other; so this is plug flow vessel equipment. Now, we are already derived what is our design equation for a plug flow equipment for a reaction like this. We already said this dF_A by dV of F_A is r_A . What are we saying here what we are saying is that dF_A by dV is what rate of change of moles of A per unit volume per unit time. So, rated which moles are changes per unit volume per unit time that must be equal to the rate of chemical reaction.

It is stating the obvious what is trying put across to you that every equation that is written here is that stating the obvious. It is in a form that we can understand from first principles nothing new being said. Now, we can represent this F_A from our stoichiometry; what we are said in our stoichiometry we said in our stoichiometry F_A is equal to F_{A0} multiplied by of 1 minus of x_A . This is how it is defined this conversion itself defined like this. Therefore, we are able to substitute for F_A here and simplify and we get volume of this equipment is F_{A0} ; what is the entering here this is F_{A0} what is the entering what is leaving. So, volume of the equipment is F_{A0} multiplied by this integral dx_A by minus of r_A . Please recognize that we had stirred tank $C_{A0} \int_0^{x_A} \frac{dx_A}{-r_A}$.

On other words, here we have an algebraic equation here we have an integral. So, there is a difference. So, this is the differential equation whereas, this algebraic equation. So, when we have stirred tanks we deal with algebraic equation; when we have plug flow reactors we will deal with differential equation. Let me the equation relates with variables that we change as you change the equipment from one to the other. Now, that we know the size of the equipment we can now talk in terms of what is called as residents time that is defined as volume of the equipment divided by the flow rate flow rate of the inlet. So, the way in is defined as but the residents time is defined as volume divided by the flow rate at the inlet. In other words, saying that use in that this volume flow can change has it flows through the equipment. But, the way it is defined it is defined with respect to inlet flow.

On other words residents time also involves certain reference flows at which we do our calculation its convenient its convenient do it that way. So, if you defined that is way. So, if you defined residents time is V by v_0 , then this equation becomes $C A_0$ this is the known quantity; this integral $d X A$ by minus of $r A$. What is $r A$ is rate of chemical reaction, this number comes from understanding the chemical kinetics and we will related date look at how we can determine this function that determined this function $r A$. Now, having said this let us just quickly understand these functions recognize that this volume depends on X and $r A$.

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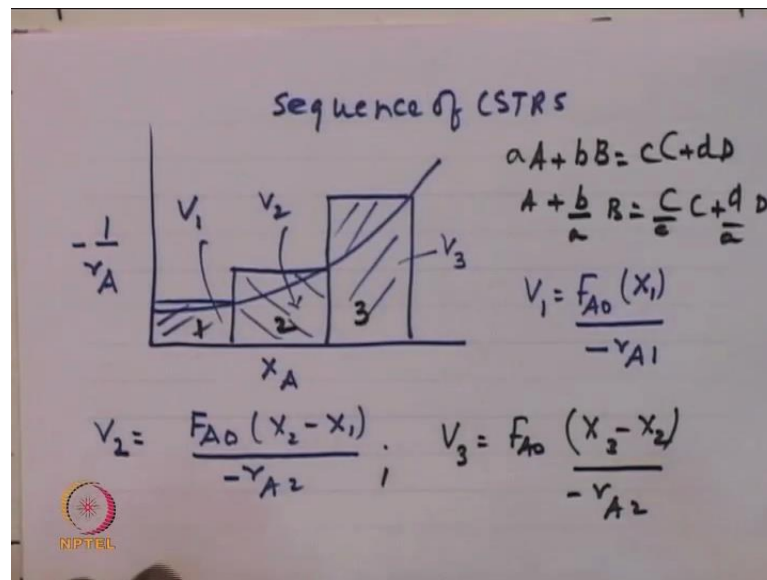


Suppose we plot have you said this suppose they make a plot of one by r_A please see this functions X and r_A . So, if I plot x versus one by r_A ; you notice that this area this area is what we are taking about. If you look at this you can make a plot of one by r_A versus X that is what I have done here one by r_A versus X . Therefore, this term X 1 by minus of r_A is simply the area under the under this curve this rectangle is the area. So, this area of the rectangle multiplied by $F A_0$ becomes the volume of the equipment. We can see here if I plot X versus r_A as I have done here X r_A you can see here this area multiplied by $F A_0$ becomes the volume of the equipment.

Now, for a plug flow reactor we said 0 to X dx by minus r_A . How do this when you plot 1 by r_A X A this integral is simply under this curve area under the curve. So on other words, what we notice here is that the way we are formulated the problem of single reaction taking place in ideal vessels; whether it is stirred tank whether it is plug flow the plots of one by r_A versus X actually it gives the way of determining size of the equipment. For example, in many cases it may be easier you to determine this function 1 by r_A versus X from your experiments the functionality r_A may not be known. But data may be available that you may have the experimental data here. You may have experimental data you may have experimental data so that you can use the experimental data and determine what is the size of the equipment?

In many cases particularly reactions, which you all many complex reactions etcetera; you will find it easier term actually make a plot of 1 by r_A versus X A . So, that we can determine the area under the curve and hence size of the equipment is very convenient that way and it is preferred most cases you will find this might be much faster way of determining size of the equipment that may be of listed. So, what we have try to say so far is that if you have a batch if you have a continuous equipment like CSTR or a continuous equipment like PFR; you can plot this 1 by r_A versus X and then find out the size of the equipment.

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Recognize that support instead of one tank you have many tanks and other words if you have several tanks in sequence by the V_3 is F_{A0} times X_3 minus of X_2 divided by r_{A2} the minus. On other words, if you have a plot of minus of 1 by r_A our reaction once again please recognize that our reaction still this there is no change. We are still looking at single reaction $A + b/a B = c/a C + d/a D$; we are still looking at this. So, if you have a sequence of tank one so, this is tank one this area this is for tank one; this is the area for tank two; this is tank three. Therefore, size of tank one two and three can be calculated by multiplying area and the curve by the flow F_{A0} .

So, this graphical procedure gives you a way determining size of the equipment size of the equipment simply by experimental data and that is great advantage of this procedure. Because you do not have to look for a functionality to determine the rate of chemical reaction the huge advantage that is the particularly for the reaction were you have you have great difficulties and understanding that what is going on experiment will be easier another point that frequently that we might want to know is suppose you have continuous equipment continuous equipment that is say.

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Actual Residence time in PFR

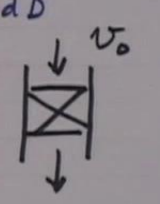
$$\rightarrow \boxed{\quad} \quad aA + bB = cC + dD$$

$$d\tau = \frac{dv}{v}$$

$$\frac{dF_A}{dv} = r_A$$


$$d\tau = \frac{dF_A}{r_A v} = \frac{F_{A0} dx_A}{-r_A v}$$

$$\tau_{\text{actual}} = F_{A0} \int \frac{dx_A}{-r_A v}$$



$$\tau = v/v_0$$

$$\tau_a =$$



Let us say catalytic for example and due to chemical reaction as you can see here in the chemical reaction there could be some volume change. How do you account for the volume change? We account for this volume change by recognizing that our equation dF_A by dv equal to r_A ; this is the fundamental statement of material balance for a plug flow vessel. We also know that the time of residence is given by dv by v . On other words if you have a plug flow equipment then the residence time at every point is given by dv by v and this is the general statement of material balance by combining these two; now we can tell that what is the actual time of residence after fluid elements inside this equipment. So, what we are trying to put across here is that if you have plug flow equipment in which there is a chemical reaction taking place.

As a result of which there is some volume change and you can take their effect into account through this equation. So, you not only can find out residence time using our residence time can be on the basis of inlet flow; we can do residence time this way or we can do residence time actual which is given by what is this. So, both forms both approaches are available and whatever is appropriate you will use in a given application. I will stop here with this we will take up this at in the next lecture; the implication of what I have said for its application to various that we would encounter.

Thank you.