Advanced Chemical Reaction Engineering
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Lecture - 29
1) Shrinking Core Ash Diffusion Model
2) Combination of Resistances

We continue with gas solid reactions, quickly I recognize what was said little earlier.

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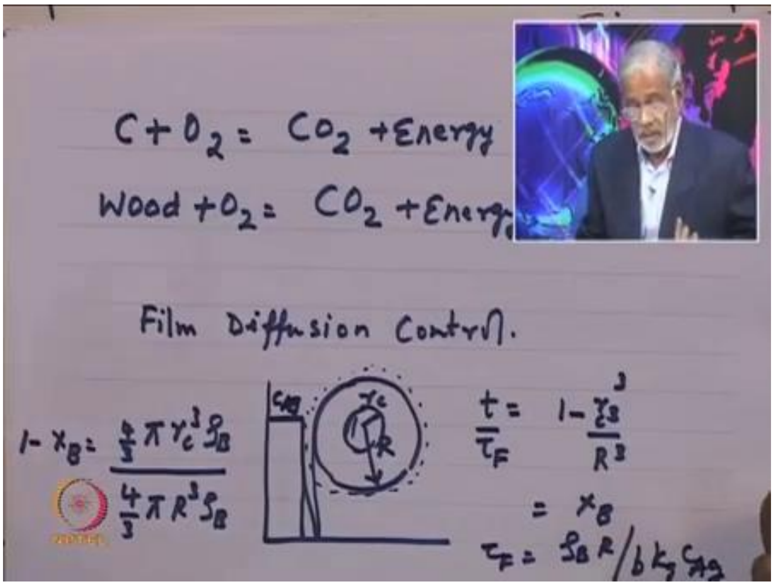

$C + O_2 = CO_2 + \text{Energy}$

$\text{Wood} + O_2 = CO_2 + \text{Energy}$

Film Diffusion Control.

$$1 - x_B = \frac{\frac{4}{3}\pi r_c^3 \rho_B}{\frac{4}{3}\pi R^3 \rho_B}$$

$$\frac{t}{\tau_F} = 1 - \frac{r_c^3}{R^3} = x_B$$

$$\tau_F = \rho_B R / b k_g C_{Ag}$$

In gas solid reactions let say we have coal reacting with oxygen giving you CO2 or say wood reacting with oxygen giving you carbon di oxide and energy. Our interest is to see that the energy its available coal or a wood is fully utilized. Therefore, we do not want to make we want to make a good use of the material that is with us so complete consumption is of concern to us, we would like to understand conditions, which we have a the complete consumption possible.

So, we talked about film diffusion control, under which we said if there is a particle. Then this particle the resistance the resistance to supply of gas is in the external film in the external film. Therefore, the rate at which chemical reaction occurs is essentially determined by this concentration, all the concentration drop occurs in the external film.

So, the rate at which chemical reaction occurs is a determined by the resistance in the external film. We will consider in this case, we found out how the time for complete time for the reaction is governed by t the relationship between am call t equal t by tau f equal to 1 minus r c cubed by r cubed, while r is the particle size and r c.

We mentioned last time is r c is the un reacted core size, really this is the un reacted core r c. Since, x b, which is extent of reaction 4 by 3 pi r c cube row b divided by 4 by 3 pi r cube row b 1 minus. That is why this becomes simply X B. This case we already discussed and we understand therefore, that for the case of film diffusion control the time for complete consumption divided by the time.

For the time for chemical reaction to complete consumption is given by this where tau f, which showed is simply row b r divided by b K g time C A g where C A g is the concentration of gas, which is outside the pallet does not change as the reaction proceeds, showing that this solid is in content to the large quantity of gas. Of course, this is an approximate situation, you have considered. We will relax all these. The second trace we considered if we use reaction control.

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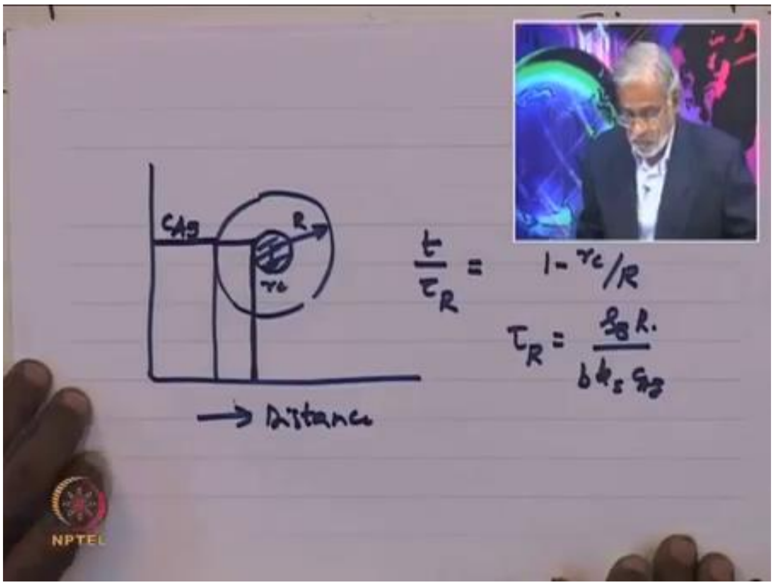

Here we have solid then we have this solid in contact with large quantity of gas. We said this un reacted core this is the un reacted core of radius r c the radius are un reacted core is r c here this is the particle is R. Then we said that the time for complete consumption divided by the time for complete consumption, time for reaction time for complete consumption we showed is equal to t by tau equal to 1 minus r c by R.

We showed this yesterday where tau r is given as row b r divided by K s C A g is to one moment yesterday I miss this term 3 here. This 3 I have missed b K s C A g. So, this case here you have the gas coming in at some concentration. This is the un reacted layer, so the real drop from the sense set the drops here. So, that all the reaction time will takes place at the un reacted surface, this is C A g. What is this? This is the distance, this two cases we considered yesterday. The third case we were talking, then we could not complete. We were that is the case of the ash.

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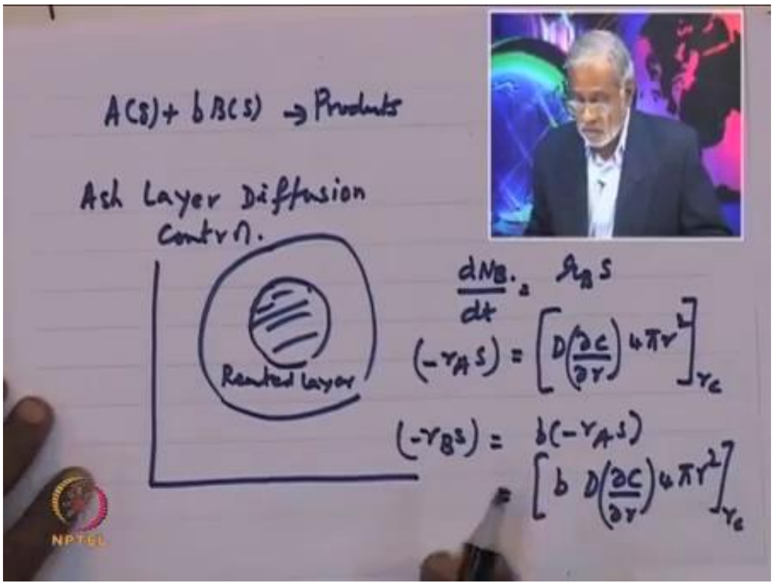

$A(g) + bB(s) \rightarrow$ Products

Ash Layer Diffusion Control.

$\frac{dN_B}{dt} = r_B s$

$(-r_A s) = \left[D\left(\frac{\partial C}{\partial r}\right)4\pi r^2\right]_{r_c}$

$(-r_B s) = b(-r_A s)$

$= \left[b\, D\left(\frac{\partial C}{\partial r}\right)4\pi r^2\right]_{r_c}$

Ash layer diffusion control, now here what we have is we have a solid and this is the solid un reacted solid. Now, what is happening is that as the solid is reacting this is the un reacted layer, this is the a reacted layer, this is the reacted layer. Now, as the reaction proceeds the extent of the reacted layer keeps changing and ash layer diffusion control moves that you know the rate at which can the reaction occurs depends upon the resistance due to the ash layer. Therefore, these resistance will keep changing as the reaction occurs, which you must take into account. This is what we would like to do.

So, let us recall that the rate at which chemical reactions occurs d N B divided by d t is r B s something that we have been saying for a long time. We also recognized that r A s, which is this is our reaction peace recognize. Our reaction is a gas plus b b solid, we will look all its this is the reaction we are taken. So, the rate at which reaction occurs is a rate at which the solid is supply rate at which the material is supplied to the un reacted surface multiplied by 4 phi r square.

This is at r equal to r c, now the point that we would like to understand here is that in the case of ash layer diffusion control the rate at which chemical reaction occurs is d del c del r 4 phi r square. That means if this is the un reacted surface the rate at which materially supplied to this surface is a rate at which chemical reaction will occurring. We also recognized that this interface keeps changing as time proceeds, which you must take into account as the reaction proceeds.

So, similarly, therefore, if you want to say what is r B s. Then we can say it is b times minus y, yes this comes from. Therefore, this is equal to b times d del c del r 4 phi r square at r equal to r c. Recognize, that we have left out this minus psi from the fix law of diffusion.

The reason why we left out this minus psi is because, in this case the direction of diffusion is opposite to the direction of positive walk. That is why we have left out the negative sign. Now, that we know the rate at which chemical reaction occurs essentially is determined by finding out, what is c how c changes with i. Essentially, we need to find way by which c changes with i. So, this is what we would like to do now by looking at the different equation, that governs the variations of concentration inside the particle. Let us see how to do this, let us as quickly recognized the situation.

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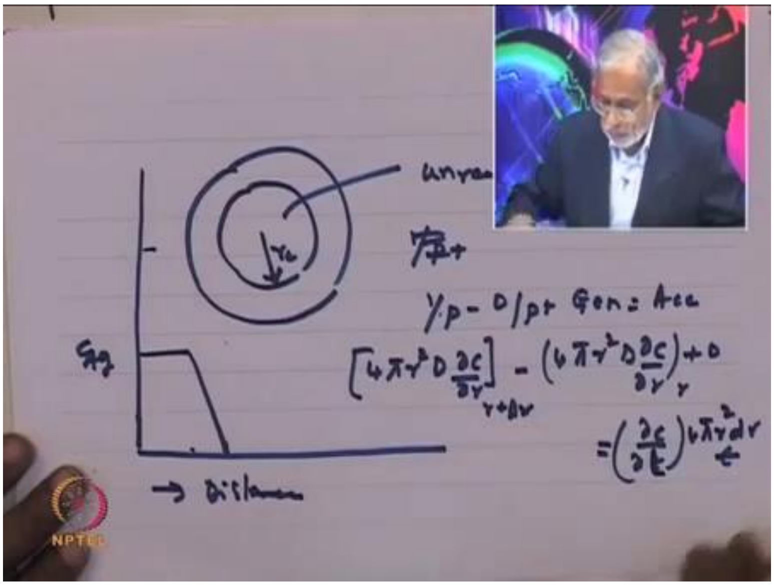

Once again how situation is this, now we have the concentration. This is the un reacted, this is r c this is un reacted. Now, what we saying is that this concentration remains I will try to here concentration and then between this point and this point. So, i will say here that is the concentration drops. This is C A g, this is distance, now how we take into account this variation of concentration drop here as the reaction proceeds. So, that the thickness of the reacted layer keeps changing to account. For that I would write the diffusion equation input minus of output plus generation equal to accumulation, this is the diffusion.

So, what is input 4 pi r square d del c del r, this is at r plus delta r 4 phi r square d del c del r at r. There is no generation and there is accumulation, which is del c del t multiplied by the 4 phi r square d r times epsilon. Showing that there is porosity that is why the fluids accumulate 4 pi r square del c del r. This is what input at r plus d r then 4 phi r square del c del r at r. So, this is what is the input, and output and generation accumulation. So, this is the material balance for and a differential element. We can simplify this you can simplify this. Take the limit as you take the limit as r tends to 0 this becomes our diffusion equation simplifies as r square del by del r square del by del r r square del by del r d del c del r equal to epsilon del c del r, where r equal to r, c equal to c naught.

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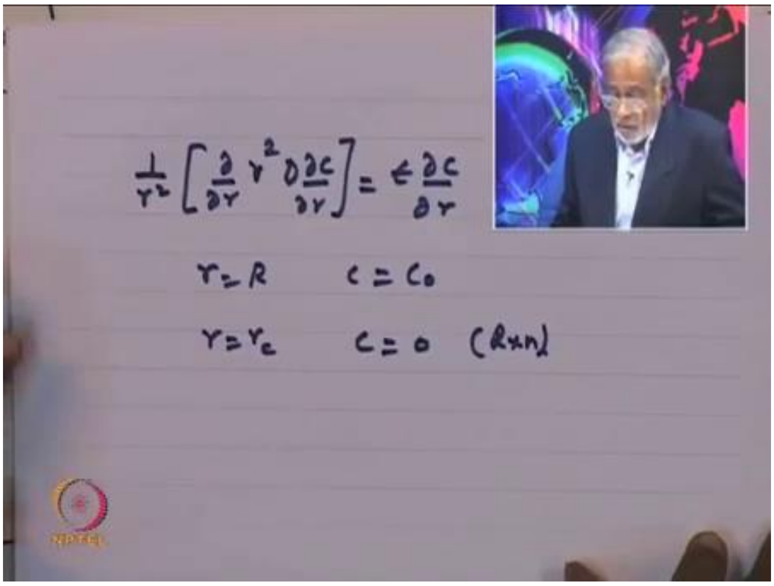

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r} r^2 D \frac{\partial c}{\partial r}\right] = \epsilon \frac{\partial c}{\partial r}$$

$r = R \qquad c = C_0$

$r = r_c \qquad c = 0 \quad (Rxn)$

We know this r equal to r c we say that c equal to 0, because of reaction. So, this is the problem that we have to solve that to find out just once again. Just recall the problem that we want to solve, we said just now we said just now, which you want to understand. What is the rate at which this particle is reacting.

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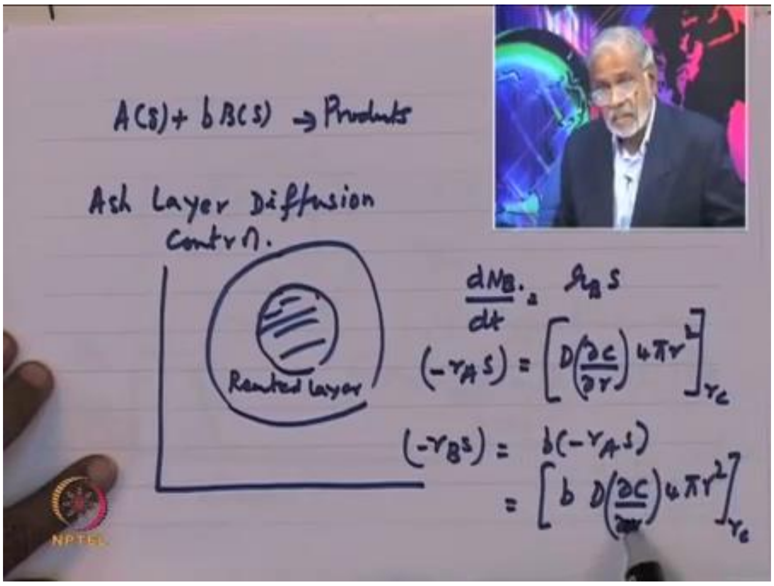

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$= \left[b\, D\left(\frac{\partial C}{\partial r}\right) 4\pi r^2\right]_{r_c}$

We must know what is the rate of chemical reaction we said that rate of chemical reaction given by v times d del c del r multiplied by 4 pi r squared at r equal to r c. On other words the rate of chemical reaction depends upon del c del r. We need del c del r through an appropriate understanding of the process. To understand, this what we have done is that, we have set up the diffusion equation an elemental volume of the cercal pallet.

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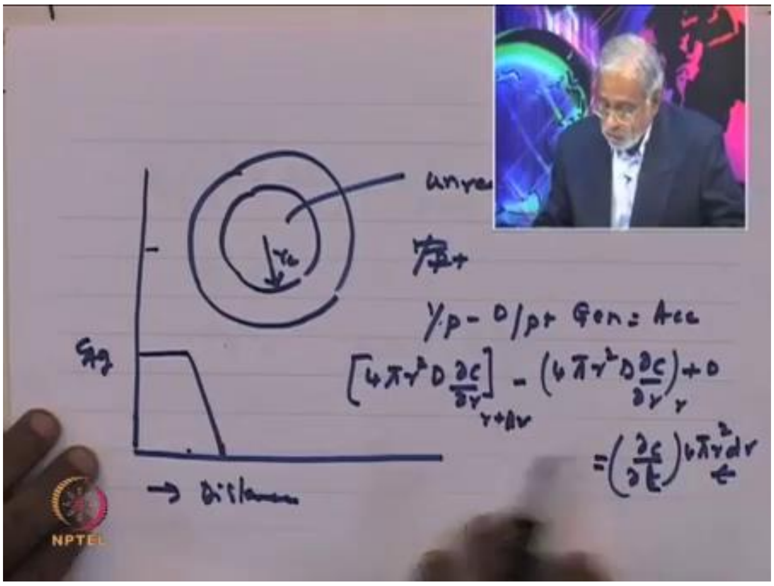

I/p - O/p + Gen = Acc

$$\left[4\pi r^2 D \frac{\partial c}{\partial r}\right]_{r+\Delta r} - \left(4\pi r^2 D \frac{\partial c}{\partial r}\right)_{r} + 0 = \left(\frac{\partial c}{\partial t}\right) 4\pi r^2 dr \, \epsilon$$

Then we have simplified this by taking the limit as delta at del is to 0.

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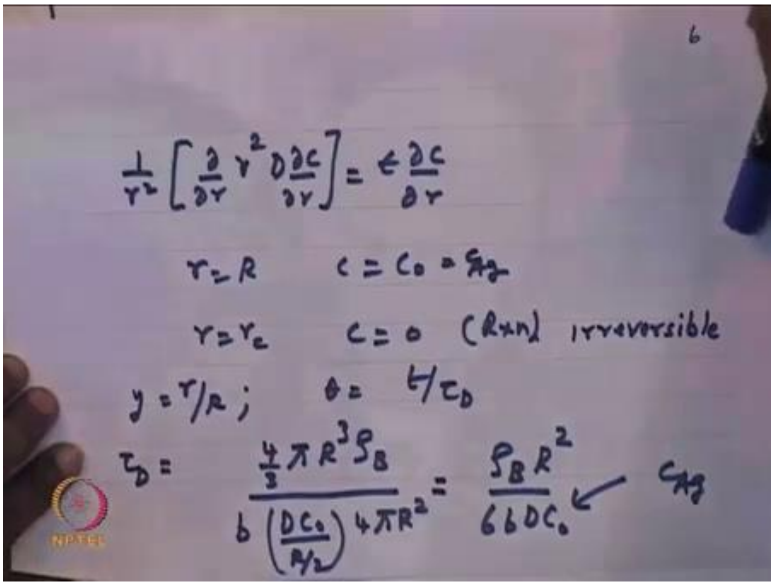

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$$\frac{1}{r^2}\left[\frac{\partial}{\partial r} r^2 D \frac{\partial c}{\partial r}\right] = \epsilon \frac{\partial c}{\partial r}$$

$r = R \quad c = C_0 = C_{Ag}$

$r = r_c \quad c = 0$ (Rxn) irreversible

$y = r/R \, ; \quad \theta = t/\tau_D$

$$\tau_D = \frac{\frac{4}{3}\pi R^3 \rho_B}{b\left(\frac{DC_0}{R/2}\right)4\pi R^2} = \frac{\rho_B R^2}{6bDC_0} \leftarrow C_{Ag}$$

Then found this is a diffusion equation that governs the process. The conditions are that this particle at the surface is particle concentration c 0 or we can call it C A g, which over is a appropriate for us. And r equal to r c, there is c 0, which means the material has been consumed, that is the point. That means all the drop in concentration occurs in the ash layers. How do you solve this? Now this c equal to c, when is c equal to c 0, which mean concentration is 0, which means there is an irreversible reaction.

Now, if it is a reversible reaction then there will be an equilibrium condition. We will look at this as you call, and we can define some non-dimensional, where if it is to make a little easier t by tau d, tau D is 3 diffusion time the time in characteristic time for diffusion we can understand this as 4 by 3 pi r cubed row b. So, this is a total amount of material that is with us and b times d c 0 by r by 2. So, this is the maximum rate of diffusion divided by the average diffusion path multiplied by 4 pi r squared. This is the maximum surface area over, which the diffusion takes place.

So, this is some way of characteristic diffusion time. So, we can simplify this title as row b times r squared divided by 6 b d times c 0. Sometimes you call this is C A g this is we mean in the same thing. Now, you can ask how is that I would made a definition of tau D in this form.

Now, actually since I know the final results I found it convenient to write it is in the form, but it does not matter even if you do not write it in this form, the answer is may will not change. So, what we are saying is that if you look at the time, time coordinate in terms of theta, which is non dimensional with respect to the diffusion time, which is defined as total amount of material divided by the maximum rate of diffusion. This is the maximum rate of diffusion, this is the diffusion path.

So, this gives you the diffusion the moles of materials that is coming in. So, this gives you total amount of material divorce moles material by unit time, gives you an idea of the diffusion characteristic time. If you non dementionalized with respect to diffusion characteristic time, we get some feel for the time constant of the process. So, with these non demensionalization our equation diffusion equation you can be simplified. So, let us do that now. So, let us simplify the diffusion equation. So, please recognize that I am going to replace this r in terms of y replace time in terms of this is time in terms of theta.

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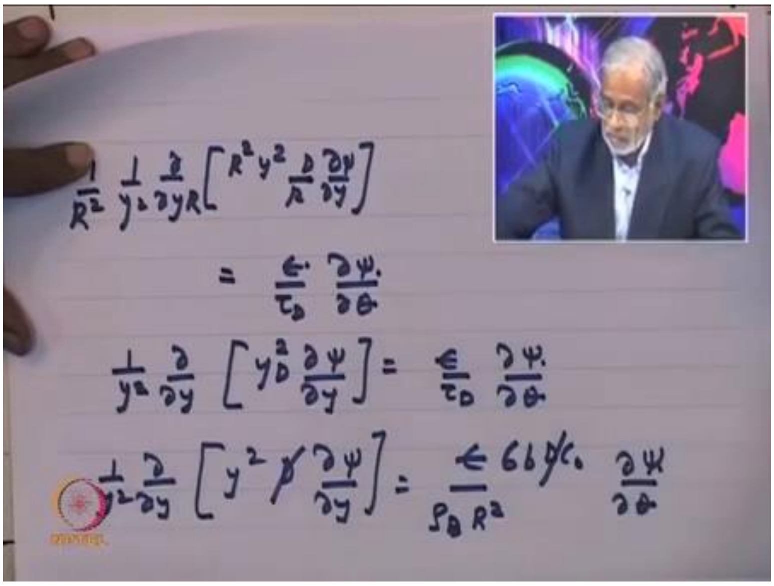

$$\frac{1}{R^2}\frac{1}{y^2}\frac{\partial}{\partial yR}\left[R^2y^2\frac{D}{R}\frac{\partial\psi}{\partial y}\right]$$

$$=\frac{\epsilon}{\tau_D}\frac{\partial\psi}{\partial\theta}$$

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left[y^2D\frac{\partial\psi}{\partial y}\right]=\frac{\epsilon}{\tau_D}\frac{\partial\psi}{\partial\theta}$$

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left[y^2\cancel{D}\frac{\partial\psi}{\partial y}\right]=\frac{\epsilon\,6b\cancel{D}C_0}{\rho_B R^2}\frac{\partial\psi}{\partial\theta}$$

So, when I do that we get 1 by r squared 1 by y squared del by del y within brackets of r squared y squared or y squared d by r del psi by del y.

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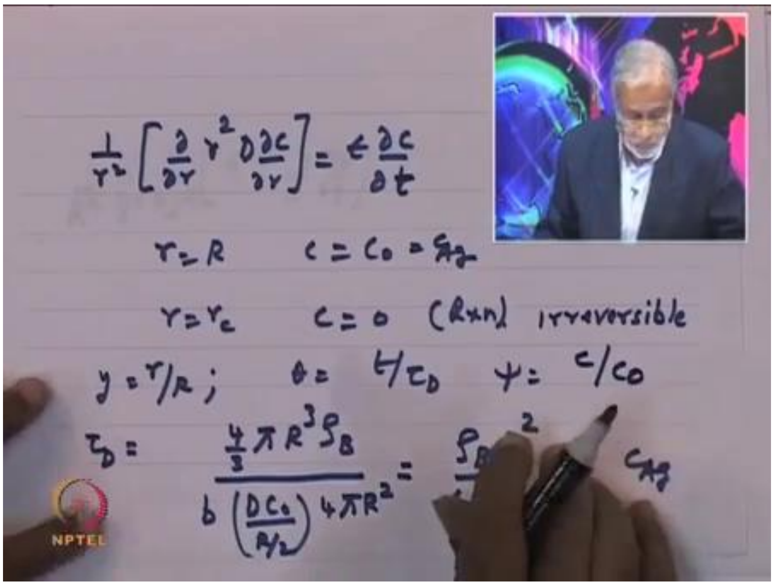

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}r^2D\frac{\partial c}{\partial r}\right]=\epsilon\frac{\partial c}{\partial t}$$

$r=R \qquad c=C_0=C_{Ag}$

$r=r_c \qquad c=0$ (Rxn) irreversible

$y=r/R; \qquad \theta=t/\tau_D \qquad \psi=c/c_0$

$$\tau_D=\frac{\frac{4}{3}\pi R^3\rho_B}{b\left(\frac{DC_0}{R/2}\right)4\pi R^2}=\rho_B \quad ^2 \quad C_{Ag}$$

Now, I also put 1 more psi equal to psi equal to c divided by c 0 equal to on the right hand side we have epsilon by tau D del psi divided by del theta. So, please make this file is straight forward there is nothing new being said here. So, this r squared I have written as r squared y squared dil this r I have written as r y r times yo K this r squared. I have written as r squared y squared, this r I have written as r y remains as such.

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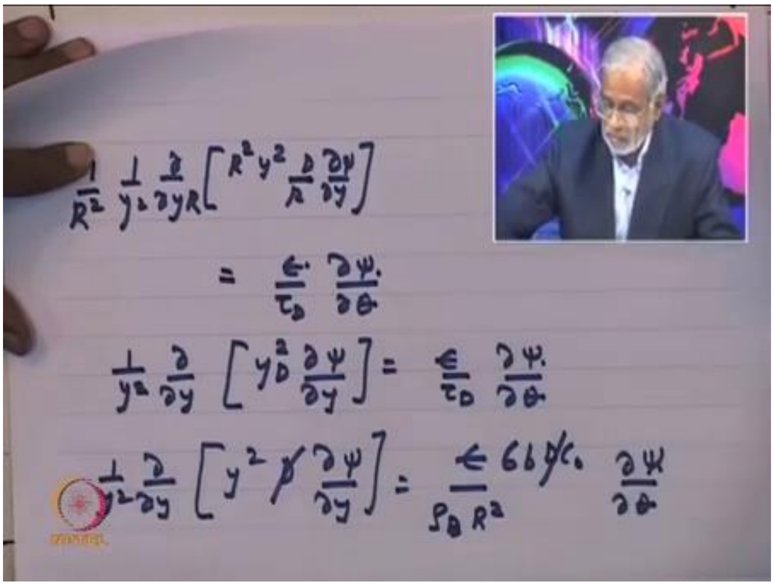

$$\frac{1}{R^2}\frac{1}{y^2}\frac{\partial}{\partial y R}\left[R^2 y^2 \frac{D}{R}\frac{\partial \psi}{\partial y}\right] = \frac{\epsilon}{\tau_D}\frac{\partial \psi}{\partial \theta}$$

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left[y^2 D \frac{\partial \psi}{\partial y}\right] = \frac{\epsilon}{\tau_D}\frac{\partial \psi}{\partial \theta}$$

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left[y^2 \cancel{D} \frac{\partial \psi}{\partial y}\right] = \frac{\epsilon\, 6 b \cancel{D} C_0}{\rho_B R^2}\frac{\partial \psi}{\partial \theta}$$

So, this c I have put as c by c 0 I am non dimentionalized on both sides. Therefore, I have put it as del psi del psi on both sides. So, nothing new has been done. So, you simplifies this if simplifies as 1 by y squared del by del y of y squared del psi by del y. I put a d here equal to epsilon by tau D del psi by del theta. Now, I will replace this tau D in terms of what we know.

So, let me just write this as del y, y squared d del psi del y equal to epsilon. I replace this tau D we already done. That tau D tau d is while this tau d, which is row b r squared by 6 b d c 0. I will just put this put this here row b r squared, then denominate numerical 6 b d c 0 del psi by del theta. I am just replacing this tau D here is in whatever definitions of tau d. Here, what we have got before now if simplifies simply some terms a i go away. So, that this diffusion equation, which describes the diffusion of the reactant gas through the ash layer. So, it simplifies like this let me just write this once again.

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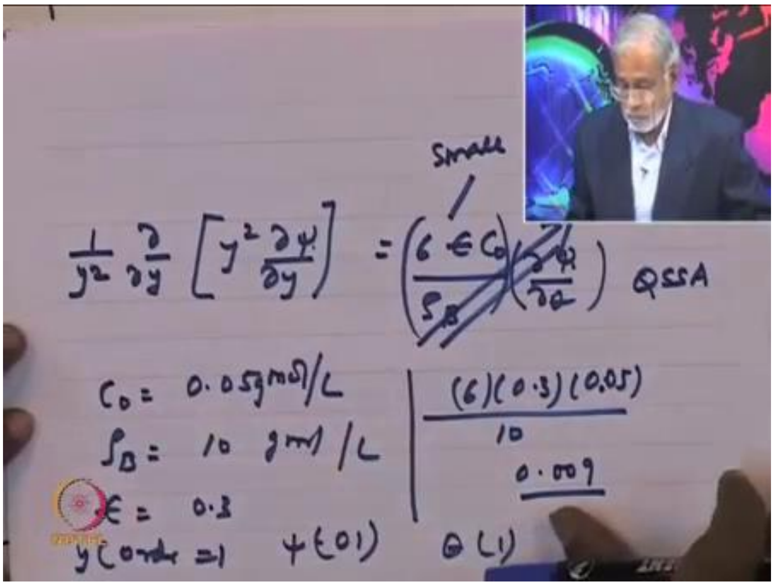

Small

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left[y^2\frac{\partial\psi}{\partial y}\right] = \cancel{\left(\frac{6\epsilon C_0}{\rho_B}\right)\left(\frac{\partial\psi}{\partial\theta}\right)}$$ QSSA

$C_0 = 0.05$ gmol/L

$\rho_B = 10$ gml/L

$\epsilon = 0.3$

y(order = 1 $\psi(01)$ $\theta(1)$

$$\frac{(6)(0.3)(0.05)}{10}$$

$\underline{0.009}$

Now, just for your sake, so equation our equation looks like this 1 by y squared del by del y of y squared d cancels off. So, del psi del y equal to 6 epsilon 6 epsilon c 0 by row b r squared. So, r squared is also got cancelled off where is this r squared, this is looks once again this r squared stays there. That is why this r squared stays there. So, this what cancels off. So, it was that, so its row b del psi by del theta. So, this is the diffusion equation that describes the diffusion of our reactants.

Now, we notice here C naught, which is typically of the order of 0.05 moles per liter. Then row b, which is the density of solids typically 10 g moles per liter, this is g mole. So, that and epsilon, let say about 0.3 typically so you put all these numbers here let we just calculate for your sake 6 epsilon is 0.3 C naught is 0.05 divided by row b 10. So, this becomes 0.05 0.15 point common. This is a 0.15 by about 0.009, so this term this whole term this is a small quantity. Notice here on this left hand side y has the order of magnitude is one order of magnitude order is one.

Then psi also the order is order is 1, then you have on the right hand side theta also order of magnitude is one. So, everywhere the order of magnitude is 1. So, this is a small quantity in view of that we can delete this term. So, this approximation is called as casuist steady straight of approximation. What it means is that the accumulation of material inside the pores of the solid is not very important from the point of view. Your understand in we can g the diffusion and how the material gets consumed. Therefore, we might assume well look at the left hand side and solve the problem. So, under the resumption that casuist steady state of approximation is satisfactory.

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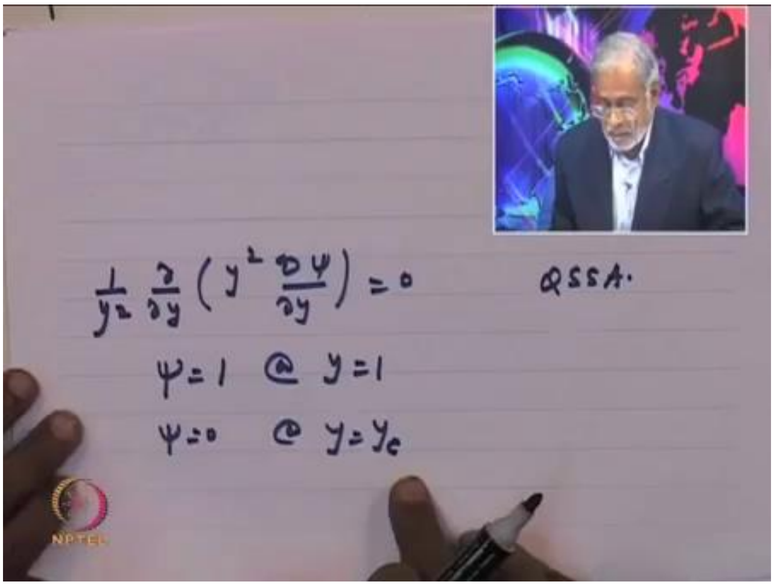

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial\psi}{\partial y}\right)=0 \qquad \text{QSSA.}$$

$\psi=1$ @ $y=1$

$\psi=0$ @ $y=y_c$

That means we can as well 1 by y squared del by del y of y squared del psi by del y equal to 0. You might as well solve this equation under the conditions that pi equal to 1 at y equal to 1 and psi equal to 0 at y equal to y c. What we are saying is that to understand the ash layer diffusion problem under the casuist steady state approximation, where in the accumulation of material inside the pores of the solids are not very important from the point of view. Our understanding the material balance therefore, the diffusion equation can be say, I mean dissolution to the diffusion equation can be approximately gotten by looking at the left hand side alone by del 1 by y squared del by del y y squared del psi by del y equal to 0. This is the casuist steady state approximation.

Now, this is a fairly simple differential equation and the conditions are also pi equal to 1 and y equal to 1 psi equal to 0 y equal to y c, what is this mean? It means that all the reaction occurs in the ash layer. There is no material left as the un reacted layer, it is reached. This is the meaning of this condition, so let us solve this. So, fairly simple problem to solve. So, we have when we integrate this well, well when we integrate this and this integrating is write now.

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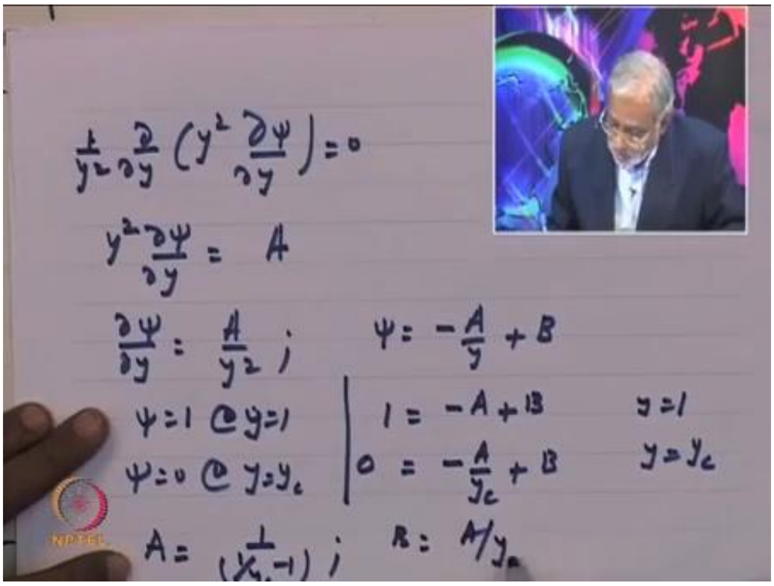

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial \psi}{\partial y}\right) = 0$$

$$y^2\frac{\partial \psi}{\partial y} = A$$

$$\frac{\partial \psi}{\partial y} = \frac{A}{y^2}; \quad \psi = -\frac{A}{y} + B$$

$\psi = 1$ @ $y = 1$ | $1 = -A + B$ $y = 1$

$\psi = 0$ @ $y = y_c$ | $0 = -\frac{A}{y_c} + B$ $y = y_c$

$$A = \frac{1}{(1/y_c - 1)}; \quad B = A/y_c$$

So, maybe I write it once again just make it easier 1 by y squared del by del y of y squared del psi by del y equal to 0. Let me integrate once when I do that integrate ones M when a do that y square del sigh by del y equal to a constant. We integrate once again del sigh by del y equal to a by y square I integrate this. We integrate this we say sigh equal to minus of a by y plus bi put this are conditions are sigh equal to 1 at y equal to 1 sigh equal to 0 at y equal to y c, if put the two condition. So, get the here sigh equal to 1 equal to minus a y plus b. Then that is the row y equal to the 1. Then we have the 0 equal to minus of a by y c plus b that is a y equal to y c. So, we can find out word a and b, which hence of the where the simply add this solution. This go through once again 1 by 1 y c minus 1 a is this and b equal to a by y c.

Therefore, we can the option substitute the a and b, this only deferential equation we can get. So, I try the final form, so what we the get let me just run through this once again without losing. So, under the Qauzi state approximation the equation that we have to solve is diffusion equation.

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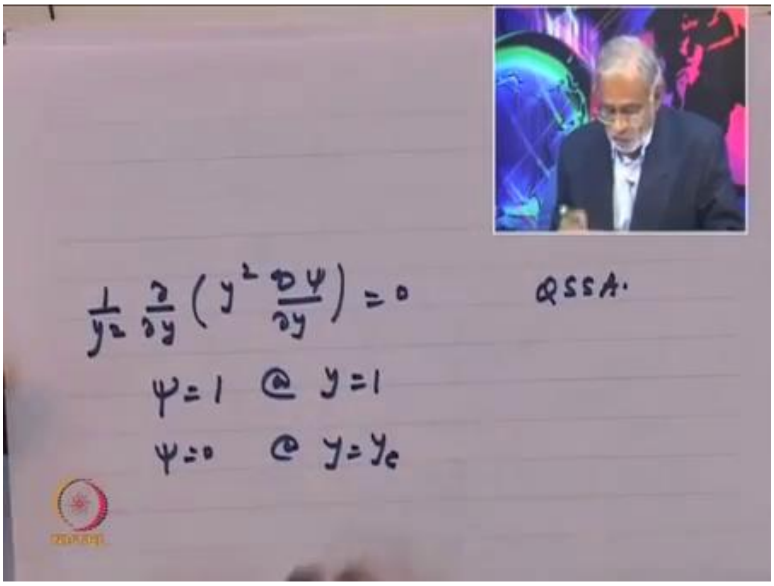

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial \psi}{\partial y}\right)=0 \qquad QSSA.$$

$\psi=1$ @ $y=1$

$\psi=0$ @ $y=y_c$

Now, this is the boundary condition sigh equal to 1 at y equal one sigh equal to 0 at y equal to y c. This condition imply all reaction take place, and as un reacted layers reach, then reactions stop. Because, there is the diffusing matter completely concerned.

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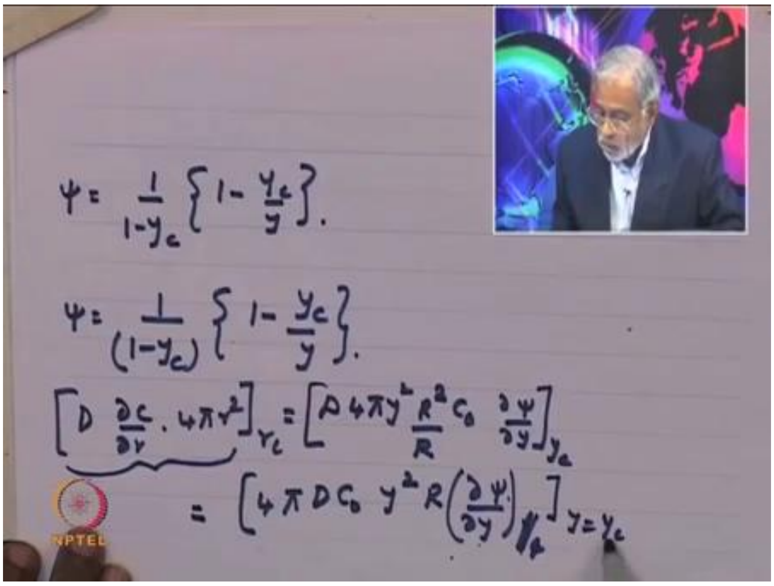

$$\psi=\frac{1}{1-y_c}\left\{1-\frac{y_c}{y}\right\}.$$

$$\psi=\frac{1}{(1-y_c)}\left\{1-\frac{y_c}{y}\right\}.$$

$$\left[\underbrace{D\frac{\partial c}{\partial r}\cdot 4\pi r^2}\right]_{r_c}=\left[D\,4\pi y^2\frac{R^2}{R}c_0\frac{\partial \psi}{\partial y}\right]_{y_c}$$

$$=\left[4\pi D c_0\, y^2 R\left(\frac{\partial \psi}{\partial y}\right)\right]_{y=y_c}$$

So, that now for solution give write this only sigh equal to 1 by 1 minus to y c 1 minus to y c by y, this is the solution. We can write it slightly more comfortable form sigh equal to 1 by 1 minus to y c with in bracket 1 minus y c by y. Now, having say this resist quickly recall what we say that earlier is that the rate at which the reaction occurs.

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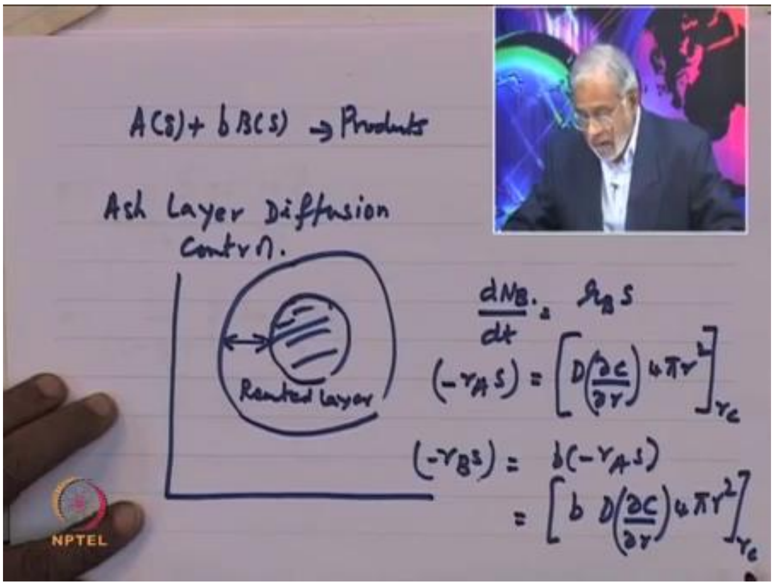

$A(g) + bB(s) \rightarrow \text{Products}$

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$$(-r_B s) = b(-r_A s)$$

$$= \left[b\, D\left(\frac{\partial c}{\partial r}\right) 4\pi r^2 \right]_{r_c}$$

We said this earlier that the rate of chemical reaction, we started I actually when we started we want to understand, what is the diffusion that this taking place in the unreacted layer. We said the rate of chemical reaction r B s is given by b times d daisy, there are 4 by R square at r equal to r c. So, what we said that once you find out what is del c del r then we know what is chemical reaction occurs at a given r into r c. What we have done is that we have got we by doing Quassi is resistance approximation. We founded what is c, what is sigh in terms of y c, we can put it terms are Qaussi it. So, let us do that right now, so we want to know find out what is the b times the del c there are 4 pi r. This is what we are interested in, because there is what resistance chemical reaction, let us try do that now.

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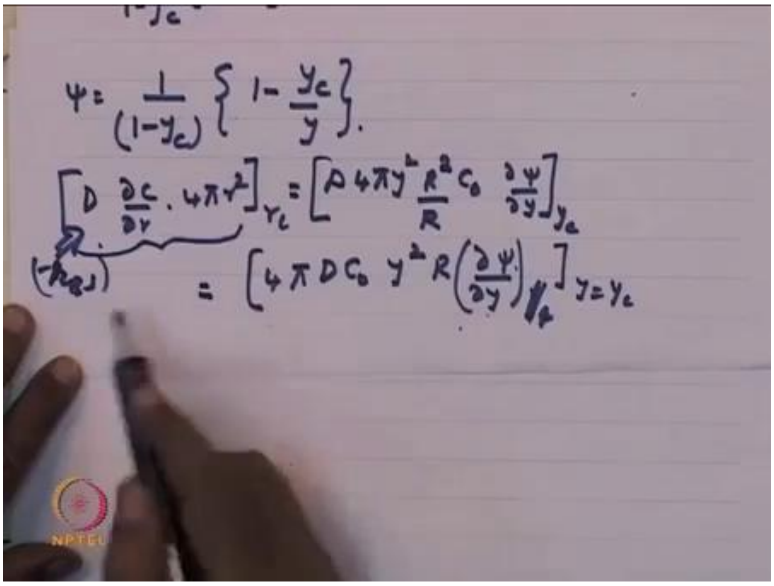

$$\psi = \frac{1}{(1-y_c)}\left\{1-\frac{y_c}{y}\right\}.$$

$$\underbrace{\left[D\frac{\partial c}{\partial r}\cdot 4\pi r^2\right]_{r_c}}_{(-r_{BS})} = \left[D\,4\pi y^2\frac{R^2}{R}c_0\,\frac{\partial\psi}{\partial y}\right]_{y_c}$$

$$= \left[4\pi D c_0\, y^2 R\left(\frac{\partial\psi}{\partial y}\right)_{y_c}\right]_{y=y_c}$$

So, what is d del c del r 4 pi r square at r equal to r c. We want this one, what is this? I want writing this in terms are the non-dimensional coordinate. We see whether I got it write 4 pi y square r square c naught del sigh del y divided by r. Let see we all the numbers of d is here now del c del r I write del c del r i put r here for the taken that 4 pi r square and 4 pi y square r square at r equal to r c see. So, this because y equal to y c for a now let us take this for 4 pi, where is d forgot on the d 4 pi d C naught y square.

This are r square so it is r del sigh del y at y c is it alright what am saying this our interest. We know that this is the term we are interested in d del c 4 pi r square comma’s calculate. We know what is size what is for I put in terms of sigh by converting r to i r than r 4 pi r square put all those terms. Now, we got everything y this whole thing we want calculated y equal to y c.

So, what is del y del sigh del y we can calculate del sigh del y here. Then evaluate this at y equal to y c, it is fairly straight forward now, we know the differentiate this and put y c. So, let me do that not doing anything re complicated. So, please let we restate what have I said already that we have got sigh in terms the y c. We already said this is the r B s we already said this is the r B s so we put r B s in terms of numbers that we know. Surely differentiate sigh with the respect to y an evaluate the whole thing advise equal to y c. Then will get chemical reacted inside that is what I am said we do now please help me now. So, I will differentiate the substitute will differentiate the substitute.

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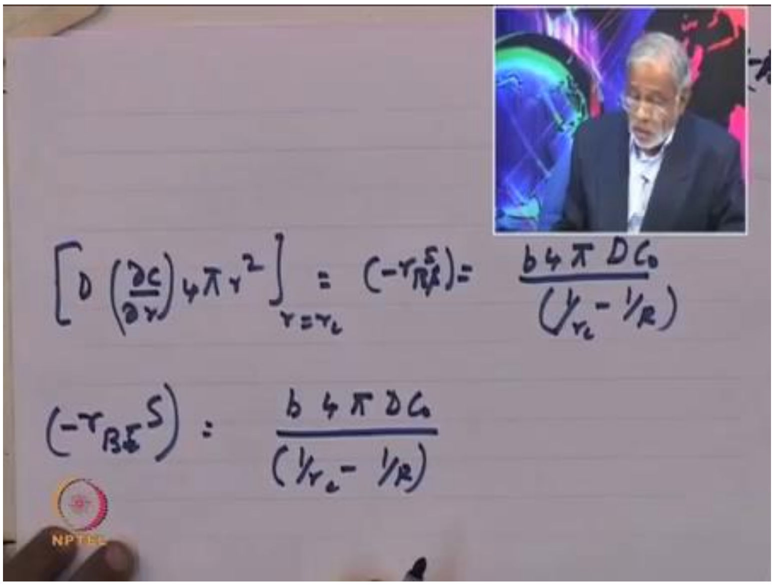

$$\left[D\left(\frac{\partial C}{\partial r}\right)4\pi r^2\right]_{r=r_c} = (-r_{Bs}^{S}) = \frac{b\,4\pi DC_0}{(1/r_c - 1/R)}$$

$$(-r_{Bs}^{S}) : \frac{b\,4\pi DC_0}{(1/r_c - 1/R)}$$

So, we get d even go through all these details straight forward 4 pi r square. So, this is the what we want at r equal to r c. Now, we are said that is also equal to minus the r B s. So, I will differentiate and then putting on the together so we get equal to minus r B s equal to b 4 pi d C naught divided by 1 by r c and 1 by r. What I have done I have differentiate sigh with respect to y put y equal to y c. Then replace to y c, we know that y c what is y c? y c equal to r c by R. So, I replace y c in these equation in terms of replace to y c by r c by y are simplified.

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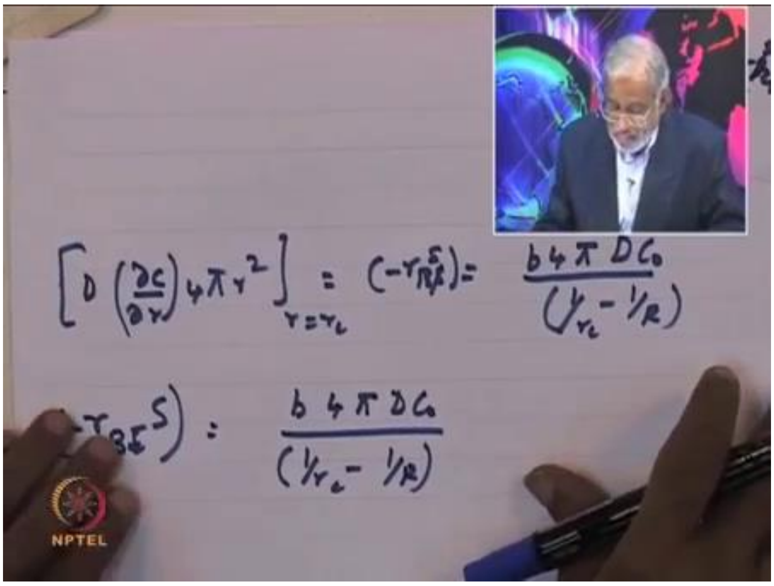

$$\left[D\left(\frac{\partial C}{\partial r}\right)4\pi r^2\right]_{r=r_c} = (-r_{Bs}^{S}) = \frac{b\,4\pi DC_0}{(1/r_c - 1/R)}$$

$$-r_{Bs}^{S}) : \frac{b\,4\pi DC_0}{(1/r_c - 1/R)}$$

And a got now the rate of chemical reaction under the case of ash diffusion control is given by. So, r b s r b s, this is equal to b. So, minus b 4 pi d c naught divided by 1 by r c minus 1 by r this is clear? Now, this is the clear the straight forward.

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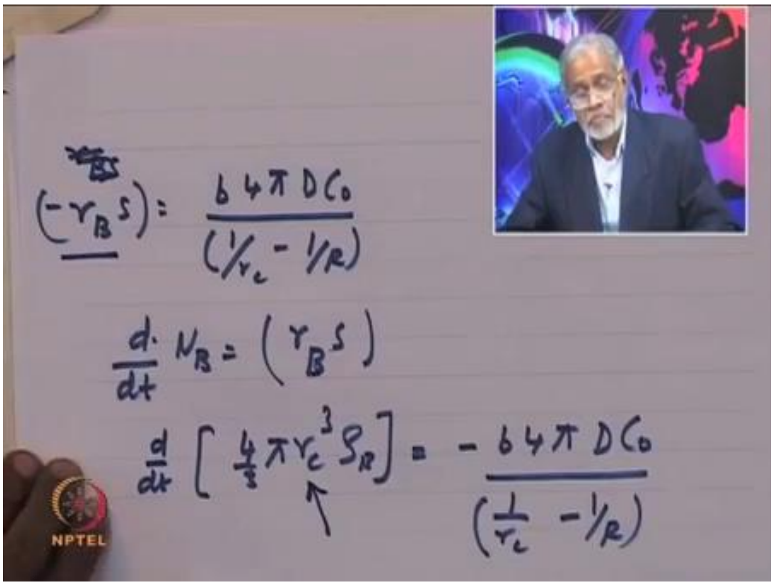

$$(-r_B s) = \frac{b\, 4\pi D C_0}{(1/r_c - 1/R)}$$

$$\frac{d}{dt} N_B = (r_B s)$$

$$\frac{d}{dt}\left[\frac{4}{3}\pi r_c^3 \rho_B\right] = -\frac{b\, 4\pi D C_0}{\left(\frac{1}{r_c} - 1/R\right)}$$

We have r B s, let me write r B s equal to b times 4 pi d C naught divided by 1 by r c minus 1 by r. We know d by d t of n b, but definition is r B s, what is the meant by d by y and d t? The rate at which the more will be the change these with time, what is the r B s? r b s refers to rate at which chemical reaction. This is a r B s rate per unit multiplied by a which is the relevant to the reaction to this kind of controlling c g. Now, left and side is was d by d t what is the n b 4 by 3 pi r c cube row b we know this, what is r b from here minus of b 4 pi d C naught divided 1 by r c minus 1 by R, is this clear?

So, what we have done we have look at the case of farcical party and going chemical reaction under reaction control. We also recognized is the thickness ash lay is keep changing with the time. Therefore, the row confusion equation for the ash layer. We recognized that as the reaction most word the un reacted surface reaction of stop because therefore, we look solution problem the understand the state problem we said and the under the state problem state the written side, which is the accumulation term become the an important gas solved reaction. Therefore, the dedicated that term said that this called Quazi steady state approximation. Therefore, the look at the equation only left and side equal 0 we solve the diffusion equations.

Then we understood that where the rate of chemical reaction occurs can now be given by the expression which is given by 4 pi d 1 r square minus 1 by r.

So this is the rate of chemical reaction occurs. Therefore, we can look at the material balance. So, the solved, which is the d b d t n v equal to r B to s. Therefore, the replace r B s in term what we derived. Then we have an equation, which tells how the un reacted core changes with time. So, this how simplify the problem. Let us go for the next problem. We have now d b d t, so the left hand side we have in the left hand side. See here left and side we get d by d t and depreciable its 4 pi r c square.

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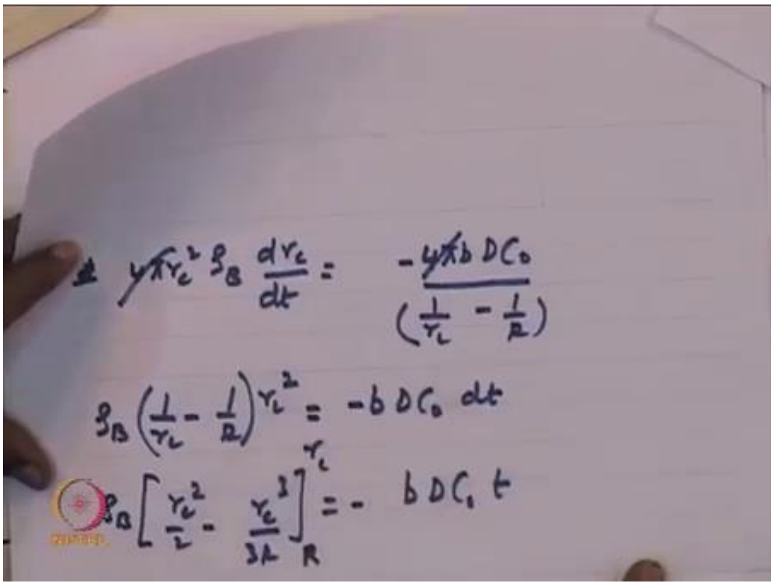

$$\cancel{4\pi} r_c^2 \rho_B \frac{dr_c}{dt} = \frac{-\cancel{4\pi} b D C_0}{\left(\frac{1}{r_c} - \frac{1}{R}\right)}$$

$$\rho_B \left(\frac{1}{r_c} - \frac{1}{R}\right) r_c^2 = -b D C_0 \, dt$$

$$\rho_B \left[\frac{r_c^2}{2} - \frac{r_c^3}{3R}\right]_R^{r_c} = - b D C_0 t$$

So, the differentiated for the row be d r c by d t there left hand side equal to minus of 4 pie b d C naught divided by 1 by r c minus 1 by r or I will put this in this form row b 1 by r c minus 1 by r, r c square 4 pi I can cancel off equal to b d C naught d t, is it okay?

Now, we have got 1 by are minus 1 by r this the minus sign d r taken on the terms integrate. Now, row b this integrated r c square by 2 this is the come r c cube by 3 r you are go for integration from r to r c equal to minus b d C naught t. It is clear all of you what I am saying, what have done what have done is nothing complicated as been done. Using the Quazi steady state approximation. We have derived the expression for the rate of chemical reaction in terms of the un reacted core radius and the initial radius so on. The ready the material balance for the solid we have found out how r c changes with time and this is how r c changes in time, now do the integrations. Let’s do that now, now we have to the take limits from r 2 r c. So, when I do that, so let me write it once again.

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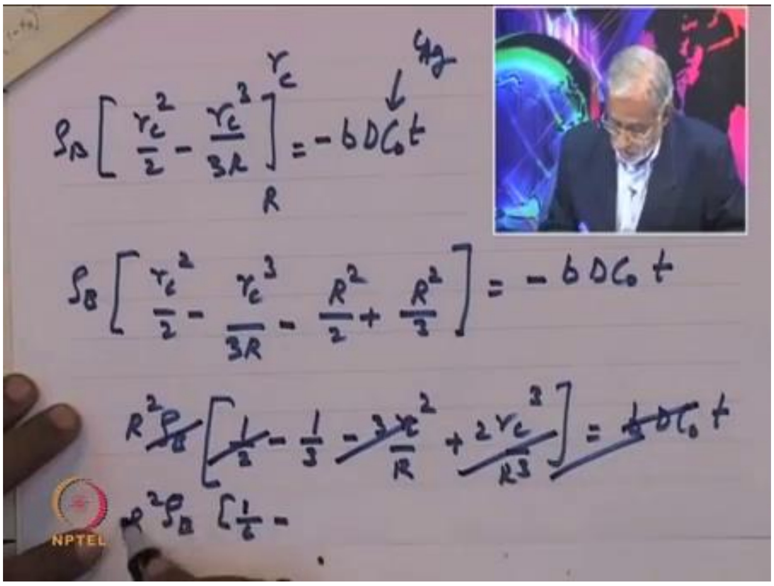

$$\rho_B\left[\frac{r_c^2}{2}-\frac{r_c^3}{3R}\right]_R^{r_c} = -bD\overset{C_{Ag}}{\overset{\downarrow}{C_0}}t$$

$$\rho_B\left[\frac{r_c^2}{2}-\frac{r_c^3}{3R}-\frac{R^2}{2}+\frac{R^2}{3}\right] = -bDC_0t$$

$$R^2\cancel{\rho_B}\left[\cancel{\frac{1}{2}}-\frac{1}{3}\cancel{-\frac{3r_c^2}{R}}+\cancel{\frac{2r_c^3}{R^3}}\right]\cancel{= bDC_0}t$$

$$R^2\rho_B\left[\frac{1}{6}-\right.$$

Row b with in brackets r c square by 2 minus r c cube by 3 r going from r to r c equal to minus b d C naught t c naught sometimes called C A g, both are the same. Now, we lets go through the limits row b within the bracket r c square by 2 minus of r c cube by 3 r minus r square by 2 plus r square by 3. So, let me take r square row b outside equal minus to b d c naught of t. So, take this out r square what happened I remove the sign also here.

Therefore, it becomes 1 by 2 minus of 1 by 3, now this become minus 3 r c square by r plus 2 r c cubed by r cubed equal b d c naught t. Let me see I got it right, so it is this is 3 minus 2, 1 by 6, so r square row b, r square row b, this is 1 by 6. So, let me do this again I am not happy with this, let me just write that step again.

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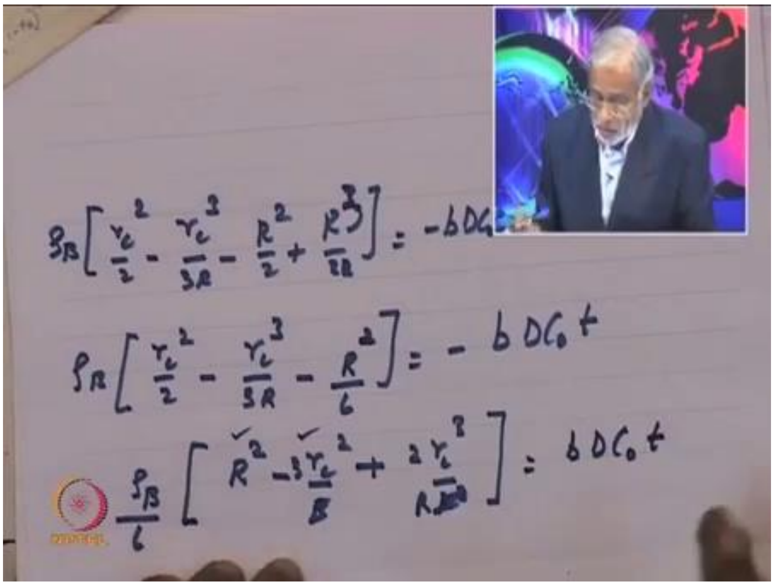

$$\rho_B\left[\frac{r_c^2}{2} - \frac{r_c^3}{3R} - \frac{R^2}{2} + \frac{R^3}{3R}\right] = -bDC$$

$$\rho_B\left[\frac{r_c^2}{2} - \frac{r_c^3}{3R} - \frac{R^2}{6}\right] = -bDC_0 t$$

$$\frac{\rho_B}{6}\left[\overset{\checkmark}{R^2} - \overset{\checkmark}{3r_c^2} + \frac{2r_c^3}{R}\right] = bDC_0 t$$

Row b terms r 6 c square by 2 minus r c cube by 3 r minus r square by 2 plus r cube by 3 equal to minus b d C naught t. So, this is what we have, now let me see simplify this let we simplify this by row b. So, this is r c square by 2 that then r c cube by 3 r I will take it like that. So, this to simplify these to r c square, so r c cube by 3 r. So, this become r square by 6 equal to minus b d C naught t. So, now the r cancel of change is no problem. Now, I will take r square row b I simplify the this by taking 6 denominator. So, and then this sign also for take it way. So, this become r squired minus of r c square by 3 r c square correct plus 2 r c cube by r cube.

So, this is by r this is alright r c cube by r all got it right and I have taken 6 outside. So, it becomes this is term is okay, is second term is minus sign. There ask this term is there become plus and therefore, becomes twice fine equal to b D C naught t. Now, let me right form, so I am doing now is there going take this r square common.

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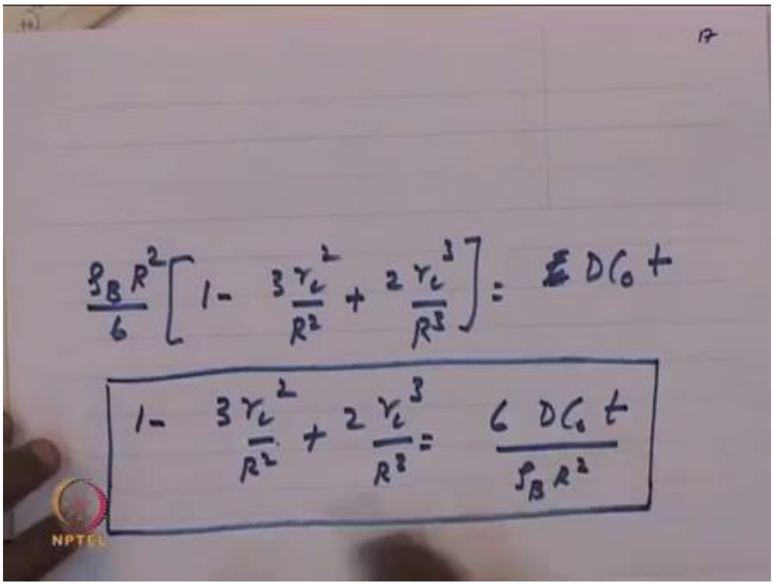

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$$\frac{\rho_B R^2}{6}\left[1 - \frac{3r_c^2}{R^2} + \frac{2r_c^3}{R^3}\right] = D C_0 t$$

$$\boxed{1 - \frac{3r_c^2}{R^2} + \frac{2r_c^3}{R^3} = \frac{6 D C_0 t}{\rho_B R^2}}$$

So, I get row b r square by 6 1 minus 3 r c square by R square plus 2 r c cube by R cube equal to 6 D C naught t or 1 minus 3 r c square by R square plus 2 r c cube by r cube equal to. So, 6 is, so it becomes 6 D C naught t divided by row b r square. So, for the case of ash layer diffusion we have that time dependence of the un reacted core radius is given by this relationship. Now, we notice here that time t equal to r c 0, what is the r c equal to 0 means that the particular becomes freely consumed, which means r c equal to 0.

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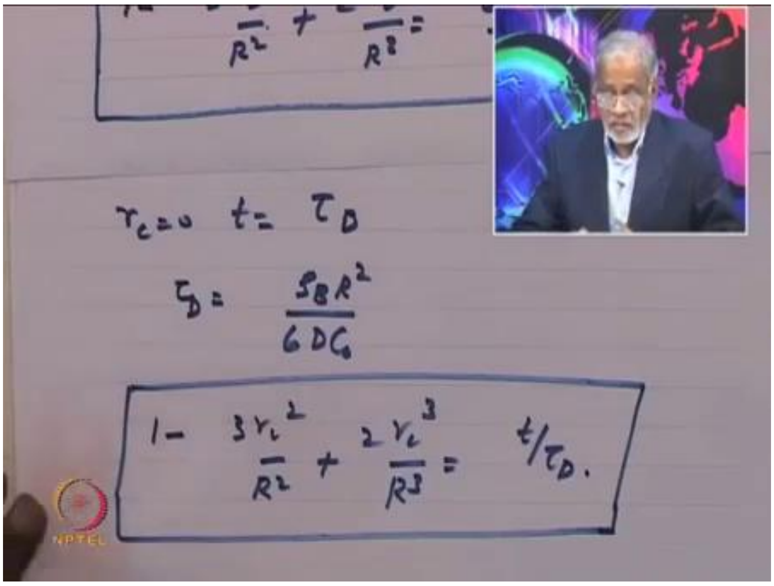

$r_c = 0 \quad t = \tau_D$

$$\tau_D = \frac{\rho_B R^2}{6 D C_0}$$

$$\boxed{1 - \frac{3r_c^2}{R^2} + \frac{2r_c^3}{R^3} = t/\tau_D.}$$

We have r c equal to 0 equal to 0 t equal to tow d are called this time complete consumption of the particle. So, let this calculate the r c equal to 0 whole thing is off. Therefore, tow d becomes row b r square 6 D C naught row B R square 6 D C naught this alright. Therefore, we can substitute, so this we get 1 minus of 3 r c square by R square plus twice r c cube by R cube equal to t divided by tow D. This is the point that we got mention. So, going through reasonable amount simplification, so on what will be the able to do they fine how the unreacted core radius changes, with time for the case of the ash layer diffusion.

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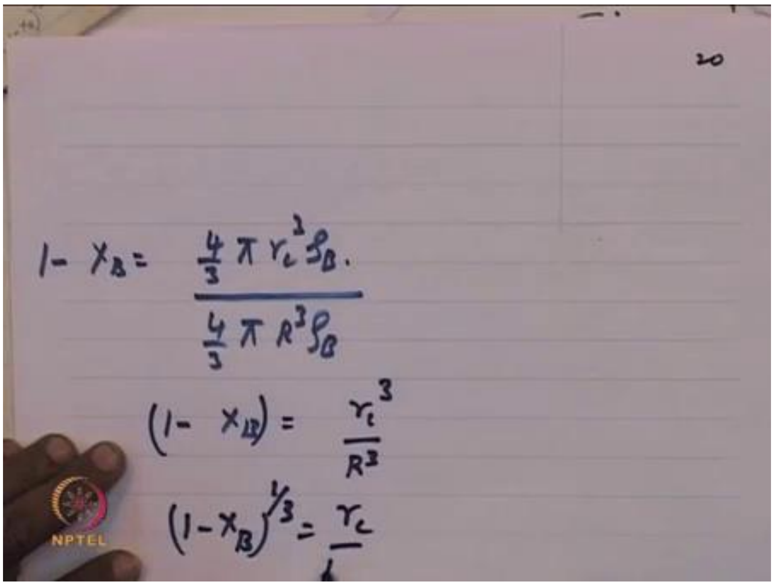

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$$1 - X_B = \frac{\frac{4}{3}\pi r_c^3 \rho_B}{\frac{4}{3}\pi R^3 \rho_B}$$

$$(1 - X_B) = \frac{r_c^3}{R^3}$$

$$(1 - X_B)^{1/3} = r_c$$

X B by definitions is 4 by 3 pi r c cube time row b divided by 4 by 3 pi r cube row b 1 minus. Therefore, r c cube essentially time says is that at r c cube by 1 minus b equal to r c cube by r cube or 1 minus of X b it to the power of 1 by 3 is r c by R.

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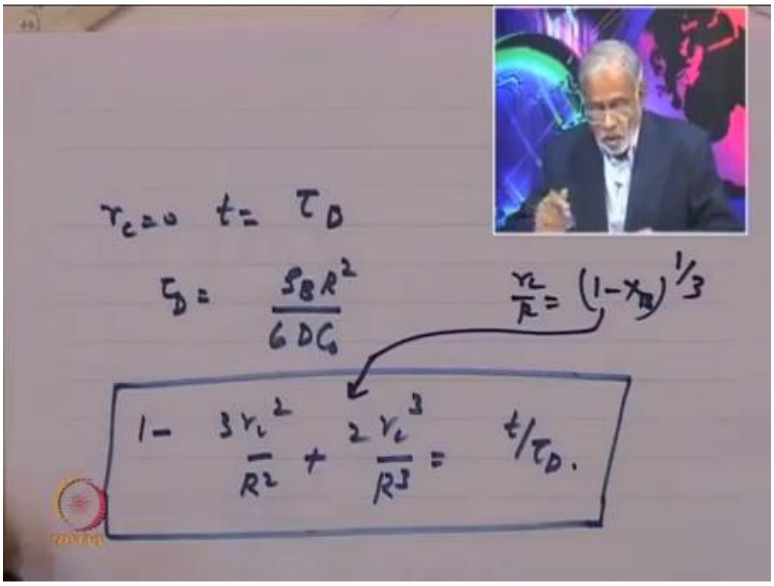

$r_c = 0 \quad t = \tau_D$

$\tau_D = \frac{\rho_B R^2}{6 D C_0}$

$\frac{r_c}{R} = (1 - X_B)^{1/3}$

$$\boxed{1 - \frac{3 r_c^2}{R^2} + \frac{2 r_c^3}{R^3} = t/\tau_D.}$$

Therefore, these terms r c by R r c by r can be put into the terms of X b, which is measurable quantity. See in a experiments it is more difficult to determine the core radius unreacted core radius, but the extent of reaction can determine by the chemical method. So, r X b is more easily measurement for quantity. Therefore, it is more convenient to express this in terms of r c by notice that r c by R is actually 1 minus x b to the power 1 by 3. Therefore, represent approximately substitute r c b r in terms there all expression in put into terms X b they left and side can be determine experimentally, whole the right and side t tow d K determine. And that is how you can actually quantify in the chemical and said this the question that arises is that if we have in general case we may had there is a general problem and this the whole thing once again.

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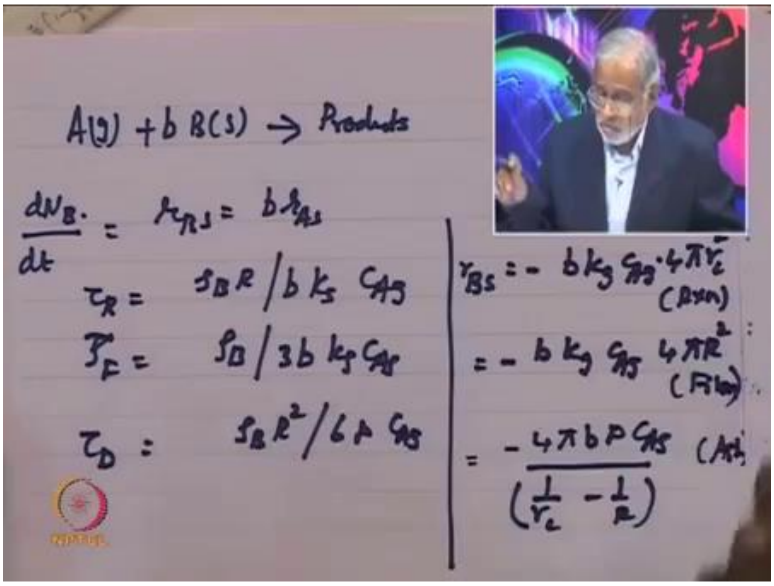

$A(g) + b B(s) \rightarrow$ Products

$\frac{dN_B}{dt} = r_{BS} = b r_{AS}$

$\tau_R = \rho_B R / b k_s C_{AS}$

$\tau_F = \rho_B / 3 b k_g C_{AS}$

$\tau_D = \rho_B R^2 / 6 D C_{AS}$

$r_{BS} = - b k_s C_{AS} \cdot 4 \pi r_c^2$ (Rxn)

$= - b k_g C_{AS} 4 \pi R^2$ (Film)

$= \frac{- 4 \pi b D C_{AS}}{\left(\frac{1}{r_c} - \frac{1}{R}\right)}$ (Ash

We have d N v b by d t equal to r B s equal to b times r s. This is the reaction A gas plus B solve going to products. Now, we know what we done reaction row b r divided by b terms K s C A g we know that tow f equal to b are divided by 3 times b K g C A g and tow d you just the derived j c row b r squared divided by 6 d cg c a 0, this is what we so for seen. Now, what we want to recognized right here just lose the thread r B s equal to minus b K C a g into 4 by r c square.

This is for reaction equal to minus b K g C A g 4 pi r square for film diffusion control equal to minus of 4 pi b d C A g divided by 1 by r c minus 1 by R. This is for ash control this is ash, this is film, this is reaction. So, what are saying is that we have considered, so for cases in which only reaction in controlling only film diffusion is controlling, only ash decision control. Now, in the given situation you mind have more than one situation under control. So, handle the general case is of all the reaction mechanism being operated how we handle that by recognizing that resistance equal to we call this omega potential divided by rate.

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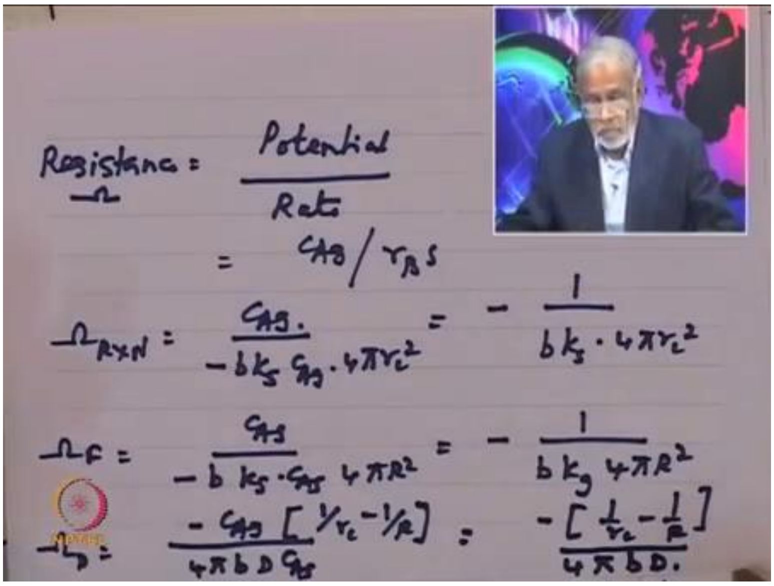

Resistance Ω = $\frac{\text{Potential}}{\text{Rate}}$

$= C_{Ag}/r_{Bs}$

$\Omega_{Rxn} = \frac{C_{Ag}.}{-bk_s C_{Ag}\cdot 4\pi r_c^2} = -\frac{1}{bk_s\cdot 4\pi r_c^2}$

$\Omega_F = \frac{C_{Ag}}{-b\, k_s\cdot C_{Ag}\, 4\pi R^2} = -\frac{1}{b k_g\, 4\pi R^2}$

$\Omega_D = \frac{-C_{Ag}[1/r_c - 1/R]}{4\pi b D C_{Ag}} = \frac{-[\frac{1}{r_c}-\frac{1}{R}]}{4\pi b D.}$

So, in this case potential c A g and rate is r B s and we have potential c A g are other words c A g divided by r B s. So, let us right, so resistance reaction equal to c A g divided by minus of b K s c A g into 4 pi r c square, which is equal to minus of 1 by divided by b K s times 4 pi r c square. So, this is clear. Therefore, the resistance can be c A g divided the r B s there will be minus of 1 divided by b tense K s times 4 by r c square. This is for reaction, omega film is c A g divided the minus of b K g c A g 4 pi r square. That is equal to minus 1 divided by the b K g 4 pi r square.

Similarly, diffusion c A g divided by you have I write like this 4 pi b d and this term concern numeric that is 1 by r minus 1 by r. This how the term comes have c A g b become here for the comes as that the minus sign minus half 1 by r c minus 1 by r divided by 4 pi b and d. So, what we said here what we done is that based on whatever we have done so far we are able to determine to what is rate of chemical reaction occurs.

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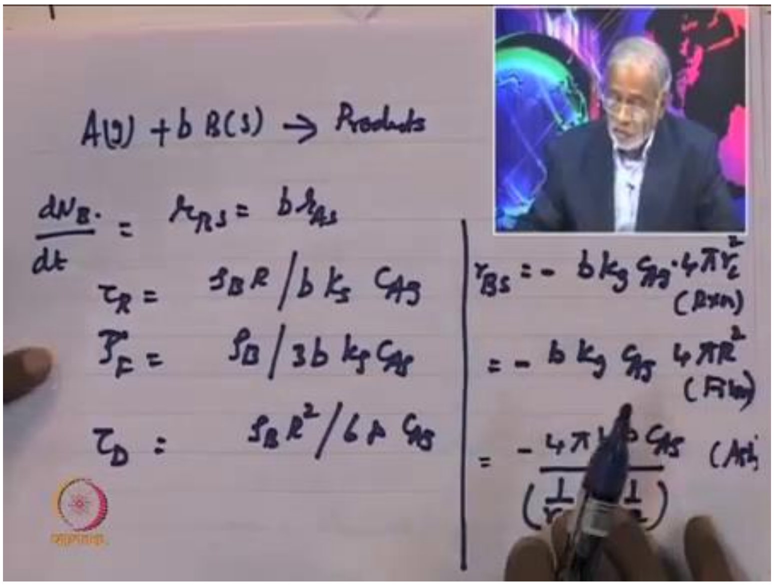

$A(g) + b\,B(s) \rightarrow$ Products

$\frac{dN_B}{dt} = r_{Bs} = b\,r_{As}$

$\tau_R = \rho_B R / b\,k_s C_{Ag}$

$\tau_F = \rho_B / 3b\,k_g C_{Ag}$

$\tau_D = \rho_B R^2 / 6 D\, C_{Ag}$

$r_{Bs} = -\,b\,k_g C_{Ag} \cdot 4\pi r_c^2$ (Rxn)

$= -\,b\,k_g C_{Ag}\, 4\pi R^2$ (Film)

$= \frac{-\,4\pi \ldots C_{As}}{\left(\frac{1}{r_c} - \frac{1}{R}\right)}$ (Ash

We done that already r B s expression only therefore, we define what is called this resistance they, which is the diffusion c A g divided by that at be r B s that be potential divided by rate. Therefore, we get next expression for resistance part the 3 cases. Now, if than situation we find that the more one the resistance the important, we simply have to and the resistances.

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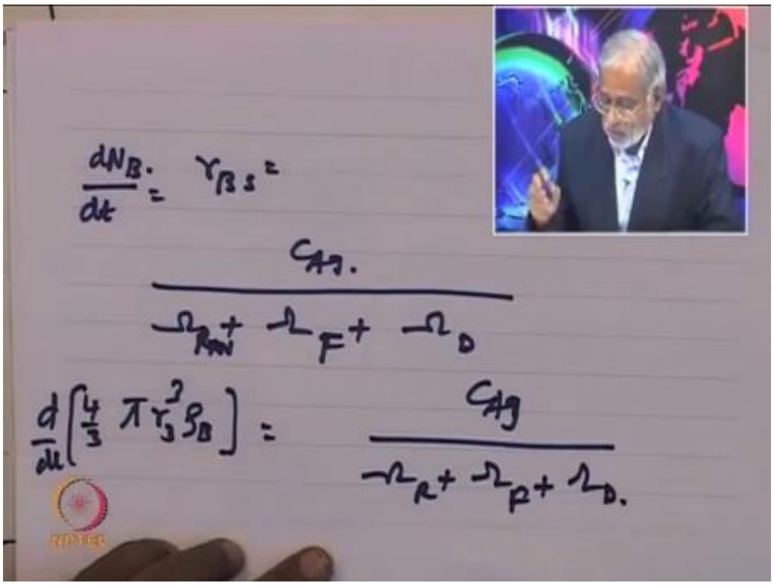

$\frac{dN_B}{dt} = r_{Bs} = \frac{C_{Ag}}{\Omega_{Rxn} + \Omega_F + \Omega_D}$

$\frac{d}{dt}\left[\frac{4}{3}\pi r_c^3 \rho_B\right] = \frac{C_{Ag}}{\Omega_R + \Omega_F + \Omega_D}$

So, in the given situation let us say all the where all the control operating which means d N B by d t which is the r B s in the presence of all the reaction, all the controlling mechanism operating, which is omega 1 plus omega 2 plus omega 3 or omega this is film and this is reaction. So, in a general situation where all the where all the reaction film as well diffusion is important you simply have to add this resistance, which means what the rate at which chemical reaction occurs. Now, we let me replace that slide to lessee algebra, but let us do this properly. So, d N B by d t we know this d n b by d t what left and side what 4 by 3 pi r c cube and row b of the differentiate this. That is the left and side the right and side C A g divided by reaction film and diffusion. So, we can do that now.

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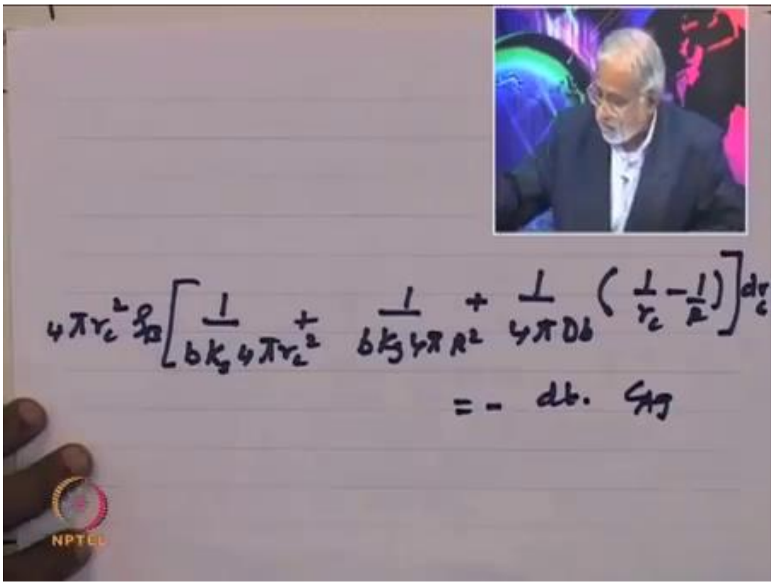

$$4\pi r_c^2 \rho_B \left[\frac{1}{b k_s 4\pi r_c^2} + \frac{1}{b k_g 4\pi R^2} + \frac{1}{4\pi D_b}\left(\frac{1}{r_c} - \frac{1}{R}\right) \right] dr_c$$

$$= - dt \cdot C_{Ag}$$

4 by 4 pi r c square row b. So, I just r c square row b this differentiate, this is time 3 cancel of the 4 pi r c square row B row B d r c. So, write this multiplied this by we should the multiplied that is resistance term by term. So, that resistance 4 pi r c square row b multiplied 1 by b K s 4 pi r c square plus 1 by b K g 4 pi r square plus 1 by 4 by pi D b with in brackets 1 by r c minus 1 by r d r c equal to minus to d t times A g.

I hope we have not part see this is this go through once again because, term nothing new is has been said. Please recognized that d n b d t are r B s. These are the terms omega reaction and omega film, omega diffusion all that we have already derived here. Simply this cross multiplying cross multiplied. So, that is what be done 4 pi r c square row B all this terms one have taken a problem minus sign also be there.

Now, we can integrate and integration goes from are integration goes from what capital R to r c it is clear. On other words it notice here each of these terms is exactly term that will be when through when the did there whole analysis independently now they all occur together, now we can integrate then get the answer. Now, we can integrate find let me integrate this. So, I am just going to integrate, which each of the terms going from r r to r c r to r c are r to r c. Now, see here done of all these before. So, I will go through the whole thing once again instead simply do the integration quickly.

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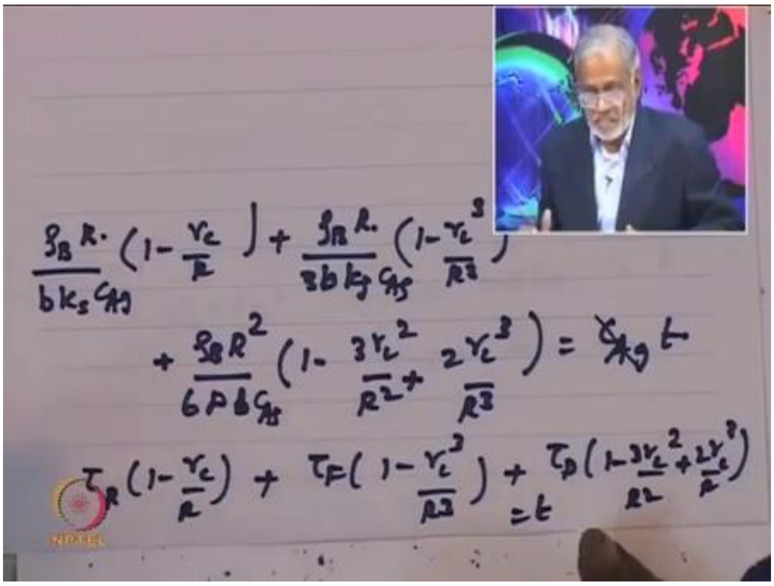

$$\frac{\rho_B R}{b k_s C_{Ag}}\left(1-\frac{r_c}{R}\right)+\frac{\rho_B R}{3 b k_g C_{Ag}}\left(1-\frac{r_c^3}{R^3}\right)+\frac{\rho_B R^2}{6 D b C_{Ag}}\left(1-\frac{3r_c^2}{R^2}+\frac{2r_c^3}{R^3}\right)=\cancel{C_{Ag}}\,t$$

$$\tau_R\left(1-\frac{r_c}{R}\right)+\tau_F\left(1-\frac{r_c^3}{R^3}\right)+\tau_D\left(1-\frac{3r_c^2}{R^2}+\frac{2r_c^3}{R^3}\right)=t$$

What I get here on the left hand side it is row b r divided by b K s times 1 minus r c by r, this is one term, row b r divided by 3 b kg into 1 minus r c cube by r cube. This is the film diffusion. The next term is row b r square divided 6 B d 6 B d into 1 minus 1 minus 3 r c square by R square plus twice r c cube by cube equal to C A g. Now, we put the C A g here and recognized is that if puts C A g here. Then it simply this that the term multiplied the do that I put that C A g and put here C A g here. I will put C A g here notice here that this whole thing here, written as tow reaction 1 minus r c by r plus tow film 1 minus r c cube by r cube plus tow diffusion 1 minus trice r c square r square plus twice r c cube by r cubed equal to t. What we saying now?

What we said this way if you have a particle in which all the resistance here important. Then the time is the required for give the extent reaction is simply the additive of the effect each of the three reaction film and diffusion. That means whatever previously when we only one now the resistance the operating.

So, we had only this term, because could t when it reaction control are this term t, when a film diffusion control or third term equal to d. When it was diffusion control, I am ash diffusion control now all these term simply addition of the three resistance. What we say that when the problem is linear we can have this additive defects. So, just to cut this long story short what we have said is that if you all three of them controlling. Then the results you will get is the results you gotten for the individual cases, but they are all added together.

Thank you.