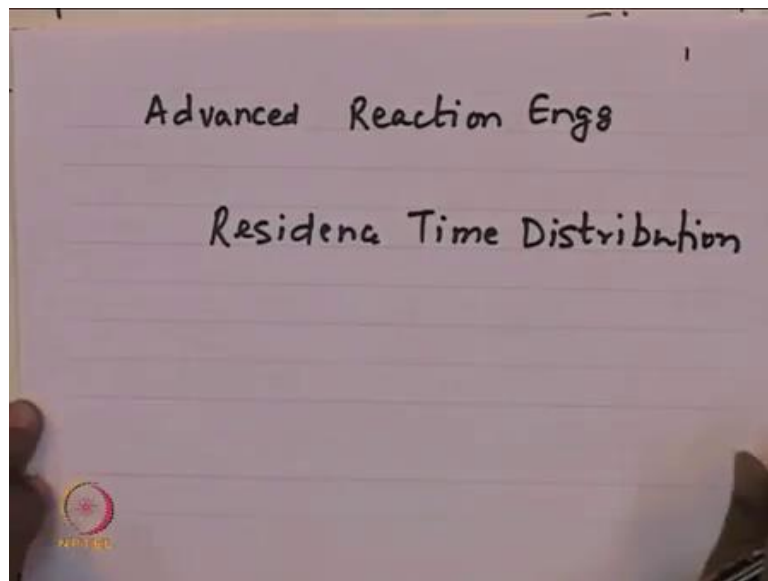


Advanced Chemical Reaction Engineering
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Lecture - 26
Residence Time Distribution Method

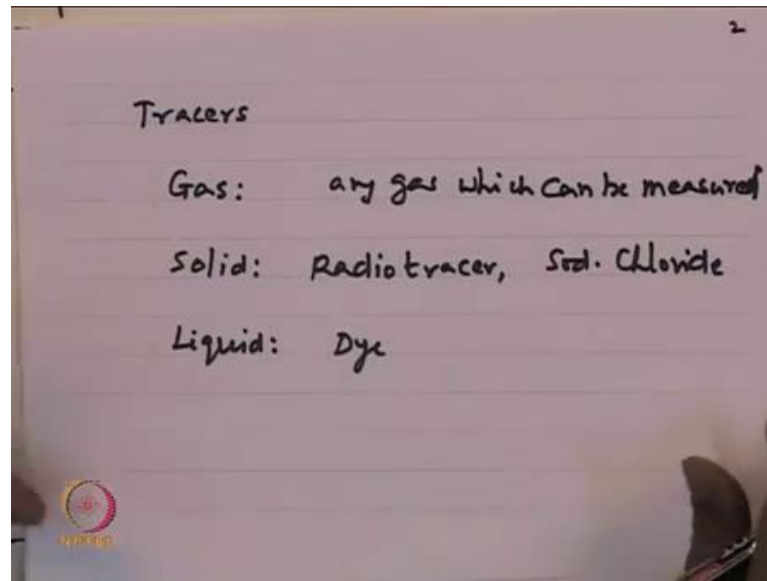
Today will be looking at residence time distribution.

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Residence time distribution. Now, why have we looking at residence time distribution? We know from our understanding of real life that, the longer system, I mean a fluid elements pens in the equipment and undergoes a greater amount of reaction. And therefore, if you know the residence time we will be able to understand how the reaction system is performing. So, what we want to do here is to setup methods by which we can understand how long a fluid element pens in the equipment.

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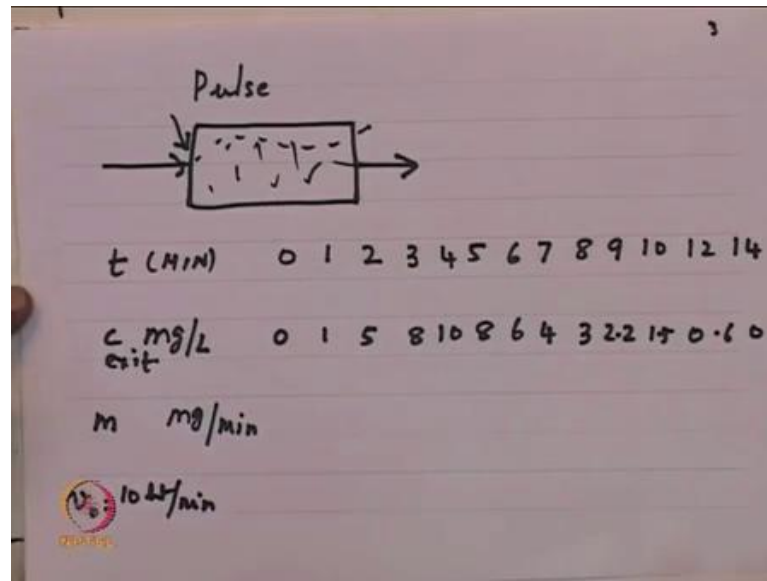


Now for this we use tracer, tracers, what are the tracers that are used? Tracers used in residence time distribution technique are those that, enter the equipment and behave exactly like the fluid elements that would be entering, whether it is solid liquid a gas, but does not undergo any chemical reaction. Therefore, it gives you the time, the fluid elements pass in the equipment. So, examples are let us see if we have a gas, if we have a gas we can have tracers which you can measure. Any gas tracer that you can I mean, put into the equipment which can measure is a useful gas tracer.

Solid, so various types of suppose you have equip like a fluid bed for example, if you won't understand solid residence time distribution. We can put tracer, it can be a radio tracer, it can be a simple things like sodium chloride or potassium carbonate. Anything that has similar density as that as the particles of the fluid bed and does not undergo the chemical reaction is what is appropriate. See, liquid it can be a colored dye which behaves just like the other fluid, similar densities and so on. Therefore, this is able to tell you the distribution of residence times for the liquid.

So, it can be a dye here, it can be a radio tracer or it can be let us say, sodium chloride. Important thing is that, the density must be, so it can be any gas which can be measured accurately. So, this is so, a tracer is appropriately chosen so that, we are able to put the tracer into the system and measure the concentration of the tracer, whether it can if it say tracer gas or is a tracer solid or tracer dye, it should be able to measure appropriately.

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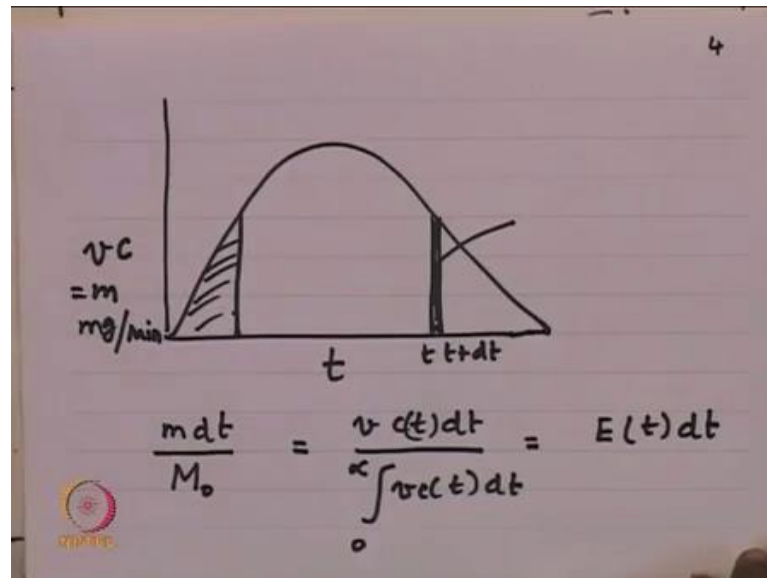


Let us say you have an equipment, fluids are coming in fluids are going out and our interest is to find out how this fluid moves. Whether it moves like this or whether it moves like this, like this, like this and goes out we don't know. So, what we do is that we put a pulse of tracer, what is meant by a pulse of tracer? A pulse of tracer is a tracer of small quantity which is very quickly or suddenly introduced into the fluid stream. And this fluid stream it is able to mix quickly and therefore, follow the path of various fluids that is going through the system. Let us, let us say we have some data I am just putting down some data, just to illustrate what we want to say.

So, this is some experimental result which I am just putting down, some measurement we have made some writing down all the numbers. So, suppose we have an experiment in which for various times this minutes, these are the measured concentration of tracer that comes out. These all done at the exit, these are all exit concentrations. So, as well you put this tracer you start measuring at the exit, this is what you have observed. Suppose, if I say that the flow is something like 10 liters per minute, if I give you the flow you can calculate how much material, milligrams per minute that is come out by simply multiplying the concentration by v .

So, you get on all the flows here so, if you want to find out the grams of tracer that is exiting the system, is simpler to multiply the exit concentration by the flow that you have also measured.

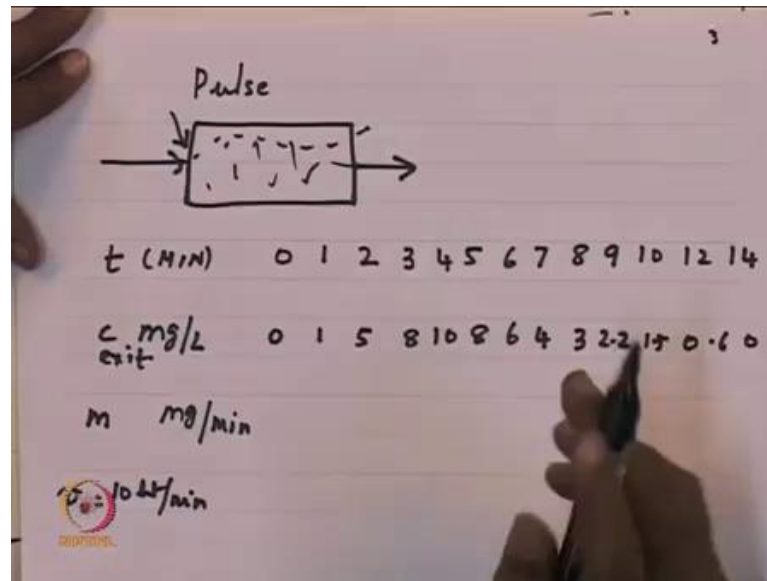
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Now let this plot, this data in this form let us say we have time versus v times c , what is v times c ? v times c by is m which is a mass. So, this will look something like this, showing that the amount of material this, what are the units of this? It is milligrams per minute. So, if I ask you at any time t , suppose this is a anytime t , what is this material? So, this refers to material that is exited the vessel, so many these area refers to the grams of material that is left the vessel. Now, suppose I ask you what is this area? What will we tell us, this area what is the meaning of this area? This is the grams of material that is left the equipment between t and t plus $d t$, if this is a time, this so many grams as left the vessel.

So, if I now ask you what is this term $m dt$ divided by m_0 , what is $m dt$? $m dt$ is v times $c dt$ divided by, what is m_0 ? m_0 is simply integral 0 to infinity of v times $c dt$. So, the denominator is the total amount of material that we have injected while the numerator is the amount of material that has left the vessel between time t and t plus $d t$ or this is the material that has spent between time t and t plus $d t$ in the vessel. This term is often termed as the $e t$, the residence time distribution function. So, let me go through this once again. What have we done, let us this done through the experiment once again. We have an arbitrary vessel to for which we want to determine the residence time distribution function.

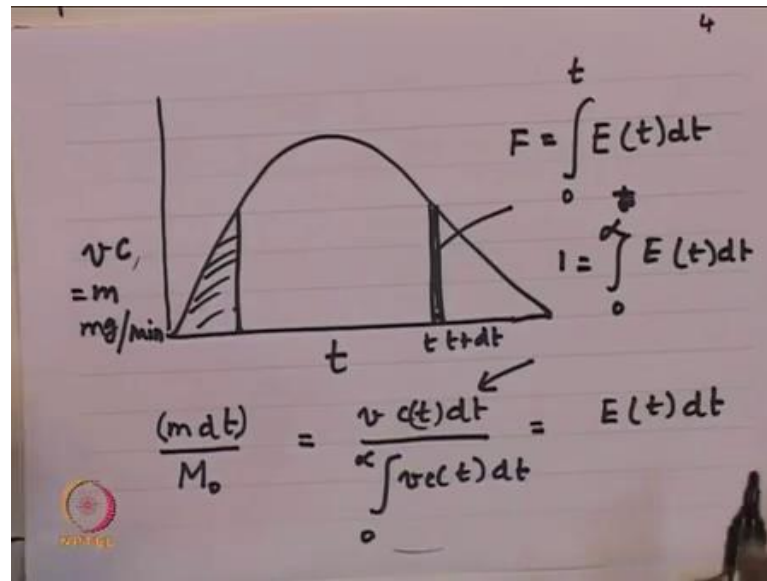
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Now what we do is that since there is a continuous flow input and continues flow output, we put a tracer appropriately chosen that it behaves exactly like the fluid, but we are able to measure it at the exit that means, at this exit point can be measured. So, we have measured the concentration of the tracer at the exit and this is the concentration that we have obtained. What we have done, we have plotted this data in this form while we say v times c is a mole of the flow rate, which is known in this particular cases, given as 10 liters per minute. In any experiment we know the flow rate any way.

So, you can plot $v c$ which is the grams of tracer per minute verses time. So, when you plot this so, from there we can get what is the $m d t$, what is $m d t$? $m d t$ is the fraction material that lives between time t and t plus $d t$, what is m_0 ? By definition is a total amount of material is injected which is 0 to infinity time $v c d t$. So, both numerator and denominator are known quantities because, it comes from an experiment.

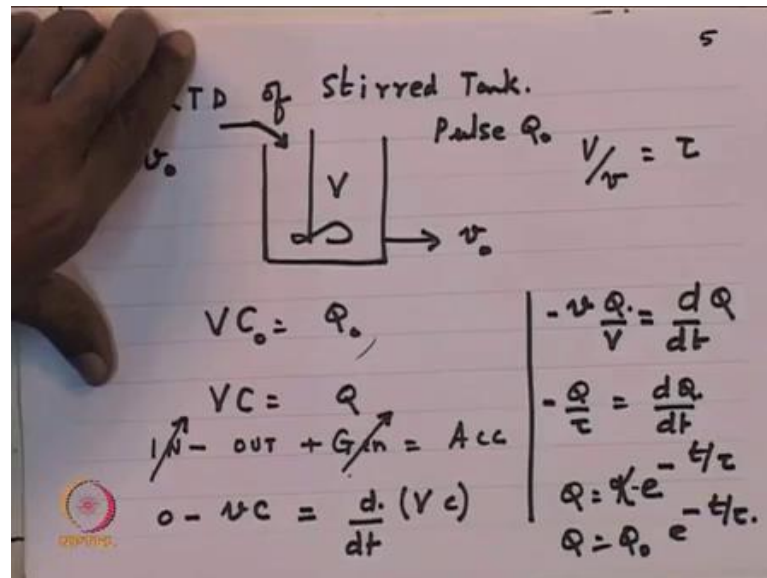
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Therefore, this ratio $v c dt$ by definition this is it the gram size material that is left the vessel between t and t plus dt . And denominator is the material that is, total amount of material is injected. Therefore, this ratio represents the fraction of material that leaves the vessel between t and t plus dt or the fractional material that spends time between t and t plus dt which means, it spends time t in the equipment. Therefore, this is the fraction which has the residence time, which is t and t plus dt and that fraction is given has $e dt$. The important message from this experiment is that, residence time distribution function that is e function is an experimentally measurable quantity.

Now let us ask one more question here, what is, what is this integral $e dt$? What is this integral between 0 to t , by definition what is the meaning? This refers to all material that is entered between time $t=0$, dt and is left. So, this is often called as F function. So, this the integral between 0 to t of e function represents total amount of material that has left the vessel between time, in this time interval t . So, another property that is obvious from the definition of e function is at $e dt$ 0 to infinity is 1 , it is very obvious from this definition that when we integrate $e dt$ 0 to infinity it is 1 , which because it refers to the total material that we have injected.

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Now, we have understood how we can obtain the residence time distribution function. So, let us look at some ideal situations this r t d of stirred tank, what is that we want to know now is, we want to determine what is the r t d of stirred tank. So, we have a stirred tank to which we have material coming in and material going out. Let us say it is coming in at a volume flow rate v_0 and let us say it is also going out at v_0 , it is a well stirred vessel. We are putting a pulse of tracer, a pulse of tracer let us say Q_0 milligrams of tracer we have put into the equipment.

So, we should expect that the instant let us say volume of the equipment is V , at the instant that we have put into the equipment. So, must by definition we should have the total amount of tracer that we have injected, must be equal to the volume of the fluid in the equipment multiplied by concentration, this by definition. Therefore, by definition at any other instant of time you should have that any of, the quantity of tracer in the equipment is where C is the concentration of tracer V is the volume. Therefore, any other instant of time we should have the amount of tracer in the equipment is V times C . Where, C is a concentration of tracer and there was total amount of tracer is V times C equal to Q .

So, what is Q_0 ? Q_0 is a total amount of tracer that we have injected at 0 time, because of the flow you will find that the Q_0 keeps decreasing as time proceeds. Therefore, at any instant of time the amount of tracer in the equipment is V times C which is Q and Q will be less than Q_0 by common sense. Now let us write a material balance to find out what is

the residence time distribution. So, let the material balance this input minus of output plus generation equal to accumulation. So, input of amount of tracer that is input, amount of tracer is going out, amount of tracer that is generated equal to the amount of tracer that is accumulating.

There is no generation of tracer because, there is no chemical reaction. Therefore, input of tracer minus output of tracer plus generation of tracer equal to 0. Now are we putting continuously any tracer into the system? Our tracer is coming in as a pulse therefore, there is no continuous input of tracer, the, what we have is 0 and output. So, what is 0 minus what is the amount of tracer going out? It is v time c equal to d by $d t$ of v times c . So, this is a statement of material balance for the tracer, how did we get this? We said that if you put q_0 grams of tracer into the system, by definition q_0 equal to v time c naught, at any instant time $v c$ equal to q and material balance for tracer is input output generation equal to accumulation.

There is no generation of tracer because, there is no chemical reaction, there is no continuous input of tracer. Therefore, there is no input therefore, we have 0 minus of $v c$, $v c$ is the output equal to d by $d t$ of $v c$. So, this is a differential equation it governs the variation of tracer in the equipment. Now we know that v times c is q so, we have v and by definition c is given by q by v , equal to d by $d t$ of v time c is q . Now we know that v by v equal to residence time. So, we have here q by τ equal to $d q$ $d t$ or q equal to e to the power of minus t by τ , with some constant of integration.

This is solution of, this is this and then a time t equal to 0 q is q_0 therefore, the solution is q equal to $q_0 e$ to the minus of t by τ . So, what have we saying now that if you have a continuous stirred tank ray then, if have a pulse of tracer that the introduce at zero time and then the variation of the holdup of tracer in the equipment, which time is given by this equation q equal to q_0 times e to the power of minus t by τ . Now our interest is what is the meaning of residence time distribution? What is the meaning of residence time distribution?

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Handwritten derivation on a slide:

$$Q = Q_0$$

$$RTD = \frac{v c(t) dt}{Q_0} = E(t) dt$$

$$= \frac{v c(t) dt}{Q_0} = E(t) dt$$

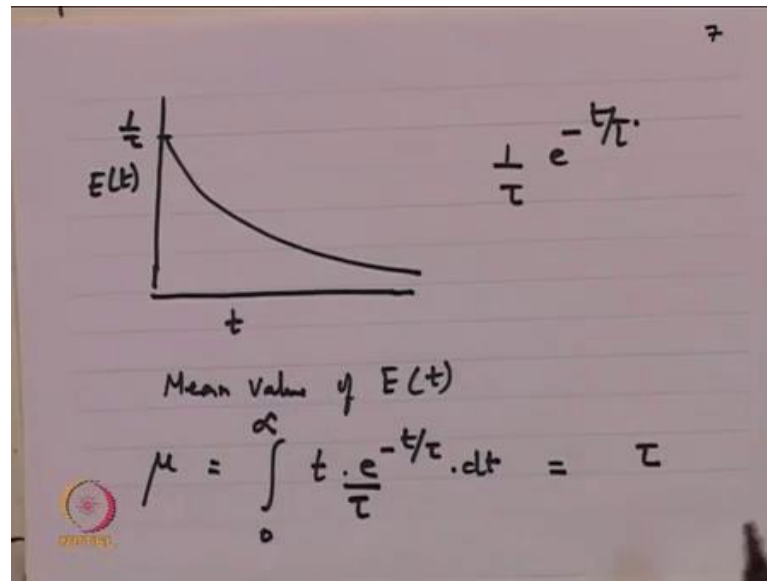
$$= \frac{v Q(t)}{V Q_0} dt = \frac{1}{\tau} e^{-t/\tau} dt$$

$$E(t) dt = \frac{1}{\tau} e^{-t/\tau} dt$$

By definition, residence time distribution (RTD) is what? By definition is $v c(t) dt$ divided by Q_0 , this definition is important. What is the RTD? By definition is, what is the amount of material that is leaving the equipment between time t with t plus dt . So, between time t and t plus dt this amount of material will leave the equipment. What is a total amount of tracer that we have introduced in system is Q_0 . Therefore, residence time distribution by definition is the amount of material that leaves between the time t and t plus dt which is, which is this divided by the total amount of material which is injected.

Therefore, this is by definition the $E(t)$ function. So, we have v , what is $c(t) dt$ divided by Q_0 is also equal to v , c is different by Q at time t divided by capital V times Q_0 . Therefore, this is 1 by τ Q by Q_0 , Q by Q_0 we have just now shown it is Q by Q_0 is e to minus t by τ . Therefore, I am putting it as minus of t by τ , this is the $E(t) dt$, I have forgotten the dt here. So, the $E(t) dt$ is 1 by τ e to the power of minus t by τ times dt . So, this is the RTD for stirred tank. So, if you want to remove the dt here and dt here, we can say that the E function for a stirred tank is 1 by τ e to the power of minus t by τ . How does this function look like? We will just see quickly how this function looks like.

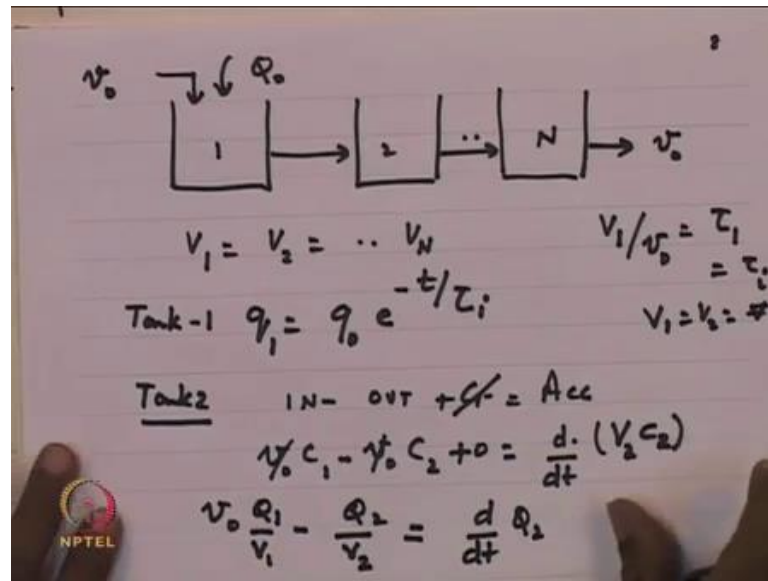
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Suppose I plot, suppose I make a plot of t versus $e^{-t/\tau}$ function so, at you can see here our $e^{-t/\tau}$ function is $\frac{1}{\tau} e^{-t/\tau}$. Therefore, at time t equal to 0, the function value is $\frac{1}{\tau}$ and as time t increases, the function value decays exponentially. So, this is the $e^{-t/\tau}$ function, let me ask you one more question. So, the next question we want to answer is, what is the mean value of $e^{-t/\tau}$? How do you define a mean? What is the meaning of mean? Mean is always defined as t times $e^{-t/\tau}$ divided by τ integrated from 0 to infinity, this is what is called as mean.

The first moment of the distribution function is called as the mean, this is a very simple thing, you can integrate and check for yourself that the mean is τ . So, the mean of the $e^{-t/\tau}$ function is a very simple integration, you can calculate and check for yourself. So, the mean of this can be found out to be τ . Now this next question we would like to answer is.

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Let us say we have a sequence of stirred tanks, we have a sequence of stirred tanks into which fluids are coming in, fluids are going out, fluids are going out, tank 1, tank 2 say its tank n. There is an n tank into which you have fluids are coming in and therefore, the fluids are going out. If you put a pulse of tracer here then, what happens to, what happens to the tracer at any instant of time in this sequence of stirred tanks. Now this is something that we can do very quickly, very quickly let us try to do that because, it has some value in terms of understanding real vessels.

Let us for the movement assume that v_1 equal to v_2 equal to up to v_n . We make this generalization for the sudden advantages, which you will see shortly after going through this exercise, we will see the advantage of these simplifications. So, you have tank 1 we have already done this tank 1 while we have said q_1 equal to q_0 , $e^{-t/\tau}$ because, I am assuming that v_1 because they are equal therefore, v_1 by v_0 is τ_1 which is also equal to τ_2 equal to τ , on other words the residence time in each of these tanks are the same.

Let us see assumption the advantages we will see shortly. So, based on this you already derive this q_1 equal to q_0 , it is already done from the previous exercise. So, we won't do for tank 1 for we will try to do for tank 2 that means, you want to write a material balance for tracer and trine tank 2. So, let me write input minus of output plus generation equal to accumulation. What is the amount of material entering this one, tank 2 which is

v naught time c_1 , what is going out? v naught time c_2 , there is no generation equal to d by $d t$ of v times which is v_2 . So, I am just calling all the v same volume v that means v_1 v naught is all have the same value taken it has v , v times c_2 .

So this is, this is the volume that means have taken v_1 equal to v_2 equal to v . So, that is the volume that mentioned here that is v_1 equal to v_2 up equal to v , all have the same volume or we can keep it has v_2 for the movement, there is nothing wrong with that. Now if I divide throughout by v naught, so what do I get? Let me write this as, how should I write this,? I want to write this has q by v . So, v naught times q_1 by v minus q_2 by v equal to d by $d t$ of q_2 is it okay? I have put this c_1 is q_1 by v c_2 is q_2 by v , I can write this has v_1 , I can write this is v_2 and v_1 equal to v_2 . So, there is no all right.

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The image shows a handwritten derivation on a slide. The equations are as follows:

$$\frac{Q_1}{\tau_i} - \frac{Q_2}{\tau_i} = \frac{dQ_2}{dt}$$

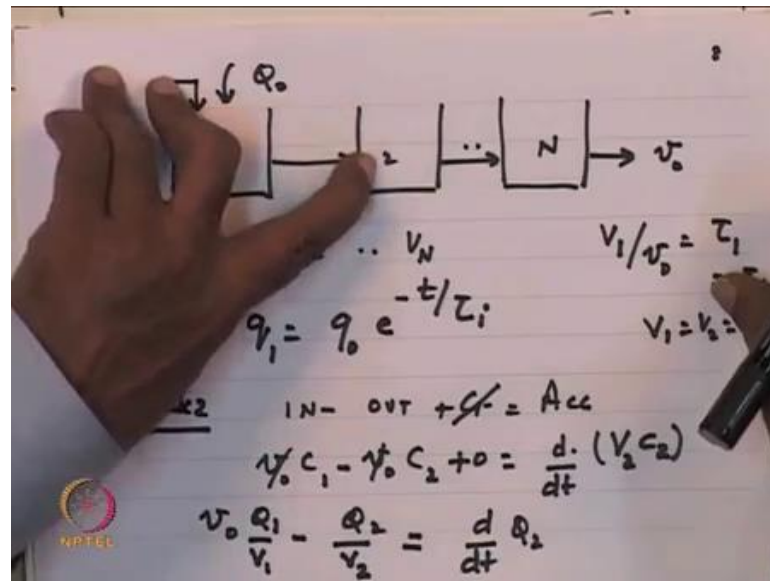
$$\frac{Q_0 e^{-t/\tau_i}}{\tau_i} - \frac{Q_2}{\tau_i} = \frac{dQ_2}{dt}$$

$$\frac{dQ_2}{dt} + \frac{Q_2}{\tau_i} = \frac{Q_0}{\tau_i} e^{-t/\tau_i}$$

$$Q_2(t=0) = 0$$

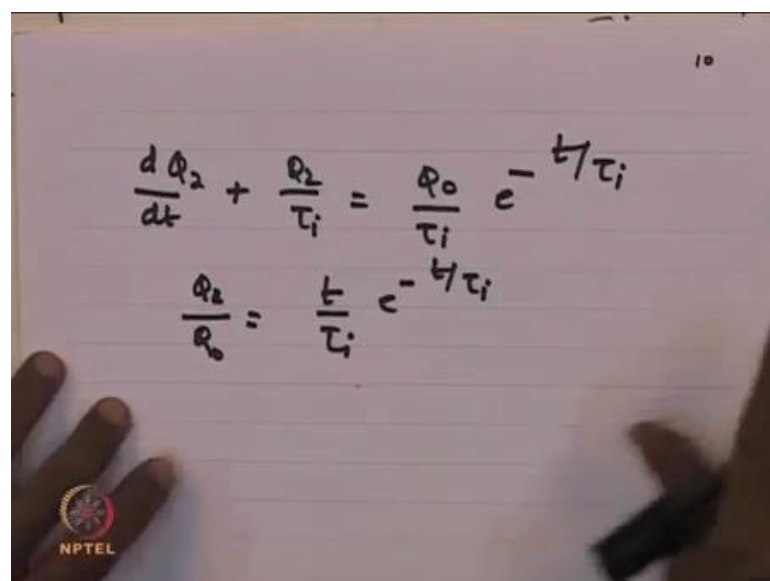
Let us, let us take this argument further and say that this becomes q_1 divided by τ_i minus of q_2 divided by τ_i equal to d by $d t$ of q_2 . q_1 is what q_0 , e to the power of minus of t by τ_i by τ_i minus q_2 by τ_i equal to $d q_2$ by $d t$, all right. Let me write it in this form $d q_2$ by $d t$ plus q_2 by τ_i equal to q naught e to the minus t by τ_i by τ_i . Let me just see whether got all the things properly. So, you will $d q_2$ by $d t$ plus q_2 by τ_i equal to q_1 by τ_i which is, which is known from the previous. So, this for you to solve this, you only need to know what is q_2 at time t equal to 0, which you know as 0.

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Because, at time t equal to 0 your tracer we have put into, our initial condition is that the tracer was put into tank 1. Therefore, in in tank 2 the amount of tracer at time t equal to 0 is 0. Therefore, q_2 equal to this, this is the initial condition for this problem. So, you can solve this there is a felly elementary. So, this nothing very difficult about solution of this equation, solving this we will get, I will write the solution because, it is not very complicated. Therefore, solution to this is that q_2 so, let me write the question once again just to remind you this is the equation that we have q_2 by τ_i equal to q_0 by τ_i e to the power of minus of t by τ_i solution is, let me write the solution.

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Solution looks like . So, q_2 divided by q_0 equal to t by τ_i e raised to the power of minus t by τ_i . So, this is a solution for the case of, for the second tank. Now let us just write the equation for the third tank so, that will make it a little easier to understand what we are trying to prove, what we are trying to prove will be quickly write down for third tank.

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Tank 3

$$IN - OUT + G = Acc$$

$$v_0 c_2 - v_0 c_3 + 0 = \frac{d \cdot V c_3}{dt}$$

$$v_0 \frac{q_2}{v_2} - v_0 \frac{q_3}{v_3} = \frac{d \cdot V c_3}{dt}$$

$$\frac{q_2}{\tau_2} - \frac{q_3}{\tau_3} = \frac{d \cdot q_3}{dt} \quad \tau_2 = \tau_i$$

$$\frac{q_2}{\tau_i} - \frac{q_3}{\tau_i} = \frac{d \cdot q_3}{dt}$$

Now our third tank input minus of output plus generation equal to accumulation. So, as so what comes into our second, third tank, this is for tank 3, we are writing about tank 3, input this is the output there is no generation equal to d by $d t$ of v times of c_3 . So, this is v naught q_2 by v_2 and then v naught q_3 by v_3 equal to d by $d t$ of v times c_3 . So, we can write this as q_2 by τ_2 minus q_3 by τ_3 equal to d by $d t$ of q_3 . Now all this τ 's τ_3 equal to τ_i therefore, I can write this is q_2 by τ_i minus of q_3 by τ_i equal to d by $d t$ of q_3 . And what is the initial condition, let me just said it out once again.

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Tank 3

$$\frac{Q_2}{\tau_i} - \frac{Q_3}{\tau_i} = \frac{d \cdot Q_3}{dt}$$

$$Q_3(t=0) = 0$$

$$Q_3 = \frac{Q_0 t^2}{2 \tau_i^2} e^{-t/\tau_i}$$

$$E_3 dt = \frac{v c_3(t) dt}{Q_0} = \frac{v Q_3 dt}{Q_0}$$

$$E_3 dt = \frac{Q_3 dt}{Q_0 \tau_i} = \frac{Q_0 t^2}{4 \tau_i^2} e^{-t/\tau_i}$$

Our differential for tank 3 continued, we have Q_2 by τ_i , Q_3 by τ_i equal to $d \cdot Q_3$ by dt . At time $t=0$, Q_3 is equal to 0. As we have please recall, in our stirred tank, in our stirred tank we have put our tracer only into the first tank. Please recognize that our tracer we put into the first tank only therefore, in tank 3 at time $t=0$, the tracer is 0. Therefore, this is 0 solution of this once again is not very complicated and this write down the solutions of Q_3 , Q_3 equal to $Q_0 t^2$ by τ_i^2 e^{-t/τ_i} .

So, what is E_3 , E_3 by definition is v times c_3 dt divided by, the total amount of material that we have put in or this is v , which is Q_3 divided by Q_0 or E_3 equal to v by Q_0 of Q_3 by τ_i dt . If you want to put you can put Q_3 dt , you see E_3 . So, this becomes Q_0 Q_3 dt so, Q_3 is Q_0 we have just now mentioned a Q_3 Q_0 . So, Q_3 by Q_0 we can put write away as straightway substitute from here as t^2 by $2 \tau_i^2$, multiplied by this is τ_i . Therefore, another τ_i is coming here e^{-t/τ_i} .

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Tank 3

$$E_2(t) dt = \frac{t^2}{2! \tau_i^3} e^{-t/\tau_i}$$

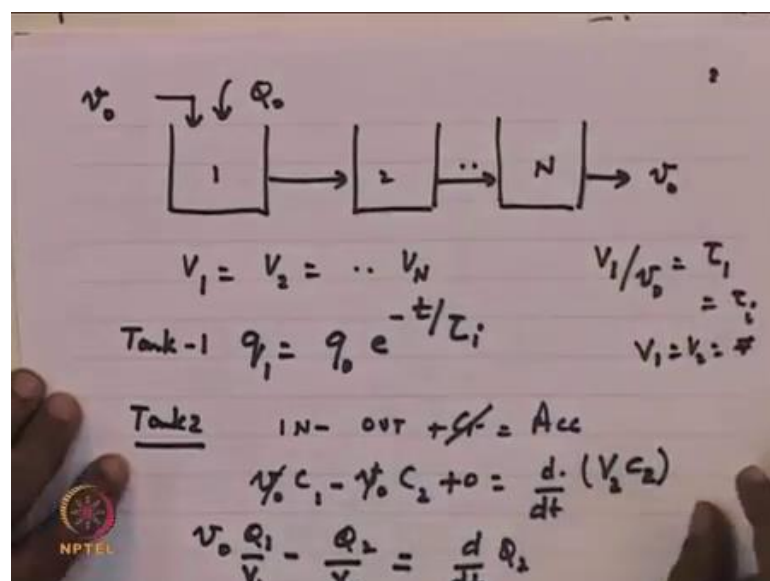
$$E_n(t) = \frac{t^{n-1}}{(n-1)! \tau_i^n} e^{-t/\tau_i}$$

$$\tau_i = \left(\frac{\tau}{N}\right) = \left(\frac{V/v_0}{N}\right)$$

$\tau = N \tau_i$

Let me write this properly once again tank 3, tank 3 continued tank 2 is e^{-3t/τ_i} equal to t^2 squared by $2 \tau_i$ cubed e^{-t/τ_i} or $e^{-n t/\tau_i}$ the n th tank, I am just writing by induction t^{n-1} . This is actually a factorial one so, for $n-1$ factorial τ_i to the power of n e^{-t/τ_i} . Where, τ_i by definition is, by definition is τ divided by the number of tanks. What is τ ? τ is v total volume divided by v_0 divided by number of tanks or $\tau = N \tau_i$. So, this is another relationship that we keep in mind. So, what we have said let me just run through this once again so that, now we have the context, the context is not lost.

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We have started by saying that there are n tanks, there are n tanks in series and volume and flow is coming into tank 1 and a tracer is put into first tank. Then, we measuring what is happening to the tracer in different tanks and by writing the material balance we found out what happens to all of them. And then, based on a definition of the e function we have derived that the r t d function for the nth tank that means, the nth tank.

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Tank 3

$$E_2(t) dt = \frac{t^2}{2! \tau_i^3} e^{-t/\tau_i}$$

$$E_n(t) = \frac{t^{n-1}}{(n-1)! \tau_i^n} e^{-t/\tau_i}$$

$$= \left(\frac{\tau_i}{N} \right) = \left(\frac{V}{v_0} \right) / N$$

$N \tau_i$

That means, whatever the fluid that comes at the end of the nth tank, we should make the measurement of r t d here. This is what we will find that e n is t to the power of n minus of 1, factorial n minus 1 tau i to the power of n e to the power of t by tau i. Where, tau i is tau divided by n where tau is simply volumetric flow to the whole, this is the volume of the entire n tank system. So, this v is n times v i, this is something we know. So, tau equal to n tau i is a result, we know from first principles, all right.

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13

Tank 3

$$E_2(t) dt = \frac{t^2}{2! \tau_i^2} e^{-t/\tau_i}$$

$$E_n(t) = \frac{t^{n-1}}{(n-1)! \tau_i^{n-1}} e^{-t/\tau_i}$$

$$\tau_i = \left(\frac{\tau}{N}\right) = \left(\frac{V/v_0}{N}\right)$$

$$\tau = N \tau_i$$

NPTEL

Having said this, let us see what is, why are we doing all this, how does it help us. To understand this, to understand this let us recognize some simple algebraic relationships.

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$$E_n(t) = \frac{t^{n-1}}{(n-1)! \tau_i^{n-1}} e^{-t/\tau_i}$$

$$\int_0^{\infty} t^n e^{-t/\tau} dt = n! \tau^{n+1}$$

$$\mu_n(t) = \int_0^{\infty} t E(t) dt = \int_0^{\infty} \frac{t \cdot t}{(n-1)! \tau_i^{n-1}} e^{-t/\tau_i} dt$$

NPTEL

Our e function for n tank sequence we have derived as factorial n minus 1 tau i to the power of n e raised to the power of t by tau i. This is we have derived, this small n and capital n are interchangeably used. Now there is one interesting relationship that we know from our basic calculus, which I will write down with a proof. See, this relationship is very useful for determining values of various integrals that we will

experience in r t v. That means, integral 0 to infinity t n e to the minus t by tau is factorial n tau raised to power of n plus 1.

Using this relationship, let us now calculate what is the first movement or that means what is called as first movement mu n, mu n what is a meaning of mu n? mu n is the first movement of the e function. First movement of the e function is called mu that means, this essentially tells us what is the mean value of the function. Now, we can do this integration right away. So, we have integral 0 to infinity t multiplied by t raised to the power of n minus 1 by factorial n minus 1 e raised to the power of minus t by tau i and then, tau i to the power of n d t. I will write d t the results do this integration because, it is important.

(Refer Slide Time: 33:12)

$$\begin{aligned} \mu_n(t) &= \int_0^{\infty} t \cdot E(t) dt \\ &= \int_0^{\infty} \frac{t \cdot t^{n-1}}{(n-1)! \tau_i^n} e^{-t/\tau_i} dt \\ &= \frac{(n!) \tau_i^{n+1}}{(n-1)! \tau_i^n} = n \tau_i = \tau \end{aligned}$$

So, mu n of t is integral 0 to infinity t e t d t, I am just substituting them here 0 to infinity t, t raised to n minus 1 factorial n minus 1 tau i to the power of n correct And then, first movement just we have got it right? t to the power of n e raised to the power of t by tau d t, you got it, you got it tau raised to the power of n. And then, e raised to the power of minus of t by tau i d t now, you can simplify this and then it is very easily seen, I will just writing down the answers here. Factorial n tau i to the power of n plus 1, factorial n minus 1 tau i to the power of n. So, I have just integrated this and using this relationship that we have taken from basic calculus.

This is a basic calculus relationship and then and it comes out to be, please help me to simplify this $n \tau_i$ equal to tau. What we are saying is that, if you have an n tank sequence where e function is given by this. If you want to find out the first movement as defined by these equation then, the first movement turns out to be n times tau i which is tau.

(Refer Slide Time: 34:58)

Handwritten mathematical notes on a whiteboard:

- Top right corner: 14
- Equation for $E_n(t)$:
$$E_n(t) = \frac{t^{n-1}}{(n-1)!} \tau_i^n e^{-t/\tau_i}$$
- Equation for τ_i :
$$\tau_i = \frac{v}{\lambda}$$
- Integral equation boxed in a rectangle:
$$\int_0^{\infty} t^n e^{-t/\tau} dt = n! \tau^{n+1}$$
- Equation for $\mu_n(t)$:
$$\mu_n(t) = \int_0^{\infty} t E(t) dt = \int_0^{\infty} \frac{t \cdot t^{n-1}}{(n-1)!} \frac{e^{-t}}{\tau_i^n} dt$$

So, essentially what it says is that first movement of the e function is tau, this is what we expect based on our understanding that v equal to, this tau equal to v by λ , that is its comes from the basic understanding.

(Refer Slide Time: 35:14)

Variance of $E_n(t)$ function.

$$\sigma_n^2 = \int_0^{\infty} (t - \mu_n)^2 E_n(t) dt$$
$$= \int_0^{\infty} (t^2 - 2\mu_n t + \mu_n^2) E_n(t) dt$$
$$= \int_0^{\infty} t^2 E_n(t) dt - 2\mu_n \int_0^{\infty} t E_n(t) dt + \mu_n^2 \int_0^{\infty} E_n(t) dt$$
$$\sigma_n^2 = \int_0^{\infty} t^2 E_n(t) dt - \mu_n^2$$

Now the second result which you would like to know is, what is the variance, variance of E_n function. Now, σ_n^2 by definition is $\int_0^{\infty} (t - \mu_n)^2 E_n(t) dt$. This is how variance is defined as $\int_0^{\infty} (t - \mu_n)^2 E_n(t) dt$. Now, this is also not a very complicated integration so, we just do some simple things so that, I am just expanding this $(t - \mu_n)^2$. I will put dt afterwards then, plus $\mu_n^2 E_n(t) dt$ therefore, first term becomes $\int_0^{\infty} t^2 E_n(t) dt$. Second one is $\int_0^{\infty} -2\mu_n t E_n(t) dt$ is $-2\mu_n \int_0^{\infty} t E_n(t) dt$. So, this becomes equal to $\int_0^{\infty} t^2 E_n(t) dt - 2\mu_n \int_0^{\infty} t E_n(t) dt + \mu_n^2 \int_0^{\infty} E_n(t) dt$. So, this becomes equal to $\int_0^{\infty} t^2 E_n(t) dt - 2\mu_n \int_0^{\infty} t E_n(t) dt + \mu_n^2 \int_0^{\infty} E_n(t) dt$. So, σ_n^2 is simply $\int_0^{\infty} t^2 E_n(t) dt - \mu_n^2$.

Now, we have once again this integral, this integral is available to us this, this integral is available to us we can use the same integral to simplify and find out what is this? The variance of this function, let us do that because, it is important.

(Refer Slide Time: 37:15)

The image shows a handwritten derivation on a slide. At the top right, the number '17' is written. The main derivation consists of three lines of equations:

$$\mu_n^2 = \int_0^{\infty} \frac{t^{n-1} e^{-t/\tau_i}}{(n-1)! \tau_i^n} dt - \mu_n^2$$

$$= \int_0^{\infty} \frac{t^{n+1} e^{-t/\tau_i}}{(n-1)! \tau_i^n} dt - \mu_n^2$$

$$\mu_n^2 = \frac{1}{\tau_i^n (n-1)!} \tau_i^{n+2} - \mu_n^2$$

So, sigma n squared equal to integral 0 to infinity t squared what is e n, the e n function? It is here, our e n function is here, our e n function is here I will write it down so, t raised to the power of n minus 1 by factorial n minus 1, e rays to the power of minus of t by tau i divided by tau i to the power of n d t. So, it is a first term I have written down minus of mu n squared. We can simplify this further which is t to the power of n plus 1 e raised to minus of t by tau i divided by factorial n minus 1 tau i to the power of n integral 0 to infinity d t minus of mu n squared.

Now we can use this integral once again, the same integral the t n t n e to the minus t by tau is factorial n, n plus 1 tau. So, that same result we can use here when I do that the result terms out as. So, I will expand this and i get this sigma n squared equal to it becomes, 1 by tau i to the power of n factorial n minus 1 denominator states numerators factorial n plus 1 tau i to the power of n plus 2 minus of mu n squared. So, t tau to the power by n plus tau comes out so, it is it is factorial n plus 1 and then tau I to the power n plus 2, it is quite simple. So, this can be simplified further, let me simplify this further.

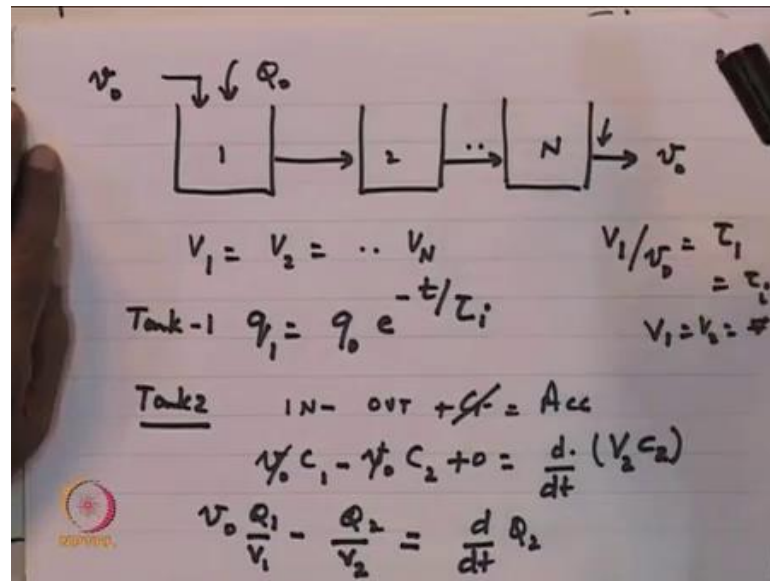
(Refer Slide Time: 39:00)

The image shows a whiteboard with handwritten mathematical derivations. At the top right, the number '19' is written. The main derivation starts with the equation $\sigma_n^2 = n(n+1)\tau_i^2 - \mu_n^2$. To the right of this, it says $\mu_n = n\tau_i$. The next line shows $\tau_i^2 n(n+1) - (n\tau_i)^2$. This is followed by $\tau_i^2 [n^2 + n - n^2] = n\tau_i^2$. The next line shows $\sigma_n^2 = n \left(\frac{\tau_i}{n}\right)^2 = \frac{\tau_i^2}{n}$. At the bottom, there are two boxed equations: $n = \frac{\tau_i^2}{\sigma_n^2}$ and $\mu_n = n\tau_i$. A small logo is visible in the bottom left corner of the whiteboard.

So, sigma n squared just help me now so, sigma n squared equal to, it simplifies, I would simplify this. So, tau i n plus 2 tau I squared will stay on the numerator factorial n plus 1 factorial n minus 1. So, n and n plus 1 will stay on the numerator so, it becomes n into n plus 1 tau i squared minus of mu n squared. Now, n into n plus 1 and then n into n plus 1, is it correct? What is mu n squared? mu n squared in, mu n is, you have written mu n has n tau i. So, it is minus of n tau i whole squared mu n by definition is n tau i. So, that is what is written here.

So, this is tau i squared so tau i squared within brackets of n squared plus n minus of n squared is equal to n tau i squared or n times tau i is, tau i by n whole squared or tau squared by n is equal to sigma n squared or n equal to, n equal to tau squared by sigma n squared. In fact this is the result for which we have done all this homework, result is the very important result that if you have a n tank sequence then the, then the mean mu n, mu n is n tau i. This is one result and then, sigma squared n is given by tau squared by n. So, let just look back at what we have done.

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What we have done is, we have taken an \$n\$ tank sequence, we have put our tracer into first tank. We have measured the output at the end of the \$n\$ tanks and then we have plotted that function and found out the mean and variance of the response that we have got. In other words what we saying is that, the response of the end of these \$n\$ tanks is something that you and I can measure. So, that response, from that response we have determined what is the mean and variance of the response that we have got.

(Refer Slide Time: 41:38)

The derivation shows the following steps:

$$\sigma_n^2 = n(n+1)\tau_1^2 - \mu_n^2$$

$$\tau_1^2 n(n+1) - (n\tau_1)^2$$

$$\tau_1^2 [n^2 + n - n^2] = n\tau_1^2$$

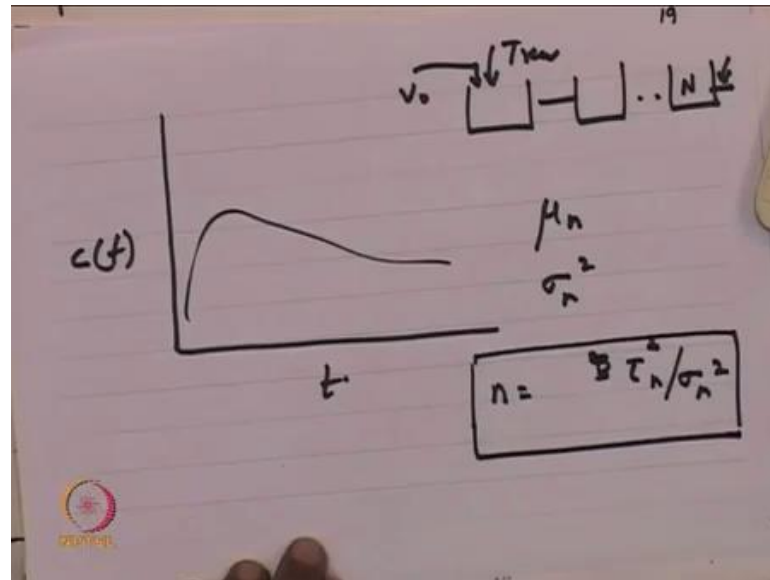
$$\sigma_n^2 = n \left(\frac{\tau_1^2}{n} \right) = \frac{\tau_1^2}{n}$$

Final boxed results:

$$n = \frac{\tau_1^2}{\sigma_n^2} \quad \mu_n = n\tau_1$$

This is the variance of the response, this is the mean of the response, both mean and that means μ_n , μ_n and the variance σ_n^2 , both are experimentally measured quantities.

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Let me repeat what we saying is that, we have determined from for our n tank sequence we have number of tanks, number of tanks, we have put our tracer here v naught. And then we measured our response at the end away n tanks and then, our response in the form of concentration at the end of verses time, we have got this response. Based on that response, some response we got, we found out the mean, we have found out the sigma squared or mean of the response, the variance of the response we got. And then, we said that n equal to τ_n squared by σ_n squared. This is what we have got that the number of tanks n equal to τ_n squared by σ_n squared. So, this is the result that we have got from our tracer.

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$$\sigma_n^2 = n(n+1)\tau_1^2 - \mu_n^2$$

$$\tau_1^2 n(n+1) - (n\tau_1)^2$$

$$\tau_1^2 [n^2 + n - n^2] = n\tau_1^2$$

$$\sigma_n^2 = n \left(\frac{\tau_1}{n} \right)^2 = \frac{\tau_1^2}{n}$$

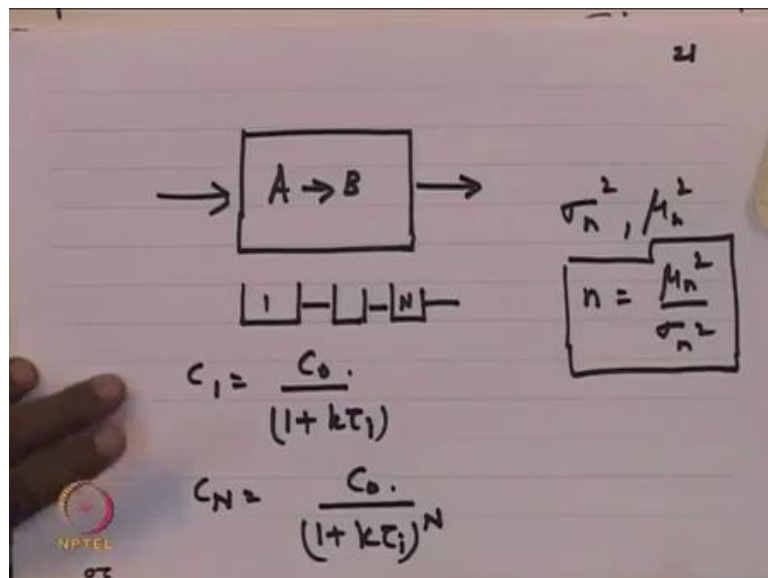
$$\mu_n = n\tau_1$$

$$n = \frac{\tau_1^2}{\sigma_n^2}$$

$$\mu_n = n\tau_1$$

We got this result, we got this n is equal to tau squared by sigma n squared. So, which is mu n squared by sigma n squared. So, if you do an a tracer experiment, if you do a tracer experiment you are able to determine the number of times that are required to describe the performance of this equipment. And that number of tanks that you determine from a tracer experiment, is simply mu n squared by sigma n squared, both are experimentally determined quantities., this is the important result. Let us see how to make use of this result to understand performance of an equipment, we will do that it shortly.

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Let us say now we have an arbitrary vessel, we have a arbitrary vessel into which fluids coming in, fluids going out and there is a reaction taking place, A going to B. Now, we won't know how this reaction will perform in this arbitrary equipment. If it is a first order reaction and let us say what we have done now, let us say we divide this, assume that you know this tank can be divided into n such tanks, 1 to n. So, we have determined the r t d of this, for which we know sigma squared n and then v squared mu n squared. Both are experimentally determined quantities and on this basis, we also know that mu n squared by sigma n squared equal to the number of tanks.

That means, this particular arbitrary vessel can be described using the tracer curve, that number of tanks that will take care of this arbitrary flow is given by mu n squared by sigma n squared. So, we want to use the result to understand this, let us do how to do this say, c 1 we know is c naught divided by 1 plus k tau I, this is for a first order reaction. Similarly, you know that c n equal to c naught divided by 1 plus k tau i to the power of n. This also we know for a first order reaction, all right. Now that we know this as a first order reaction, what can we do with this?

(Refer Slide Time: 45:10)

The image shows a person's hands writing on a piece of lined paper. The first equation written is
$$c_N = \frac{c_0}{(1 + k\tau_i)^N}$$
 The second equation, which is an algebraic manipulation of the first, is
$$\frac{c_0}{[1 + (k \frac{\tau}{N})]^N} = \frac{c_0}{1 + k \frac{\tau}{N}}$$
 The person is using a black marker to write the equations. In the bottom left corner of the paper, there is a small logo for NPTEL.

What is being said by this model of understanding this non-ideality that this c n is c naught divided 1 plus k tau i to the power of n. We also say this equal to c naught 1 plus k, what is tau i? It is tau by n tau by n to the power of n. Now we also write this as c naught divided by 1 plus k what is tau? Tau we say is mu n, what is number of tanks?

Number of tanks we have just said from our understanding of r t d theory, this can be approximated has mu n squared by sigma n squared. So, I will replace this capital n as mu n squared divided by sigma n squared, okay.

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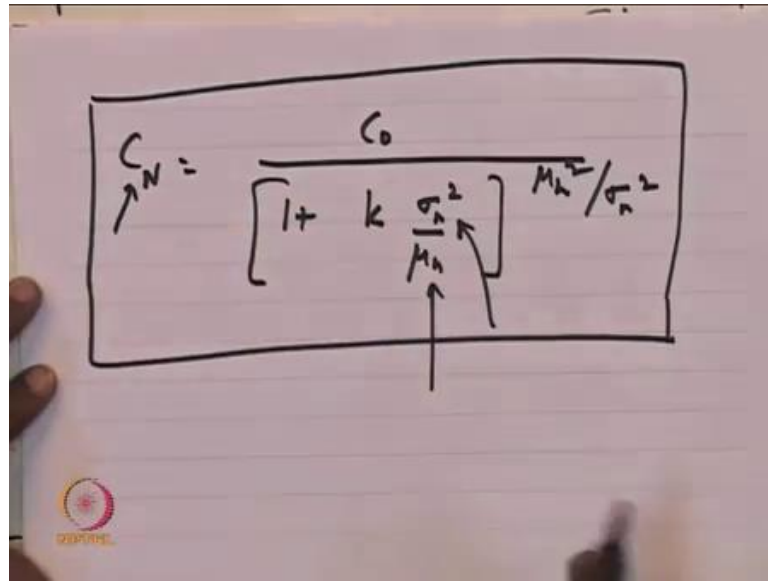
$$C_N = \frac{C_0}{(1 + k \tau_i)^N}$$

$$\frac{C_0}{\left[1 + \left(k \frac{\mu_n^2}{\sigma_n^2}\right)\right]^N} = \frac{C_0}{\left(1 + k \frac{\mu_n^2}{\sigma_n^2}\right)^N}$$

$$C_N = \frac{C_0}{\left(1 + k \frac{\mu_n^2}{\sigma_n^2}\right)^N} = \frac{C_0}{\left(1 + k \frac{\mu_n^2}{\sigma_n^2}\right)^N}$$

And to the power of n, to the power also to be determined is sigma mu n squared by sigma n squared. On other words what we are saying is, let me write it once again c n equal to c naught divided by 1 plus k mu n divided by mu n squared sigma n squared to the power of n, which is mu n squared divided by sigma n squared. So, this simplifies has c naught divided by 1 plus k mu sigma n squared by mu n to the power of mu n squared by sigma n squared. We may write it once again, we can write very neatly.

(Refer Slide Time: 46:53)



A handwritten equation on a piece of lined paper. The equation is $C_N = \frac{C_0}{\left[1 + k \frac{\sigma_n^2}{\mu_n}\right]}$. The variables C_N , C_0 , k , σ_n^2 , and μ_n are all written in black ink. There are arrows pointing from the labels C_N and μ_n to their respective terms in the equation. The equation is enclosed in a hand-drawn rectangular box.

So, we have c_n equal to c_{naught} divided by $1 + k$, τ is σ_n squared by μ_n to the power of n , which is μ_n squared divided by σ_n squared. Now this result has the advantage that, μ_n and σ_n are experimentally determined, experimentally determined quantities which comes out our experiments using simply the rtd theory. In other words, if we have an arbitrary vessel we can simply do a tracer test to find out the number of tanks that are required to describe the flow field and that comes from the rtd theory. That is described in terms of μ_n and σ_n so, once you know μ_n and σ_n and you know the rate Constant you can say, what will be the extent of reaction or what will be the concentration at the end of c .

Therefore, conversion is simply $1 - \frac{c_n}{c_{not\ so}}$, this is the conversion. So, we can tell based on rtd theory what is the extent to which the reaction will take place in this arbitrary vessel, without knowing the details over the flow field. So, it is no longer needs to be an ideal reactor, it can be an arbitrary reactor. Only thing is that we have treated it as a sequence of stirred tanks and we determined the number of tanks that are required to equivalently describe this arbitrary vessel. So, that is the advantage of this treatment. I will stop there and we will take up other things in the afternoon.

Thank you.