

Advanced Chemical Reaction Engineering
Prof. H. S. Shankar
Department of Chemical Engineering
Indian Institute of Technology, Bombay

Lecture - 21

Illustrative Example: Stability of Exothermic Stirred Tank

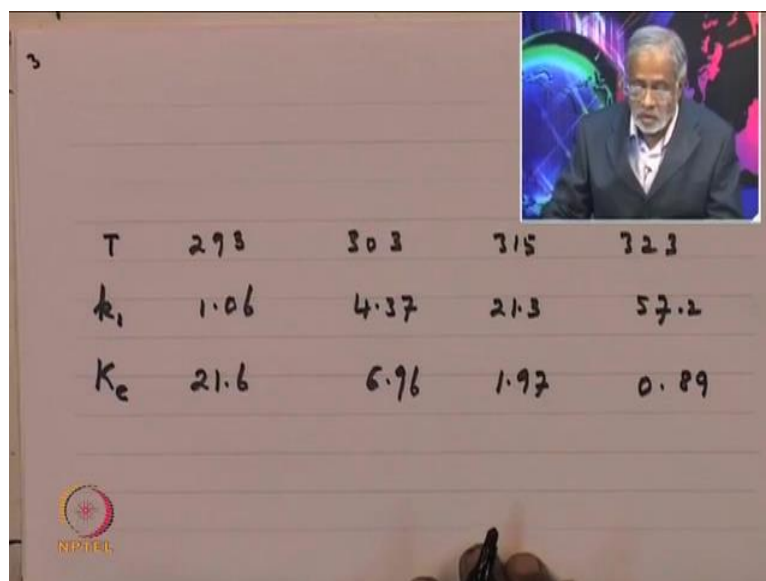
So, we look at practice problem in energy balance in this lecture. Let me first describe the problem to you.

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$C_{A0} = 1.66 \text{ kmol/m}^3$
 $T_0 = 21^\circ\text{C}$
 $v_0 = 0.6 \text{ m}^3/\text{hr}$
 $E_1 = 25000 \text{ kcal/kmol}$
 $T_c = 10^\circ\text{C}$
 $h(\text{coolant-reactant}) = 1000 \frac{\text{kcal}}{\text{m}^2\text{hr}}$
 $\Delta H_{(A \rightarrow B)} = -20,000 \text{ kcal/kmol}$

So, that we understand what the situation is? We have here a stirred tank and it has a cooling coil, all right. A reaction A going to B and it is quite reversible. So, this is the kind of reaction taking place. Set v_0 and then, composition C_{A0} and cooling temperature T_c , and feed temperature T_0 . These all this some data that is given is as follows; C_{A0} equal to 1.66 K moles per cubic meter. T_0 is 21 C, v_0 is 0.6 cubic meter per hour. Activation energy for reaction 1; this is 1, this is 2 is 25000 K cal per K mole. Cooling temperature T_c is 10 C, and heat transfer coefficient this is between coolant and reactant, this 1000 K cal square meter per hour. So, this is the data that is we have.

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T	293	303	315	323
k_1	1.06	4.37	21.3	57.2
K_e	21.6	6.96	1.97	0.89

Now, some more data we would require because the reaction is reversible. Therefore, we need some data which is also available which all. So, we have T k_1 and K equilibrium constant; 293, 303, 315, 323, 1.06, 4.37, 21.3, 57.2, 21.6, 6.96, 1.97, 0.89. So, these data on reaction velocity constant and equilibrium constants are given. The question in front of us is the following. Now, there we this is very heat of reaction I have forgotten, I am just heat of reaction there is delta H heat of reaction is give as minus 20000, A going to B this is for A going to B, heat of reaction calories K cal per K mole. So, this is a reversible reaction, and plus is exothermic. So, we know that if you have a reversible reaction, which is exothermic. It has some interesting features show in where it has a maximize reaction rate and so on, which you have already in our discussed.

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$$A \xrightleftharpoons[k_2]{k_1} B$$

(1) Derive locus of max rates

$$r_B = k_1 C_A - k_2 C_B$$
$$= k_1 C_{A0} (1-x) - k_2 (C_{B0} + C_{A0} x)$$
$$\left(\frac{\partial r_B}{\partial x} \right)_T =$$

NPTEL

So, let us look at first part of the exercise is; 1 is derive locus of max rates. What is the locus of max rates, how did you do this? So, suppose I say r_B is a rate of formation this is our reaction A going to B, and this way this is 1 and this is 2. So, r_B is what? $k_1 C_A$ minus $k_2 C_B$. Now, since there is no volume change, I write this is C_{A0} times $1-x$ minus $k_2 C_{B0} + C_{A0} x$. And, we take this as 0. So, essentially it is $k_1 C_{A0} (1-x) - k_2 (C_{B0} + C_{A0} x)$, ok. Now, locus of maximize rates is understood like this; $\frac{\partial r_B}{\partial x}$ at constant T, this is what we to find out. That means; $\frac{\partial r}{\partial x}$ at constant T so, del sorry, del r del T at constant sorry I am sorry. This is, are we wanted to know how the rate of formation of component B changes with temperature.

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$$r_B = k_1 C_{A0} (1-X) - k_2 C_{A0} X$$

$$\left(\frac{\partial r_B}{\partial T} \right)_X = \frac{k_1 E_1}{RT^2} C_{A0} (1-X) - \frac{k_2 E_2}{RT^2} C_{A0} X = 0$$

$$(1-X_m) \frac{k_1 E_1}{RT^2} C_{A0} = \frac{k_2 E_2}{RT^2} C_{A0} X_m$$

$$\left(\frac{X_m}{1-X_m} \right) = \frac{k_1 E_1}{k_2 E_2} = K \frac{E_1}{E_2}$$

Let us let us do that; we just go through that very elementary manipulations. So, we have r_B equal to $k_1 C_{A0} (1-X) - k_2 C_{A0} X$. We want to find out $\frac{\partial r_B}{\partial T}$ at constant X . So, we have to differentiate k_1 with respect to T that $k_1 E_1$ by RT^2 at constant X . Therefore, we do not change this similarly, we differentiate k_2 is respect to temperature we get $k_2 E_2$ divided by RT^2 C_{A0} times X . Now, we have shown that exothermic reversible reaction r_B goes through a maximum, we have shown that earlier.

And therefore, at the point of maximum we should have this. I will call this X as X_m so, $1 - X_m$ divided by $1 - X_m$ multiplied by $k_1 E_1$ by RT^2 C_{A0} equal to $k_2 E_2$ by RT^2 C_{A0} times X_m . or, X_m divided by $1 - X_m$ equal to X_m divided by X equal to k_1 by $k_2 E_1$ by E_2 , or some equilibrium constant times E_1 by E_2 . This is something that we all know based on the elementary principle ((Refer Time: 07:52)).

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$$r_B = k_1 C_{A0} (1-x) - k_2 C_{A0} x$$

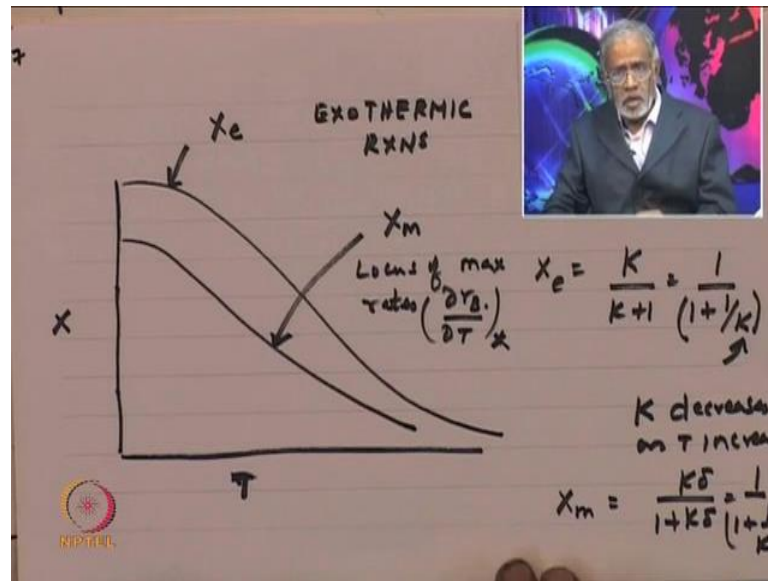
$$\left(\frac{dY_B}{dT}\right)_x = \frac{k_1 E_1 C_{A0} (1-x) - k_2 E_2 C_{A0} x}{RT^2} = 0$$

$$(1-x_m) \frac{k_1 E_1 C_{A0}}{RT^2} = \frac{k_2 E_2 C_{A0} x_m}{RT^2}$$

$$\left(\frac{x_m}{1-x_m}\right) = \frac{k_1 E_1}{k_2 E_2} = K \left(\frac{E_1}{E_2}\right) \Rightarrow x_m = \frac{K\delta}{K\delta + 1}$$

Now, suppose you look at the same reaction at equilibrium, which is you have r_B equal to $k_1 C_{A0} (1-x)$ minus $k_2 C_{A0} x$. At equilibrium we have r_B equal to 0. Therefore, we should have $k_1 C_{A0} (1-x)$ equal to $k_2 C_{A0} x$ or x divided by $1-x$ equal to k_1 divided by k_2 . Or, also means x equal to if I call this as k you know equal to k , x equal to k by $k+1$. At equilibrium we have the equilibrium conversions given by k by $k+1$, ok. Now, we also said that that x_m by $1-x_m$ is equal to therefore, this also means x_m equal to if I call this term as δ , we call this as δ this becomes that means this is δ so becomes $k\delta$ divided by $k\delta+1$, is that clear.

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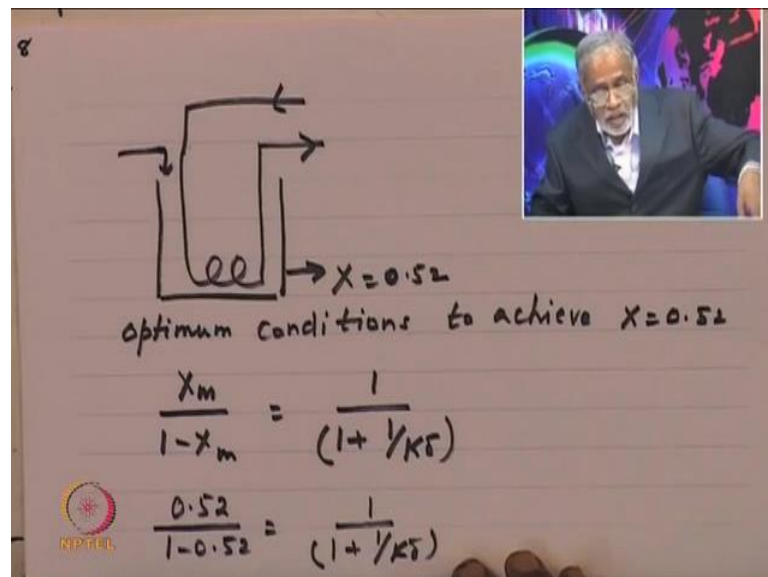
So, if we want to make a plot of what happens to X for different T. On other words; X e equal to K by K plus 1 or it is also equal to 1 by 1 plus K. Now, for a reaction which is exothermic, for exothermic reaction we know that k decreases as T increase. We know that this is familiarly is principle therefore, we have as K decrease as T increases. So, when K decreases what happens to this denominator this increases, correct. So, as T increases you know that x keeps on decreasing. Therefore, equilibrium if this is the equilibrium line, I hope you understand this. I will go throw this once again; as T increases K decreases for exothermic reaction. We are talking about exothermic reaction. For exothermic reactions as T increases K increases, when k sorry, as T increases K decreases therefore, this quantity keeps on increasing. So, 1 plus the denominator keeps on increasing therefore, X e keeps on decreasing as T increases that is what I have drawn.

Similarly, you have this X m which is given by k delta divided by 1 plus k delta and they this also can write as 1 by 1 plus 1 by k delta. Therefore, the behavior of X m also we looks something like this. This is X m, what is X m? X m by definition is the locus this is the locus of max rates. And, what max are we talking about del r B by del T at constant X. So, this is the locus we are talking about. Therefore, in our design by an large you would be interested in ensuring that the reaction rate that we are attain in the equipment is as highest possible. What is if rational for this I mean if the reaction rates are very large then, clearly the equipment there is required to do that processing would be the

smallest. And therefore, it sort of makes sense to have your process operating at the highest reaction rate.

So, what we are saying now is; that this is the first part of the question. It says the question let me state the question once again, question is recall first page when did it 1 this. We said that we have here we want this X says specify best conditions required get a conversion of X equal to 0.52. So, the problem that you would like to address now is what are the best condition that is required to get an outlet conversion X from reactor 1 equal to point 52. So, this is the question.

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The slide contains a handwritten diagram of a stirred tank reactor with a coil. Arrows indicate fluid flow: inlet on the left, outlet on the right, and a return line at the top. Below the diagram, the text reads "Optimum conditions to achieve $X=0.52$ ". Two equations are written:

$$\frac{X_m}{1-X_m} = \frac{1}{(1+1/K\delta)}$$
$$\frac{0.52}{1-0.52} = \frac{1}{(1+1/K\delta)}$$

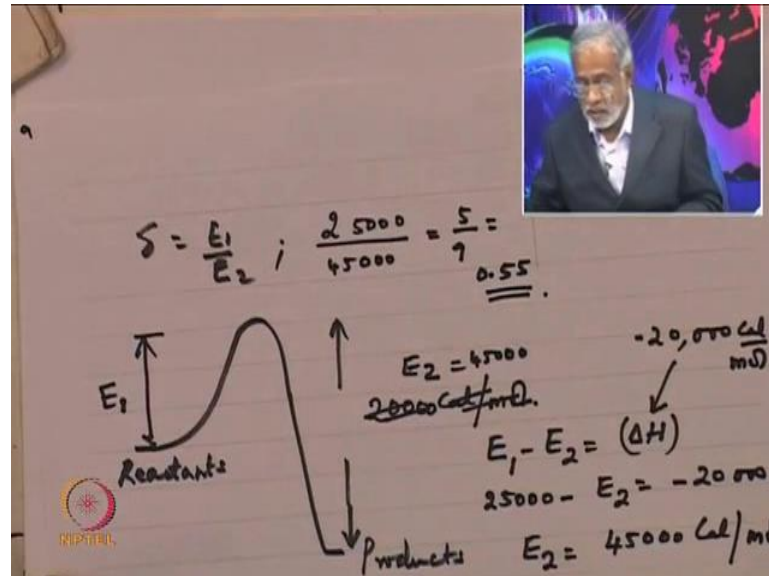
A small inset video shows a man speaking. A logo for NIPTEIL is visible in the bottom left corner of the slide.

Now, how do you address this? You address this recognizing so let us just state that question once again. We have a stirred tank, we have a coil, this is coil coming in and going out, fluids coming in and fluids going out. Now, you want this X to be 0.52 number 1. Number 2; we also want that the reaction rates shows in here be the highest or the size of the equipment for this choice should be the smallest. That means; we want to get the highest reaction optimum conditions to achieve X equal to 0.52.

What are the optimum conditions achieve X equal to 0.5. Now, we said X m divide by 1 minus of X m equal to 1 by 1 plus K delta. That means; the choice of the conditions should be such that this condition be satisfied. Now, if I put X m as 0.52 by 1 minus of 0.52 equal to 1 by 1 plus 1 by K delta. Essentially, this equation defines what should be

the value of K, or equilibrium constant at which I should operate the process. You can solve this and then, find the value of K that gives you, that satisfies the any quality, ok.

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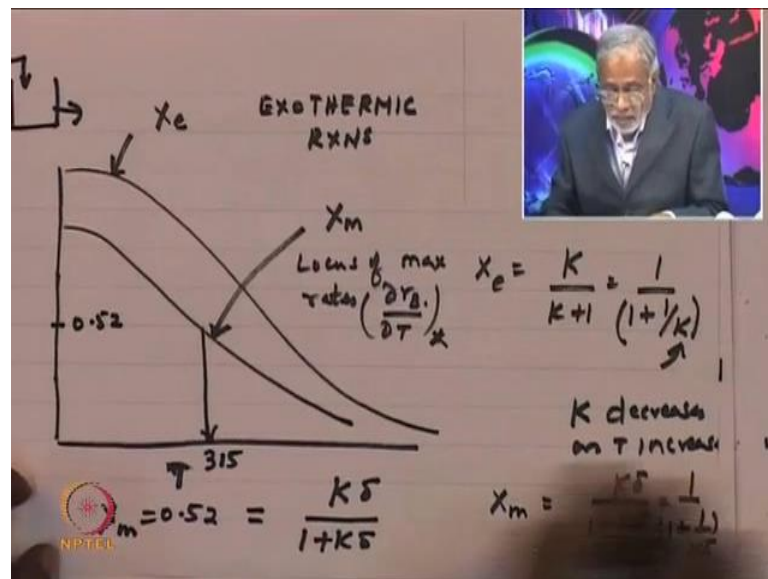
Now, to be able to do that; what is that we require to be able to do that delta, the way we have define delta is E 1 by E 2. Which means; that if this is the reactance these are reactance these are products. And, now this is given as heat of reaction is given as; how much is the heat of reaction? Heat of reaction is given as 20000. So, this is 20000 or 20000 calories per mole, it is given. What is this? This is E 1, and this is E 2. Therefore, E 1 minus of E 2 equal to delta H. This is given as this is equal to 20000 this is what is given I am sorry; I am not able to not write. E 1 minus E 2 is and E 1 is given as E 1 in the problem statement is given as 25000 minus of E 2 equal to this is exothermic heat of reaction minus 20000.

Therefore, E 2 equal to 45000 calories per mole. What we are saying is that in the problem that is specified in front of us; the active energy for the forward reaction is given as 20000. And, heat of reaction is also given as 20000, but it is exothermic. So, I put all those conditions here, and we find E 2 which turns out be 45000. So, we have in this reaction that E 1 is 20000 E 2 is 45000. Therefore delta; so, what is delta equal to 25000 divided by 45000 so, this is a heat of reaction. Is this 5 by 9, correct that is equal to 0.5 about 0.55 so, that is delta value. So, we recognize that to be able to so what we saying is the following; that our process runs in such, should run in such a way that the

divided by 1 minus of 0.52 equal to K. We do not know this and then, 25000 E 1 45000 is E 2. So, this gives you a K value of 1.97, this is what we get.

So, that means if we choose a temperature; so, choose T of reactor so, that equilibrium constant K becomes equal to 1.97. Now, how do we do this? See we have to look at the data that we have after all this data available to us it is given to us, it is given our data for this reaction is already given corresponding to 1.97. We find there is a temperature of the reactor must be chosen as 315 degree K, this is degree K. So, what is that we have done so, when implies T equal to 315 K.

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So, what are we saying? What we are saying is that; we must operate the process in such a way we must operate this process. Which means; to be able to operate the process corresponding to the maximum reaction rate that means you want this 0.5. This is X equal to 0.52 you want yes. But you want this in such a way that this process should run here at the at the highest reaction rate. That means; the condition was to be chosen should be on the locus of maximum reaction rate, which means; what we are saying is the following.

For X m equal to 0.52 we have shown the this K must take a value of 1.97, which means; the temperature at which this is a equipment this is a reactor must be operated the temperature must at which this must be operated is T equal to 315. So, it is corresponding to if this is 315 then, we can get 0.52. Which means; if 0.52 is the

conversion which you must operate then, you have to choose temperature to be 315 so, as to satisfy the criteria that X equal to $k_1 \tau$ by $1 + k_2 \tau$, ok. So, is that means if you want to choose a temperature of T equal to 315, how do we get this? We get this by recognizing that this you can get 315 if we satisfy the energy balance which means; what the whole procedure says is that the maximum reaction rate locus defines temperature. And, energy balance defines the heat load to be handling so, as to achieve this kind of operation, ok.

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The image shows a whiteboard with handwritten notes and a small video inset of a speaker. The notes include a reactor diagram with an input flow v_0 and an output conversion $X=0.52$ at temperature $T=315$. The unknowns are residence time τ and heat load Q . The material balance is given as $F_{A0} - F_A + r_A V = \text{Accumulation}$. The steady-state material balance is $F_{A0} - F_{A0}(1-X) - (k_1 C_A - k_2 C_B) V = 0$. The simplified material balance is $F_{A0} X - (k_1 C_{A0}(1-X) - k_2 C_{A0} X) V = 0$. The simplified rate balance is $X - (1-X)k_1 \tau - k_2 \tau X = 0$. The inset video shows a man speaking.

So, have I said that let us see how we can get forward? Now, the question is that we have this reactor now, to which feed is coming in it. It is you know and this X is given as 0.52, and we have said for this X to be then, the T should be 315. So, what are the unknown quantities now? We know v naught, but we do not know that the residence time here. So, residence time τ is unknown, there is 1. And similarly, the amount of heat to be removed Q is also unknown. Therefore, once we specify τ and Q the problem is fully solved. How do we do that? To get this, we look at the material balance.

What was material balance tell us? Material balance tells us the input minus of output plus generation equal to accumulation and that steady state accumulation is nil. So, what is F_{A0} and F_A ? So, we can put all these numbers here, F_{A0} minus of F_A is simply F_{A0} times 1 minus of X , and what is r_A ? By definition is $k_1 C_A$ minus of $k_2 C_B$ times V equal to 0 . So, we can simplify this recognizing that C_A equal to C_{A0} times 1

minus of X. And, C B equal to C B 0 plus C A 0 X and then, C B 0 is 0. So, putting all these simplification; we can get this is so, F A 0 this is become F A 0 X, first term minus of k 1 C A 0 times 1 minus of X minus of k 2 C A 0 X times V equal to 0. We divide throughout becomes X minus 1 minus of X times k 1 tau minus of k 2 equal to 0.

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Material Balance

$$X(1+k_1\tau+k_2\tau) = k_1\tau$$

$$X = \frac{k_1\tau}{1+k_1\tau+k_2\tau}$$

$k_1(315) = 2.34/\text{hr}$
 $k_2(315) = 10.8/\text{hr}$

$k_1(315) ; k_2(315\text{ K})$

$$0.52 = \frac{2.34\tau}{1+2.34\tau+10.8\tau} \Rightarrow \tau = 0.11/\text{hr}$$

So, we can simplify this as will be write here. We get x equal to X multiplied by 1 plus k 1 tau 1 plus k 2 tau equal to k 1 tau, which comes directly from here. I am this collecting the coefficient of X so, X multiplied by 1 plus k 1 tau plus is it write, is this 1 minus of X is it all right, X just a minute minus is plus ok, fine. So, it is so we get X multiplied by 1 k 1 tau equal to k 1 tau. So, this is taken to as a plus k 1 tau is fine. So, this is gives you X equal to k 1 tau divided by 1 plus k 1 tau plus k 2 tau. Now, k 1 at 315, and k 2 at 315 k; we can get from the data that is already given. Therefore, on the in this equation only unknown is tau, because k 1 at this temperature 315 and k 2 at this temperature is also is given in the table.

Therefore, you can calculate on the, if you given X, you can calculate tau. So, putting numbers access 0.52, and k 1 and k 2 at k 1 at 315, and k 2 at 315, this is numbers I have calculated. And then, I will tell you this numbers k 1 at 315 and k 2 at 315, k 1 at 315 is 2.34, k 2 at 315 is a 10.08. So, I calculated these numbers units are per hour all this at per hour. All the data is per hour, I hope let this k 1, k 1 is an per hour, so per hour everything is per hour. So, you can put these numbers so, what we get is that k 1 is 2.34

tau divided by 1 plus 2.34 tau plus 10.8 tau. So, if we solve this we get tau equal to 0.11 per hour. So, this is material balance.

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v_0
 C_{A0}

$T=315K$
 $X=0.52$

Optimum conditions to achieve $X=0.52$

$$\frac{X_m}{1-X_m} = \frac{1}{(1+1/K_5)}$$

$$\frac{0.52}{1-0.52} = \frac{1}{(1+1/K_5)}$$

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What we have done? See what we have done is we have this is our problem. Our problem is we have a stirred tank, where this exit conversion is specified it is also said we must operate along the locus of maximum reaction rates which means; that we should have X_m by 1 minus of X_m satisfying this relationship. So, putting the value of X_m which is specified as 0.52; we can get $k_1 \Delta$. Δ is also known, because Δ is E_1 and E_2 are known. Therefore, k_1 therefore, all the numbers are known, and on that basis from the material balance which is given by this equation. You can find out the residence time provide the others are known, in this case others are known. That gives you a residence time of 0.11 per hour.

So, our problem now is that the residence time tau is known. In this tau is equal to this is known so, flow rates are known, all these are known, X is known here. So, only unknown quantity now is; how much heat to be added or removed to maintain the temperature here as 315 degree C 315 degree K. How do you do this? Now, we look at the energy balance, and see what the energy balance says; as far as maintaining this kind of temperature.

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$V = v_0 \tau = (0.6)(0.1) = 0.06 \text{ m}^3$
 $= 0.06 \text{ m}^3 / \text{hr} \times \text{hr}$

Energy Balance for CSTR

$$V C_p \frac{dT}{dt} = v_0 C_p (T_0 - T) + (r_1 - r_2) (-\Delta H_1^*) V + Q - W_s$$

(SS)

$$0 = 0.6 \frac{\text{m}^3}{\text{hr}} \cdot 1000 (40 - 42) + F_{A0} X (-\Delta H_1^*) + Q$$

$$+ (0.6)(1.66)(0.5)(20000) \times 0.52 + Q$$

$$Q = -3158 \text{ kcal/hr}$$

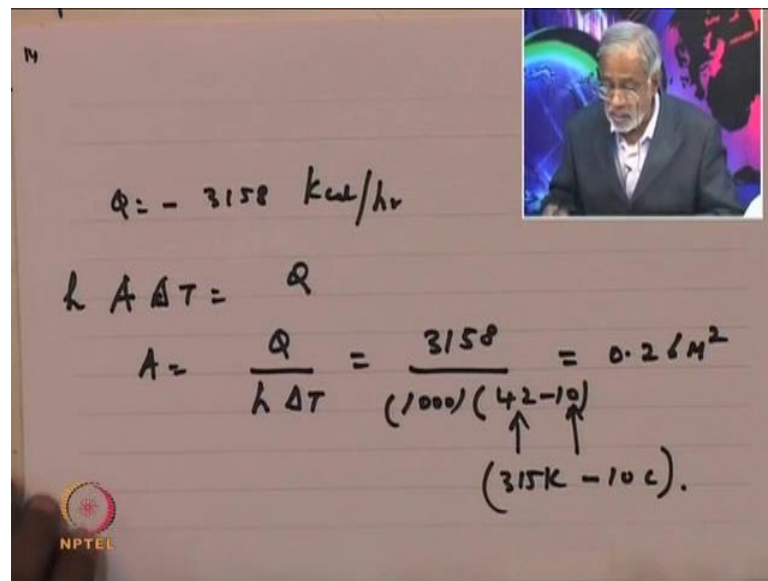
So, what is the energy balance saying? Energy balances for a stirred tank; because this is a stirred tank. So, we write the equation that we know; $d T d t$ equal to v naught $C P$ times T naught minus of T plus r_1 minus of r_2 times minus of ΔH_1 star times V plus Q minus of $W S$. So, in this case steady state therefore, this goes to 0, so r_1 minus of r_2 so everything is known here excepting Q . So, if you put all our numbers; let us do those numbers so, it is 0 equal to 0.6 and this is cubic meter per hour. $C P$ is 1000 and this is 40 minus of 42, T naught is given as 40, is it right. T naught is given as 40, ok.

And, then r_1 minus of r_2 this from our material balance we know it is simply $F A_0$ times X . I will not show this now, so it is simply $F A_0$ is what C_0 which is 0.6. C_0 is so this is simply $F A_0$ times X times minus of ΔH_1 star times V plus Q . Now, what is V ; reactor volume is equal to volumetric flow rate time's residence time. Volumetric flow rate is given as 0.6. And, then residence time; we calculate it is point 1 so, this is becomes 0.6 cubic meter per hour, is that clear.

So, we have got volume reactor volume to be 0.6 cubic meter not per hour, 0.6 cubic meters this is 0.6 this is equal to 0.06 cubic meters. So, we can put this in here and then, only unknown here is Q you can put all the numbers, I put most of the numbers. So, let us say this is 0.6 times 1.66 times 0.52 times plus 20000. And then, 0.52 is also to be put here. And, so essentially what we are saying is r_1 minus of r_2 times V is a $F A_0 X$ if is this not there, plus Q . so, this is gives you value of Q equal to minus of 3158 k cal per

hour. What is minus sign mean in the first law convention? Minus sign means; the heat is going out of the system, heat input is taken as positive in first law. Heat output is therefore, the negative sign means so much of heat will have to be removed, so as to be able to maintain the temperature of 315 degree C. So, the most important thing here is that we have been able to specify; what is the temperature at which we must operate? So, that we get conditions corresponding to the maximum reaction rates.

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The image shows a handwritten calculation on a slide. At the top right, there is a small inset video of a man with glasses speaking. The main text on the slide is as follows:

$$Q = - 3158 \text{ kcal/hr}$$

$$h A \Delta T = Q$$

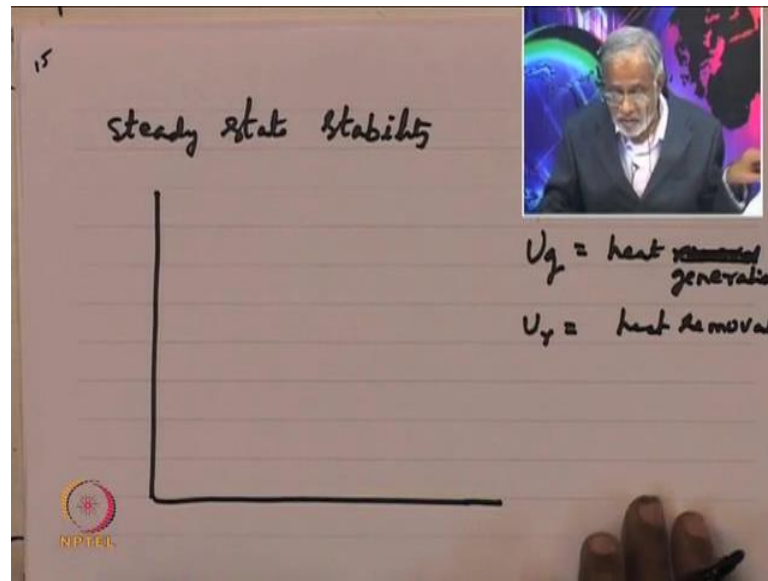
$$A = \frac{Q}{h \Delta T} = \frac{3158}{(1000)(42-10)} = 0.26 \text{ m}^2$$

Below the denominator, there are two arrows pointing to the values 42 and 10, with the text $(315\text{K} - 10\text{C})$ written below them.

Having done this, there are few simply calculation we can do after all. The design you will require you would specify things properly. So, if Q is equal to minus of 3158 kilo cal per hour so, what is the surface area heat area we have to give? So, heat transfer area is what h A times delta T equal to Q, Q is given therefore; A equal to Q divided by h delta T, correct. Q is 3158 h is thousand, what is delta T? Our delta T is temperature is 315 which is 270 and then, 315 means 42. So, it is delta T is 42 minus of 10; that is equal to 0.26 square meters, ok. This is given, this you have found out as 315 k, and this temperature is given as 10 C. So, I have that is how it becomes, what it is. So, we have we will able to calculate what is the heat load and what is the heat transfer area.

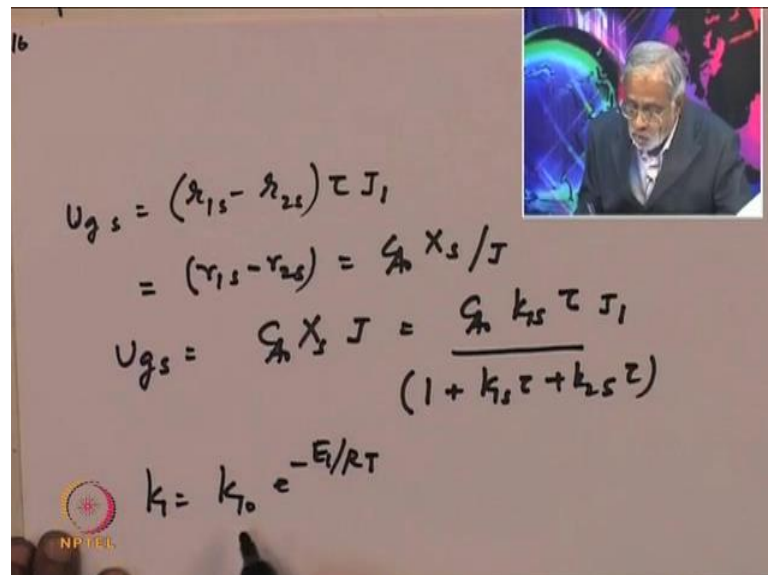
Now, the second related question in all these is to be able to see whether the process that we are running will be stable or unstable. On other words; you must check for the stability of our process, for various kinds of disturbances. That is what you would like to do know how stable is our steady state.

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So, to be able to do this; what is called as steady state stability? What we try to do is; we try to plot what is called as U_g and U_r . What is U_g and U_r ? U_g is heat generation and U_r is heat removal. And, our steady state is about the quality of heat generation, heat removal. When they are equal, we know the process is at steady state.

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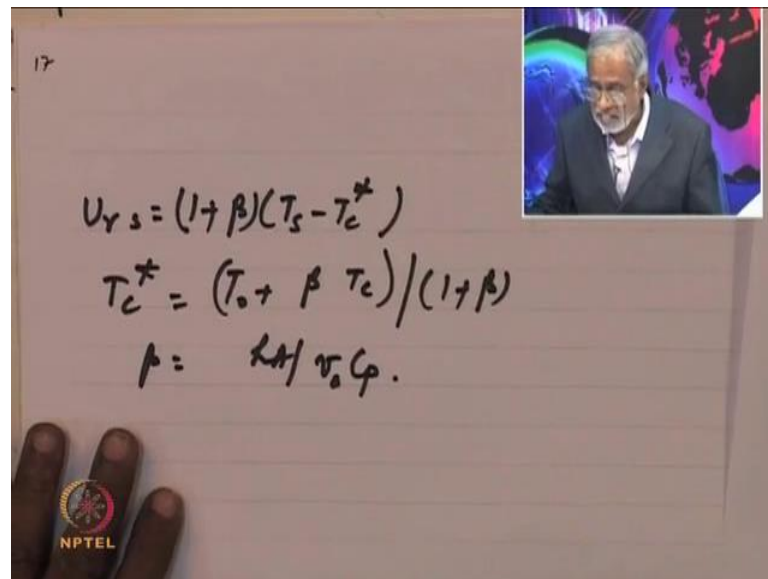


We can plot these curves U_r , just write down what is U_r and U_s ? Something we have done before is not a very complicated thing anyway. So, U_r is r_{1_s} minus r_{2_s} multiplied by τ and J_1 , ok. And then, it is we can show this is no point to doing, it

once again we know that $r_1 s$ minus of $r_2 s$ is equal to $C A 0 X s$ divided by j . I will not show this is because you already done this before. Therefore, $U g s$ equal to $C A 0 X s$ times J , which is equal to $C A 0 k 1 s \tau$ divided by $J 1, 1 \text{ plus } k 1 s \tau \text{ plus } k 2 s \tau$.

These things we have derived before therefore, this is not do it again. Notice here, that if you want to plot $U g s$; all that you need to know is what is the temperature corresponding to this $X s$. Or, in other words; you want to plot $U g s$, on this right hand side $k 1 s$ is a functional temperature. And, $k 1 s$ we know $k 1$ is what some $k 1 0 e$ to the power of minus of $E 1$ by $R T$. And, therefore since; the right hand side fully known $k 1$ is known therefore, you can plot $u g$ as a function of temperature.

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$$U_{r s} = (1 + \beta)(T_s - T_c^*)$$

$$T_c^* = (T_0 + \beta T_c) / (1 + \beta)$$

$$\beta = hA / v_0 C_p.$$

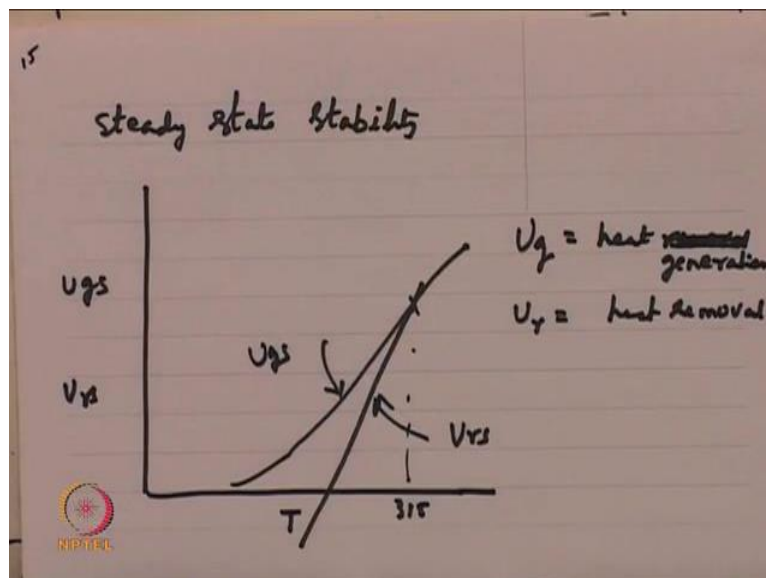
Similarly, what is $U r$? Our $U r$ by the $U r s$ equal to $1 \text{ plus } \beta$, we have done all these. Therefore, I am not doing it again $T s$ minus of $T C \text{ star}$, were $T C \text{ star}$ equal to $T \text{ naught}$ plus $\beta T C$ divided by $1 \text{ plus } \beta$. So, all these we have done where β is equal to $h A$ by $v \text{ naught } C P$ all this is known. So, what we are saying is that; we can now plot $U g$ versus temperature; we can now plot $U r$ versus temperature. Therefore, the points at which they intersect are the points at which steady state occurs. So, this is what we are trying to say, and you can plot this. How do you plot this? Show you how to plot this they are not difficult to do, but I just do some calculations to illustrate how this plotting is to be done.

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T	293	303	315	323
h_1				
h_2				
$h_1 \tau$				
$h_2 \tau$				
X_s				
$U_{gs} = C_{A0} X_s J_1$	3.32	9.9	17.2	26.1
$U_{rs} = (1+\beta)(T_s - T^*)$	-15.6	-1.29	15.9	41.6

So, let us say T and k 1, say k 2 so, k 1 tau, k 2 tau, X s, U g s and U r s. What is U g s? $C A_0 X s J_1$, what is U r s? $1 + \beta$ times $T s$ minus of $T C$ star. And, all these are known so, I will just put some say 293; I will just 21 or 2, 293, 303, 315 and 323. I am just calculating some numbers; see we put all these calculations we can find at 303, 293 these terms out to be 3.32 minus 15.6 and then, that is 293, 303 is 9.96 minus 1.29. Then this is 17.2 comes out the 15.9 and then, 26.1 and 41.6.

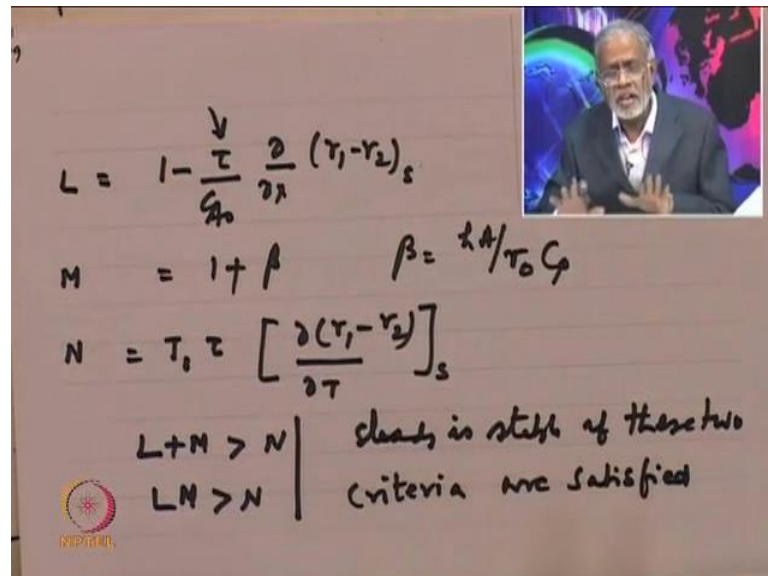
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And, other words; what we are saying is that; you can now plot see this data here, you can see here this data got this data here. U g s you can plot U g s versus temperature and then U r s. You can plot U g s versus temperature. This curve looks something like this and then, U r s versus temperature like this. And, this is the point of intersection, which is 315. So, what we are tried to say here is the following; if you have a stirred tank, in which exothermic reaction is taking place. And, then you have to operate this process at the maxima and reaction rate, which means; it specifies the temperature at you operate and so on.

Now, you can also understand the steady state by plotting U g s, this is U g s or U r s this is U r s. Or, in other words; the pointed which the intersections takes place is the point of steady state. So, we can understand steady state in various ways this is a another way of understanding steady state. Our interest now is; now that we know it is that is point of intersection; how stable this steady state is this stable, is this unstable. Now, to understand stability of steady states we have done this already. I will just quickly run throw this procedure for understanding stability of steady state.

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$$L = 1 - \frac{\tau}{C_{p0}} \frac{\partial}{\partial T} (T_1 - T_2)_S$$

$$M = 1 + \beta \quad \beta = \frac{hA}{v_0 C_p}$$

$$N = T_0 \tau \left[\frac{\partial (T_1 - T_2)}{\partial T} \right]_S$$

$L + M > N$
 $LN > N$

steady is stable if these two criteria are satisfied

We have shown in an earlier discussion that and exothermic stirred tank stability can be understood in terms of 3 numbers. L M and N where, L is defined as 1 minus of tau by C A 0 del by del X of r 1 minus of r 2. M is equal to 1 plus beta where, beta is h A by v naught C P and then, N is equal to J 1 tau within brackets del r 1 minus of r 2 divided by

del t at steady state, ok. And, stability is L plus M greater than N, L M greater than N. So, steady state is stable if these 2 criteria are satisfied. So, what is L M and N these are all number which is determine by what is given here, at residence time here is known C A 0 is known r 1 minus of r 2 at s all these are known.

So, for a given reaction kinetics L M and N is fully specified. So, what you want to do now is to illustrate to you, how you can actually calculate L M and N for this specific problem that we have in front of us. I want to calculate this in front of you. So, let me go throw this whole thing quickly. So, r 1 minus of r 2 for example; for us to calculate L M and N, we should know r 1 minus of r 2 so, that we can differentiate with respect to X and T. And, find out the value of these numbers at steady state that is what I am trying to do.

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$$r_1 - r_2 = k_1 C_{A0}(1-X) - k_2 C_{A0} X$$

$$\frac{d(r_1 - r_2)}{dX} = (-k_{1s} - k_{2s}) C_{A0} = -(k_{1s} + k_{2s}) C_{A0}$$

$$L = 1 + \frac{\tau}{C_{A0}} (k_{1s} + k_{2s}) C_{A0} = 1 + k_{1s} \tau + k_{2s} \tau$$

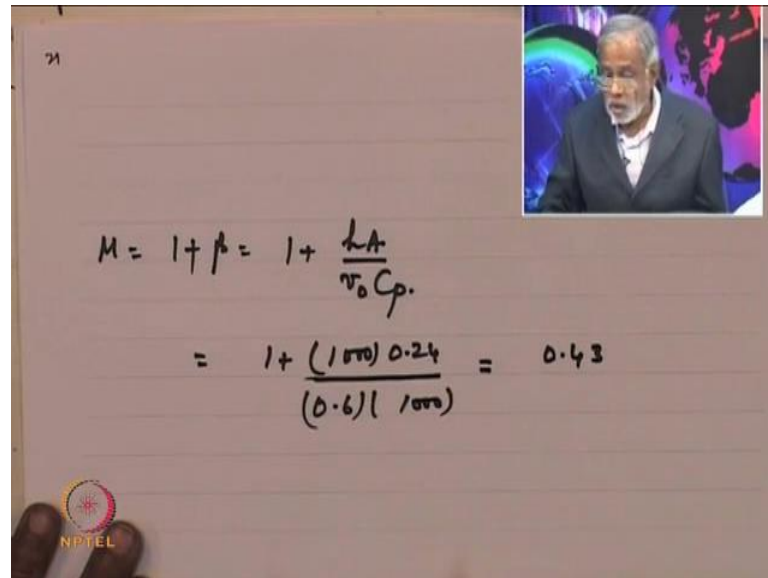
$$= 1 + (21.3)(0.11) + (10.08)(0.11)$$

$$= 4.45$$

Now, r 1 minus of r 2 is k 1 C A 0 1 minus of X minus of k 2 C A 0 X. So, del r 1 minus of del X equal to k 1 S with a minus sign minus k 2 S multiplied by C A 0, these obvious from here, ok. So, that is equal to minus of k 1 s plus k 2 s times C A 0. Therefore, what is L; we have to calculate L here; we want to calculate L, because tau is known everything else is known. So, let us calculate L; so L is equal to 1 plus tau by C A 0 within brackets of k 1 s plus k 2 s, or this is equal to that is multiplied by C A 0 is there equal to 1 plus k 1 s tau plus k 2 s tau. So, k 1 s is known k 2 s is known tau is known therefore, L can be calculated. So, will have put numbers if you have put these numbers;

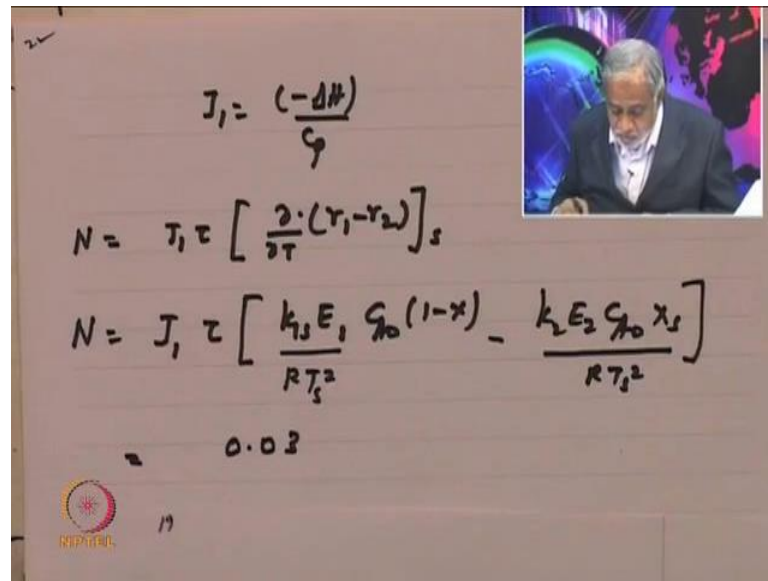
this becomes 1 plus 21.3 times 0.11 plus 10.08 into 0.11 that is equal to 4.45. The value of L for this particular problem, where tau is 0.11 per hour, k 1 and all that we have calculated L is 4.45, ok.

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$$M = 1 + \beta = 1 + \frac{hA}{v_0 C_p}$$
$$= 1 + \frac{(1000) 0.24}{(0.6)(1000)} = 0.43$$

Now you have to calculate M. What is the M? M we said M is equal to 1 plus beta which is 1 plus h A by v naught C P. That is equal to 1 plus h is let me put this numbers here, calculated it here somewhere else. H is 1000, and we are calculate this area just now we calculate area is 0.24 square meters divided by v naught C p. v naught is 0.6 and C P is 1000 kilo calories per cubic meter and so on, that comes out to be 0.43. So, L please notices here, M is this.

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$$J_1 = \frac{(-\Delta H)}{C_p}$$

$$N = J_1 \tau \left[\frac{\partial(r_1 - r_2)}{\partial T} \right]$$

$$N = J_1 \tau \left[\frac{k_1 E_1 C_{A0} (1-X)}{RT_s^2} - \frac{k_2 E_2 C_{A0} X_s}{RT_s^2} \right]$$

0.03

Now, you have to calculate M, this is J 1 how to calculate N? N equal to J 1 tau del by del t of r 1 minus of r 2 at s. So, when we do this differentiation this becomes N equal to J 1 tau del by del T becomes k 1 s E 1 C A 0 times 1 minus of X divided by R T s square that is 1. And, the other side is k 2 E 2 C A 0 X s divided by R T s square. Now, once again all these are known so, we can calculate this we can put all the numbers I just put all the numbers here. J 1 notice here, the J 1 by definition is minus of delta H divided by C p volumetric. I am putting all the numbers here, and then we do that it terms out to be 0.03. I put all the numbers here this is whole calculation terms of 0.03. Now, our steady state stability criteria our criteria of a stability; L plus M greater than N, value of N is 0.03 pleas N is 0.03.

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$L = 4.45$
 $M = 0.43$
 $N = 0.03$

$L + M > N$
 $LM > N$

Both the criteria are satisfied. Steady state is stable.

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And, L value we calculated as; L is 4.45 and, so let me let me put all the numbers together in front of you. We get L plus M so, L is summarizing here L equal to 4.45, M equal to M is 0.43, N equal to 0.03. So, L plus M greater than N, L M greater than N, we can see here, both the criteria both the criteria are satisfied. Which means what? Which means; that the steady state that we have chosen steady state is stable. By stability what we mean is that; to a disturbance the steady state would returns to its virginal state, because is the after the disturbance. By these criteria; what we really mean is that after disturbance it will take time to reach the steady state but it will reach that old steady state. That is the point this is mean by this is satisfying of these 2 criteria, ok.

Now, the question is that; this whole analysis is based on what is called as small deviations from steady state, which means that; our disturbance should not be so large. That the linear approximation that we are assumed in this steady state analysis that should be violated. So, small disturbances these are all correct; but if the disturbances are very large then, clearly our approximations in the analysis not satisfactory. There were alternative techniques should be required, I will stop there.

Thank you very much.