Medical Image Analysis
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Lecture 03

Magnetic Resonance Image Acquisition

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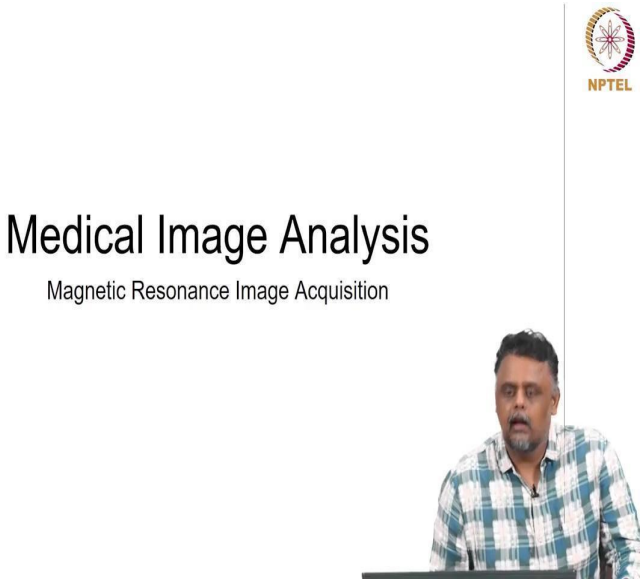

Hello and welcome back. So, in this class, we are going to talk about magnetic resonance image acquisition hardware and so-called Pulse sequences, just to give you an understanding of how images are acquired. We will also understand some of the contrast mechanisms better.

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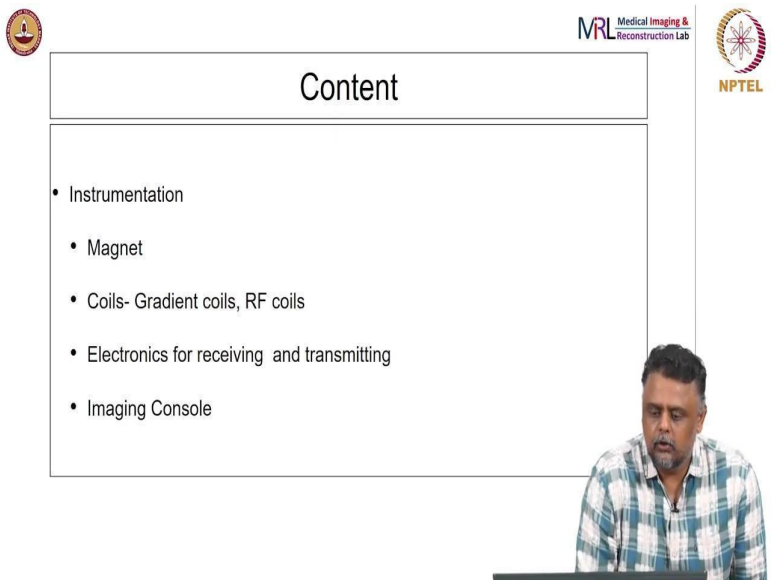

So, this is the overview of the class, we will look at some of the hardware that goes into MR imaging systems. I call it instrumentation but we will only be looking at it superficially, in a sense, just seeing what the components are. So, we will look at the main component, the magnet, the gradient coils, the radiofrequency coils, and the electronics, it contains, of course, we will not go deep down into electronics but just saying that these are some of the parts of the scanner and the imaging console.

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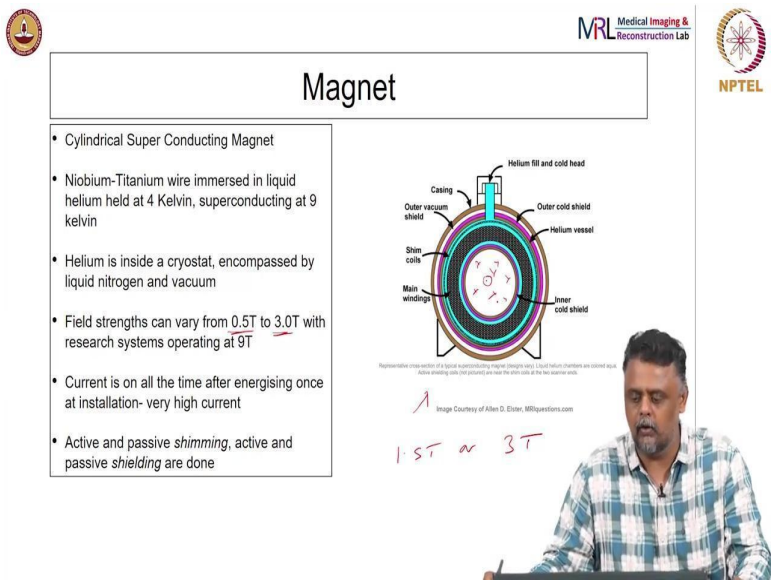

So, as far as the magnet is concerned, it is the main component and this is what is required. It is also the most expensive among the components. So, it is typically a superconducting magnet, a cylindrical superconducting magnet, and a cross-section is shown in this figure if

you see the cross-section of the magnet and it is, it basically has a superconducting wire, which in this case is niobium-titanium wire immerse liquid helium which is held at 4 Kelvin, but that particular wire is superconducting at 9 Kelvin.

The helium itself is inside a cryostat composed of a vacuum and liquid nitrogen. So, these magnets are required, you do not have to keep filling liquid helium often but liquid nitrogen refilling is required. The field strings vary from about 0.5 T to 3 T. There are of course, 9 T magnets also available, some of the higher field magnets are used in small animal imaging, for and such, biomedical imaging, and also as research magnets. So, the interesting part here is that once it is basically the wire is wound around a cylindrical core.

So, it is just say energize once, so as soon as the critical temperature is reached energize it once, which means that you inject current into it. And the current keeps on going because it is superconducting there is not much loss due to resistance, of course, over time, all of it will dissipate. So, this leads to the magnetic field being a statistical magnetic field being set up. So a static magnetic field will point along the axis of the core of the magnet, and it is typically like I said, between 0.5 to 3 T, modern clinical magnets are either 1.5 T, or 3 T.

So, I am not sure where these numbers come from, but I guess these are what is possible, given the constraints of size etc. So, like I said, when it says 9 T magnets also exist. So, one problem that you will also run study about is when you get into MRI, is that maintaining the uniformity of the field is a challenge, which means that across the cross section at every point, we have to make sure that it is 3 T and so on, and that is, that is hard to do, so there are some constraints like that.

So that is basically a superconducting wire wound around a cylindrical core. And once the critical temperature is reached using the liquid helium, you inject it with the current and that current keeps going giving rise to a magnetic field, that is the static magnetic field that you use for imaging.

There are now systems economical systems where people are trying to use permanent magnets and there are the portable systems etc., where they are trying to use permanent magnets or maybe you do not need such large magnets because if you are just imaging peripheral organs you do not need such large magnets, so which brings down the cost of the system. So, a lot of research is going on in that area.

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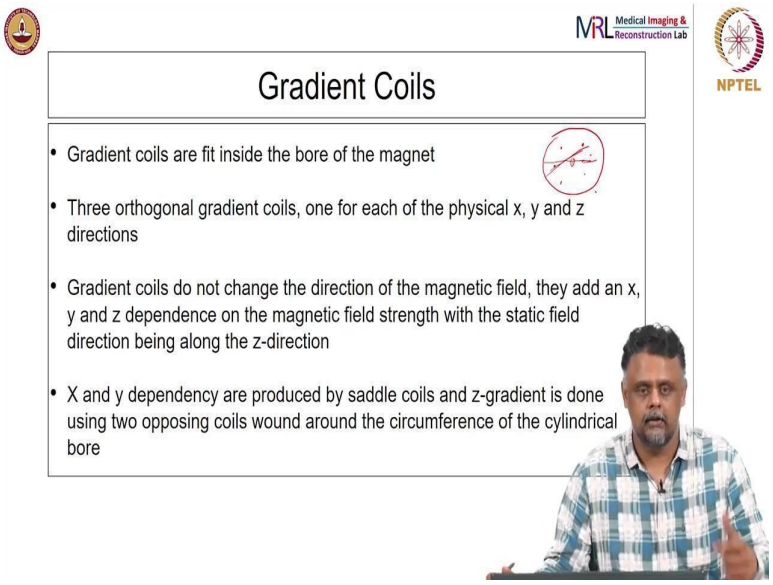

So, the gradient coil, so, we will see a little bit more about this later on. So, these gradient coils are also the basically current carrying wires, these wires give rise these current carrying wires give rise to magnetic fields. They fit inside the bore of the magnet, so there are three coils, each one of them orthogonal producing magnetic field gradients orthogonal to each other. The gradient coils do not change the direction of the magnetic field, they just add an x and y and z dependency on a magnetic field strength.

And the magnetic field which I am talking about here, the static magnetic field and still points along the z axis, the x and y dependency are produced by saddle coils and z gradient is done using two opposing coils wound around the circumference. I have some pictures I think I will show. So, what do you mean by it does not change the direction of the magnetic field?

So, we saw the, let me see if I can draw somewhere here very small. So, here this is our cross section of the core. So, the direction of the magnetic field is here. So, now we want let us say the 3 T magnets, so it will be, we would expect that let us take these random points here, we would expect the magnetic field to be 3 T everywhere. So, typically, that is true as long as we are talking about a nominal field of view.

However, normally when I say a gradient coil, what it does is the direction of the magnetic field is still the same, it is along the axis, z axis, but the value here will be different, the value which means the magnitude of the magnetic field, if you can call it that, it just creates

gradients in this. If this is the value of the b field everywhere, it just creates a gradient around each of these directions, for instance, in this case, it is along x which I have drawn here.

So, the gradient just changes the value, but not the direction of the field. The field is still along z along with static magnetic field still points along the z-axis. So, and it will not produce these gradients you need current carrying coils of different shapes configurations in order to produce these gradients. That is what I said earlier there is, for instance, x and y dependencies are produced by saddle coils, and z is done using two opposing coils found around the circumference of the cylindrical core.

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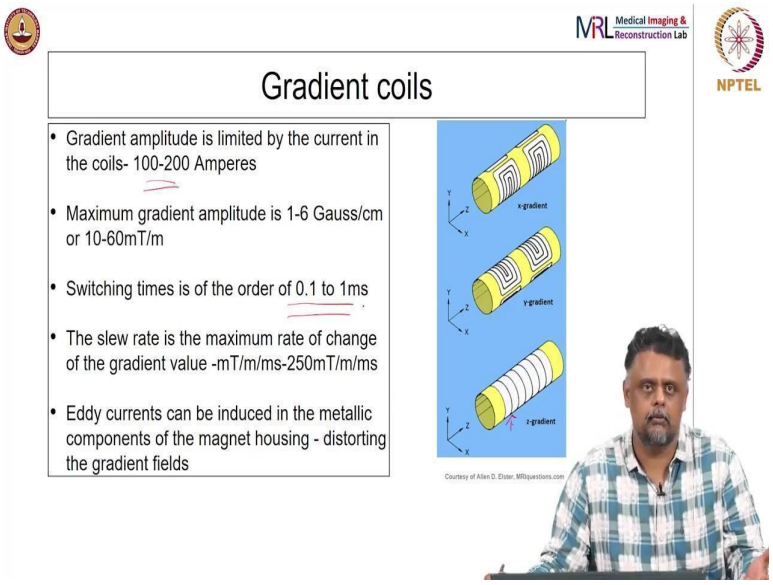

So, here is some examples right of the gradient coil. So, for instance, this is a gradient coil, which we talked about, this is two sets of wires bound in opposite directions, these are the saddle coils if we talked about kind of writing the x and y gradients. So, depending on how we make these coils the gradient is produced along each of those directions.

The gradient amplitude again is limited by the current in the coil, it is very high currents, and it is very high 100 to 200 amperes. And just to give you an idea of the order of magnitudes, the maximum gradient amplitude is 0 to 6 Gauss/cm or in this case 10 to 60 mT/m. So, I distinguish off our magnetic field. The static magnetic field is about 3 T, 1, 2, 3, T is what we talked about and this is the gradient amplitude.

The switching times this is important because when we look at Pulse sequences, later on, we will assume that these are instantaneous so this is not true, there is something called a slew rate. So, it kind of takes time for the gradient to turn on to reach the maximum value.

And because of this constant switching of currents turning on and off of current there are again Eddy currents induced in the metallic components of the magnet housing and distorting the gradient field. So, there is again shielding for this etc. So, once again, here we go deeply into hardware, these are the hardware challenges in designing a magnet for a system for imaging.

So, you can imagine that we have the cylindrical core, and you already have a superconducting wire, which is of course inside a vacuum on board say there is a vacuum so,

it does not touch anything, but, but still introducing these current carrying wires will lead to some distortion the magnetic field, and these current carrying coils are in a wound around in specific shapes, once we saw saddle coils and these are circular coils etc., and they will also produce some distorting field, how do you handle all that, that is challenging in some of the hardware design for MRI.

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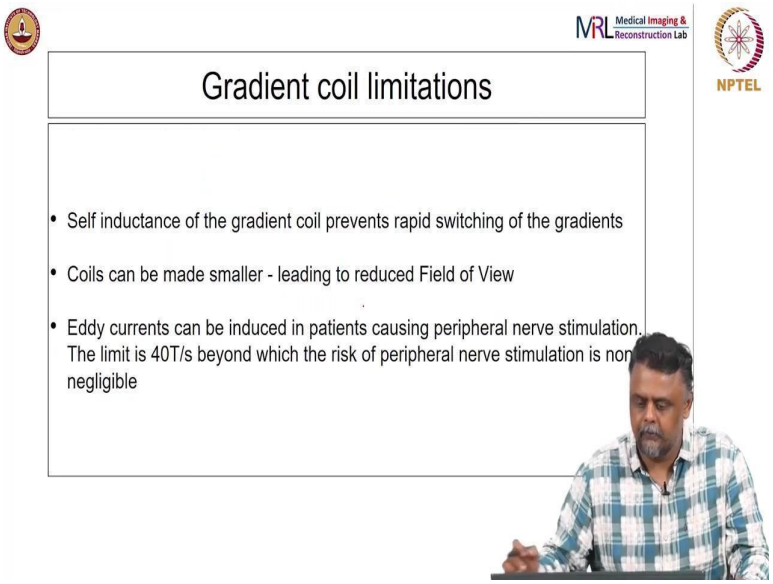

So, there are some limitations due to the gradient coil, the self-inductance of the coil, again, that is what prevents rapid switching of the gradients, and coils can be made smaller, but we made the coil smaller to reduce the field of use at a reduced rate. So, there is another actual hazard in this case to the patient because this rapid switching of the presence of currents etc. can lead to an Eddy current in the patient.

And so, there is some limit beyond which we switch speed beyond which there is some nerve damage and so that is the limitation, that you also have hardware limitation, in this case biological limitation on the hardware. So, once again these are other sets of challenges that go with designing MR Hardware.

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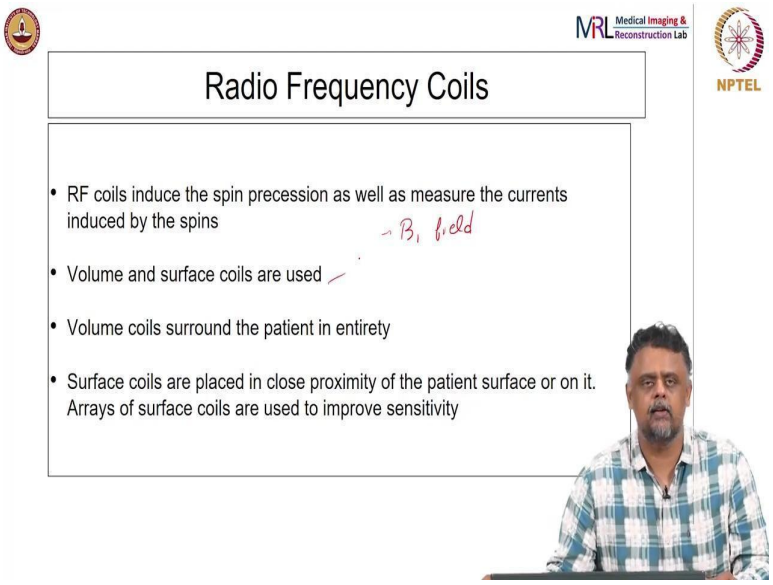

The radiofrequency coils as we have seen is basically this is these coils is where it produces that B_1 field, as we call it the B_1 field they produce the B_1 field, and this coil induces the spin precession and this coil actually also measures the current induced by the spins. So, we want to measure the echo spin. Of course, we will teach you later this is also done by the sort of coils that do both the excitation of the spins as well as the measurement.

So, basically, its axis is both transmit as well as receiver coils, there are all kinds of coils, volume coils which surround the patient surface coil you just put on top of the patient, we are basically in close proximity to the patient. And again, designing these coils, how to get the correct RF field, RF out and shape the B field magnetic etc. this is again a field of study research in itself with a lot of groups of people working on this. Once again, this is one of the hardware challenges in designing an MR scanner.

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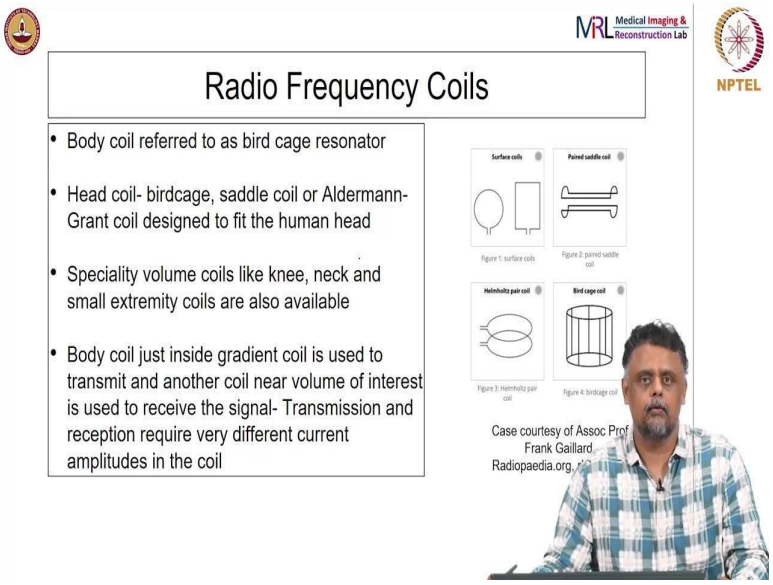

So, the radiofrequency coils again come with different configurations. I have listed some of them here. So, if you are into RF transmission etc. you should read up more about this and how to design these coils, what each one of them produces, and how to acquire electronics for these things. So, just to understand these frequencies, these RF coils once again can surround the patient or can be just in close proximity to the patient and they can be used for both transmitting and receiving RF.

The transmitted Rf is what causes the spin precession and there the induced currents are measured by the received coils. So, again, there are surface and volume piles as I said earlier. And the other thing to also keep in mind is that the transmission and reception require very different current amplitudes in the coil, because the emitted current is kind of very small, we are talking about a the spins in our body causing a current and those currents are typically much smaller than the B_1 field related currents. So, you have to worry about that also.

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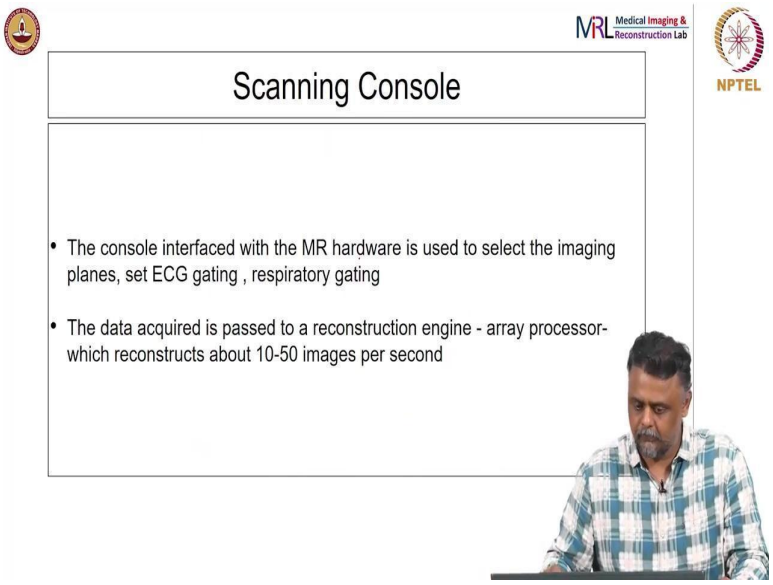

So, there is typically a scanning console like all other diagnostic devices. So, like for reference, a CT also has one. So, the console helps you interface with the MR hardware, it helps to select the imaging planes, set ECG gating, respiratory gating, etc. So, also connected to the reconstruction engine so that the acquired data is then used by the reconstruction, just maybe it is a processor, embedded system, or maybe just another workstation, which can reconstruct maybe 10 to 50 images per second.

So, MR is a real time imaging system, it just, it might, it can be an imaginary real time imagining system, it does take some time to set up but you can do very fast frames. For instance, one application of MR imaging is cardiac CT, MRIs, specialized hardware for it, you can actually image the entire volume of the heart in one cardiac cycle. So, several volumes can be acquired in one cardiac cycle.

So that kind of speed is available, of course, it comes with this as a separate protocol that has been installed in your scanner. But again, once again, this reconstruction is real time and can do 10 to 50 images or maybe more these days.

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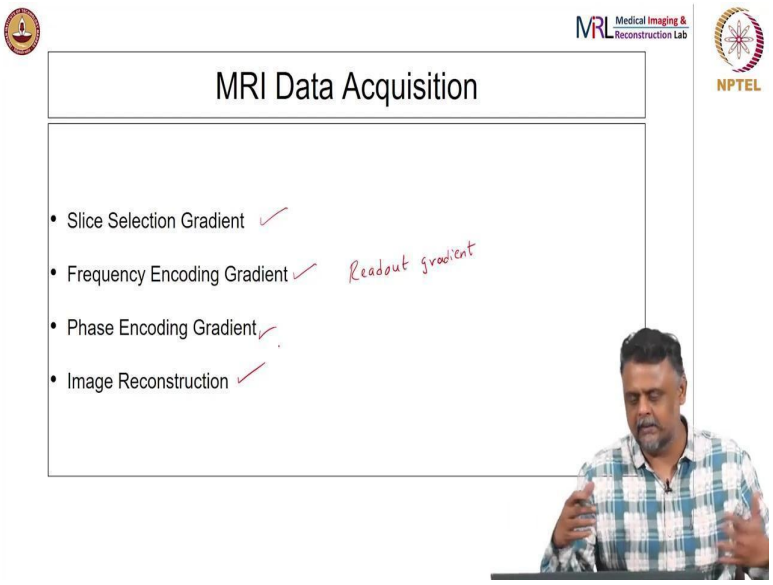

So, let us go back to how we actually acquire data. So, we saw the principles of MR signals, how they are produced, and magnetic resonance, and what does it mean? Or nuclear magnetic resonance in this case, or what, how do we understand it? How do we model it etc? But then, we are now going to look at in the context of imaging, how images are acquired using magnetic resonance, and nuclear magnetic resonance phenomena and that has to lead to something called Pulse sequences.

So, in the next maybe half hour, a few hours or so, I do not know even less will try to understand how this is implemented in an MR scanner. So, how are the image planes? How are the imaging, 3D, volumes of the patient, etc. So, what are the fundamental steps of the following, we have what is called a slight slice selection gradient, there is a frequency encoding gradient, I think this is also a read out gradient. I think this again, one more terminology.

Once again, if you are not familiar with the terminology, it is fine. If you want to ever get into this research, you should know but otherwise, it is just followed along. There is a phase encoding gradient and followed by image reconstruction, this is the four steps. So, what does each of these do and how are they used? That is what we are going to see. I will also present some so-called pulse sequence diagrams to understand the image acquisition process.

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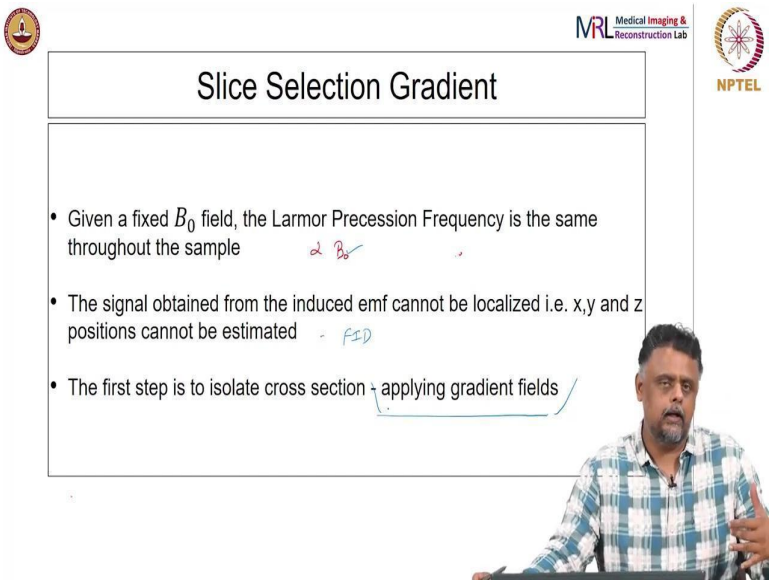

Why do we need this high selection gradient? What is it used for? So now we know that as far as the MR system is concerned, we have a fixed set B_0 field, so the Larmor precession frequency is, is proportional to be B_0 so it is the same throughout the sample. So, when we apply the RF pulse it tilts that it kind of tilts the static magnetization into the plane, and that the precession spins in the plane induces an EMF, but all of them are, the precession frequencies for all of them is proportional to B_0

And it does not really help because we will just get one FID. We saw that too we will get one FID if you can measure that, but that does not tell you where the signal is coming from, what is the spin density for instance, in a particular x, y, z, location? So, how do we go about doing this? So, the first step is to isolate the cross section by applying gradient fields, this is what we want to do, this is where the gradient coil is coming.

So, we know what the RF coil does, this is where the gradient, the gradient coils help to localize x, y and z positions, we will see how it is done.

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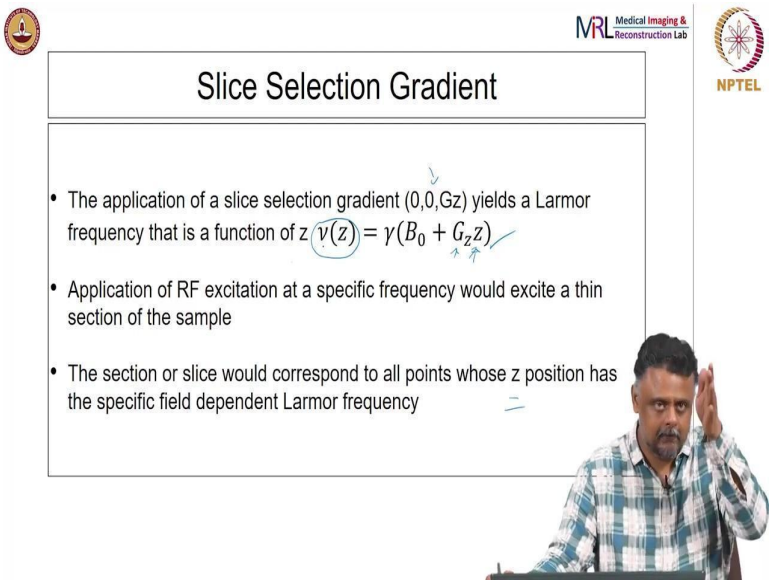

So, we apply this so-called slice selection gradient, we just basically the gradient along the z-axis. So once again, remember, the direction of the magnetic field is still along z, just at its magnitude varies by the slice selection gradient. So, what happens is, once you apply a gradient at this rate, so many T/meter or mT/meter, times z the precession frequency, this is a precession frequency becomes a function offset, you can see that here.

So, by applying a slice selection gradient in this case, that is why it is 0, 0, Gz, the x and y gradients are off, first we applied the z gradient, so it makes the Larmor pressure frequency in the volume a function of z, when we say a function of z here, this is exactly that function. So, in this case, you can see where we encoded the frequency as a function of position. So, now once we have this, at the same time, we apply the RF excitation process.

Now, since the RF excitation is at a specific frequency, there is a bandwidth we will see on the next slide, there is a bandwidth associated with the RF pulse, so it will only excite tissue in a thin section. And, what is that thin section? That thin section would correspond to all the points who's z position is the specific field dependent Larmor frequency.

So, in this case, your RF pulse has a certain center frequency and bandwidth. And whichever z position corresponds to that, center frequency and bandwidth, because now we know v is a function of z, only those pins will be flipped into the plane. So, that is the idea behind the introduction. So, this means that we are now only looking at a certain cross section of tissue.

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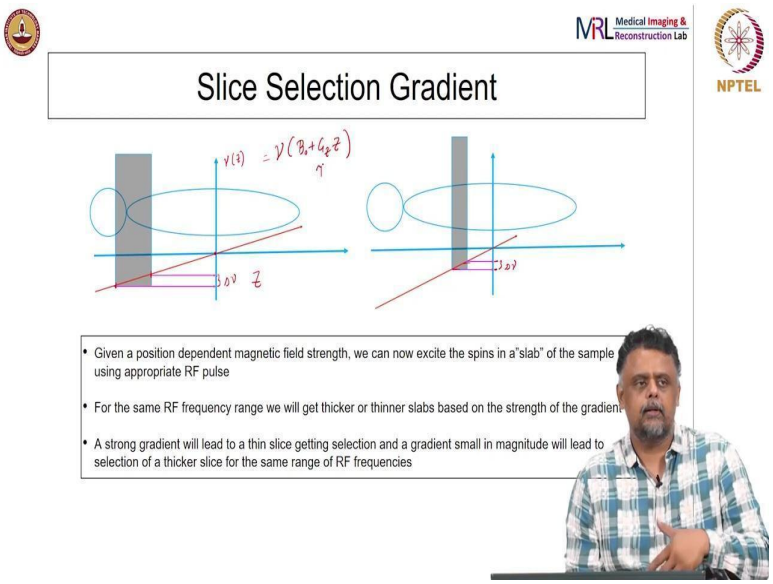

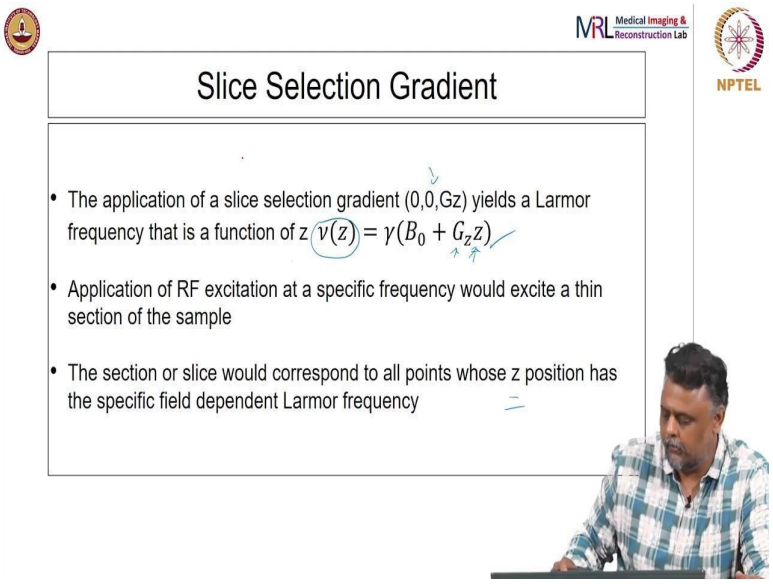

So, if we look at it better. So, if we look at this as a mockup of the patient lying on a bed, this red line is the z-axis. So, if we look at this mockup illustration, the x-axis just indicates the direction z along the length of the patient you are looking along, so this is just a side view; think of it like the side view, which is the precession frequency. This is actually a function of z, which is what this implies.

The red line that you see there shows the strength of the gradient, which means that as it moves along z on either side, my precession frequency slightly changes because of the presence of the gradient. If you go back and look at this expression, it is $\nu(z) = \gamma(B_0 + G_Z Z)$.

So, at this point, the gradient is 0, let us say and as you move to the right and the left, see the precession frequency keeps increasing. So, which is what is shown in this picture? So, the red line indicates the strength of the gradient; if it is a shallow slope, it is a very weak gradient; if it is a sharp slope, that is a strong grain, which is shown here.

Now, what does it do? Because it is shallow, going a small delta, this is the delta precession frequency, leading to a vast slice. Because the amplitude of the gradient is small, a very thick slice of tissue will have spins with pretty much the same precession frequency, which is what that formula also tells you.

So, but if you make an extreme gradient, which is what is shown here, then if you go from in a small this is the bandwidth, if you can call it in your RF pulse, that leads to a very thin as much thin as slice, for the same bandwidth, δνwe get a thinner slice because the gradient is much sharper, which means that if you go a very small delta z there is a big change in the precession frequency.

But that is what the gradient does because if G_z it is very large, let us say, a very large number, then if you move a very small distance along z, your precession frequency changes very fast that is what that is that gradient does. So which means that if you have a very small bandwidth in your radio frequency signal, that will select only a very small or thin slice of tissue.

On the other hand, if your gradient is very small, even if you go for your precession frequency to change considerably, you have to go a very longer distance, which means a thicker slice of tissue. So, for the same bandwidth in your RF signal, you will get a thicker slice on a thinner slice, depending on the strength of your gradient, so that is what is shown by this illustration.

So, this way there are two things you can do, you can select your slice thickness, so you localize your spins, and you also help to select your slice thickness. So, this is the idea behind the slice selection gradient. So, slice selection gradient and the RF pulse are kind of turned on simultaneously, we will see that in the diagram.

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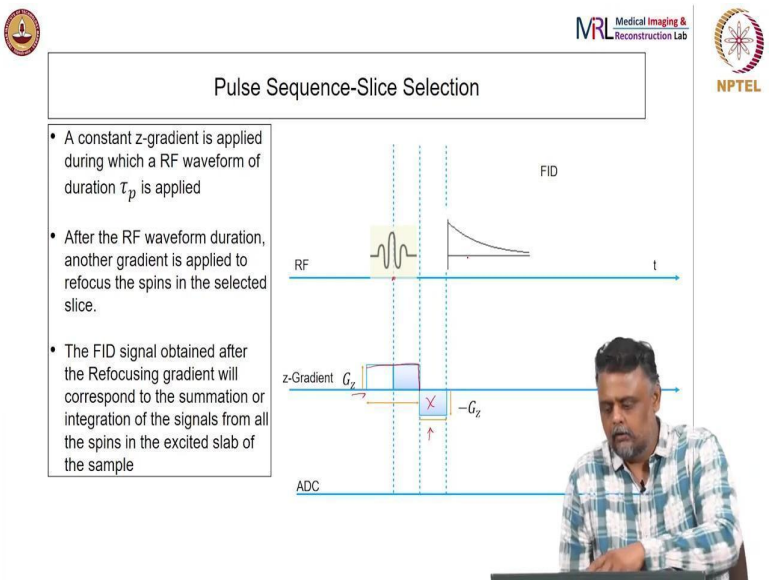

So this is a typical diagram that you will see in a lot of textbooks as well as papers etc. So, the constant jet gradient is applied, so that is what this is the time axis, the blue lines are the time axis. So, a constant z gradient is applied. Okay. And there is this, do not worry about this right now we'll talk about it later, when you get the chance at Laborde detail, this is called the refocusing gradient. So, I will just maybe tell you right now.

So, the idea is, so, once you apply these at for a specific period of time because the slice thickness is finite, and we have a finite slice thickness, there is the difference in the precession frequencies, because they are within the edges of the slice, as you go from one end of the slice to the other, there is a difference in the gradient because of the gradient and consequently, there is a difference in the pressure frequency.

So, at the end of application of this gradient, there will be a defocusing of the or defacing of these spins, so that is that is because of the difference in the magnetization or the magnetic field felt by the spins in the left side and the right side of the from the going from left side of the slice to the right side of the slice, if there is a gradient z gradient, which means that they will have slightly different frequencies which leads to the defacing.

So, to replace them, we just have to apply another negative gradient of the same amplitude but for half the time, they have brought them back into phase, so that is what this is for. So, this is just to explain the diagram in slightly better detail. So, this is when you turn on the z gradient once again, this is slightly idealistic because the z gradient cannot come on instantaneously, that is a certain time before it sets, and it comes on and it is this the time

period to which it is actually kept on. And the time t equals 0 actually starts here, which coincides with the peak of your RF pulse.

And, and then of course you have this negative gradient for refocusing gradient refacing gradient and followed by which there is the FID that is when you turn on the ADC, but if FID is actually very difficult to measure, you typically measure something called the echo, we will talk about that later, but this is the slice selection process, where you select the slice do a slice selections along with the RF pulse, you turn on the z gradient, again, the z gradient is determined by how thick which position and how you want image and how thick a slice you want image.

So, the FID signal that you obtain now, after the refocusing or a colored refacing, will correspond to the summation or integration of the signals from all the spins in this slab of tissue. So, if you go back, so whatever signal you are getting will be from here, again or from here, depending on a gradient strength, because only those will correspond to the Larmor pressure frequency, which the RF pulse will flip into the plane.

So, this is the slice selection gradient. So, even now this is just one slice, we just do not know, we still cannot figure out x, y, yet, we will just localized the signal to a particular z location or range of z location is because that is where you always talk about slice with in medical imaging, even the context of CT as well MRI, MRI because the tissue the image you are looking at even though it is flat it comes from actually a slab of tissue not just infinitely thin plane that is a bad approximation. So, the slab of tissue is what we have localized, we still have to do x and y localization.

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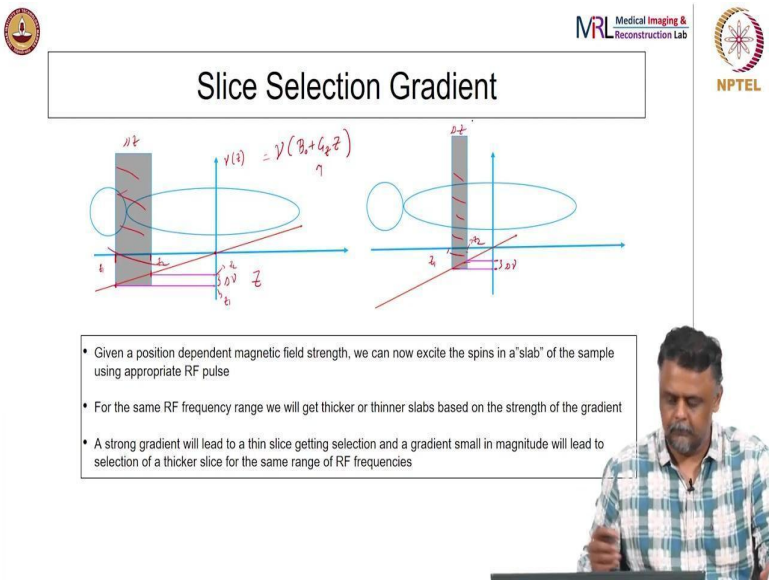

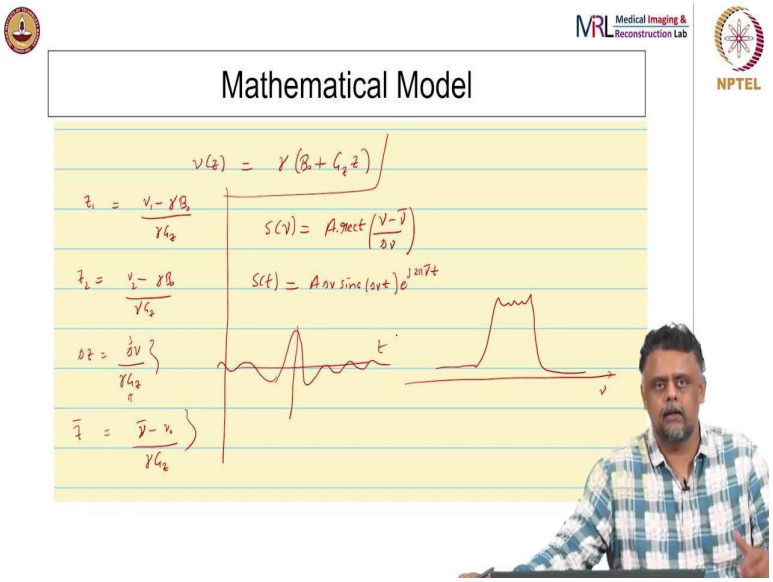

So, we will look at a couple of derivations just to understand what is going on. So, we saw that the precession frequency is a function of your $B_0 + G_Z Z$ that shows your precession frequency is now encoded by your z position at this point. So, now, if you consider two positions, Z_1 and Z_2like we saw in that picture will go back up and just show you what Z_1 and Z_2 are.

So, we can call this we can call this Z_1 This point corresponds to why I call these Z_1 and Z_2 because they actually correspond to here. So, this we call Z_1 and Z_2. So, these are the again

similarly, here you can call this Z_1 and Z_2. So, this will be some delta z this is your slice thickness.

So, now, we can actually show So, for instance, for Z_1 we can write $Z_1 = \frac{\nu_1 - \gamma B_0}{\gamma G_z}$ similarly for it to Z_2 We can write $Z_2 = \frac{\nu_2 - \gamma B_0}{\gamma G_z}$ and we can then figure out after some algebra, we can just write your slice thickness as a $\Delta Z = \frac{\Delta\nu}{\gamma G_z}$ So, this is the expression that the slice thickness tells you how we can determine slice thickness by figuring out this $\Delta\nu$ as well as by figuring out G_Z you can set the slice thickness. The slice position which is basically the average can also be written in this form, which is given by this $\bar{Z} = \frac{\bar{\nu} - \nu_0}{\gamma G_z}$ So, this is again the slice position, so all of these two are the important expressions. So, that is one set of equations.

So, another aspect is basically the RF waveform. So, what kind of RF waveform do you want? So, based on this derivation, we can say we desire a frequency signal, this frequency content is given $S(\gamma) = A.rect(\frac{\nu - \bar{\nu}}{\Delta\nu})$ Where A is amplitude and $rect$ is a function. So, which means that the signals, what do we want because we just need a rectangular pulse along the frequency axis corresponding to that particular frequency $\bar{\nu}$ of a certain width $\Delta\nu$

So, that and the signal itself you do the Fourier transform we can do that signal can be defined to be $S(t) = A\Delta\nu\, sin(\Delta\nu t)e^{j2\pi\bar{\gamma}t}$ So, this is basically what we are looking at and in this case we this is a sinc function so, we can plot it and this what we get is if you look at it the envelope of the sinc function is what I will plot.

So, which is something this is a among time axis something like this, it is a sinc function and of course, if you want the appropriate localization along the frequency axis you will have something the ideal thing to do. So, this is a longer frequency axis, this is along the time axis. So, these are the expressions which help you determine what should your RF pulse look like ideally and based on that what is your z position as well as your slice thickness. So, this is the overall model behind the slice selection gradient.

So, now we can move on and we can talk about how do we localize along the x and y direction or basically how do you do in plain localization.

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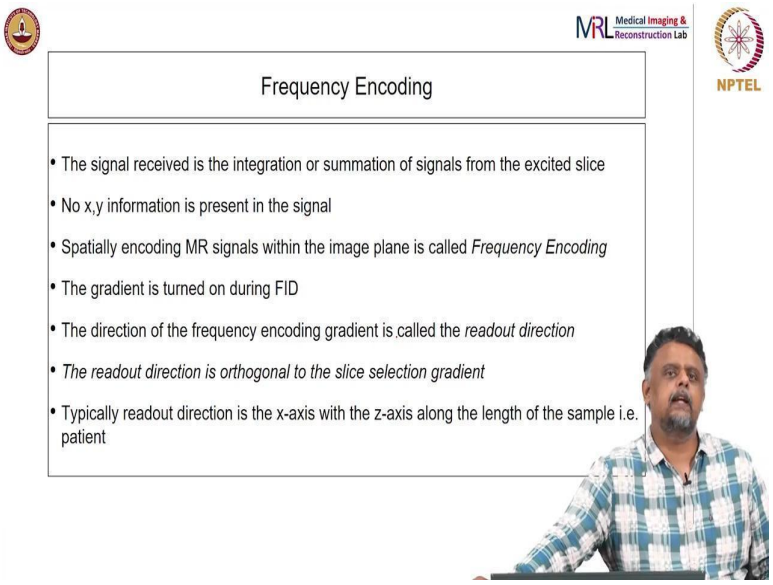

So, this is called frequency encoding, or I think sometimes it is also going to read-out frequency encoding direction is also sometimes known as the read-out direction. So, as we saw, the signal received is basically the integration or summation of signals from the excited slice. So, as we see it saw that the signal received is now restricted that is basically the spins that are flipped into the plane by the particular angle, depending on how long you have the RF pulse on is only from a certain slab of tissue, however, we do not still have localization basically no x, y information or in plane localization is present.

So, how do we spatially encode MR signals, and the way to specially encode the one fun process is referred to as the frequency encoding. In this case, there is a gradient, additional gradient which is turned on during the FID and the direction in which that gradient is turned on is referred to as the readout direction. And it is typically orthogonal to the slice selection gradient. And typically, the readout direction is the x axis, the z axis being along the length of the patient. And that is the axis along which we apply the slice selection gradient.

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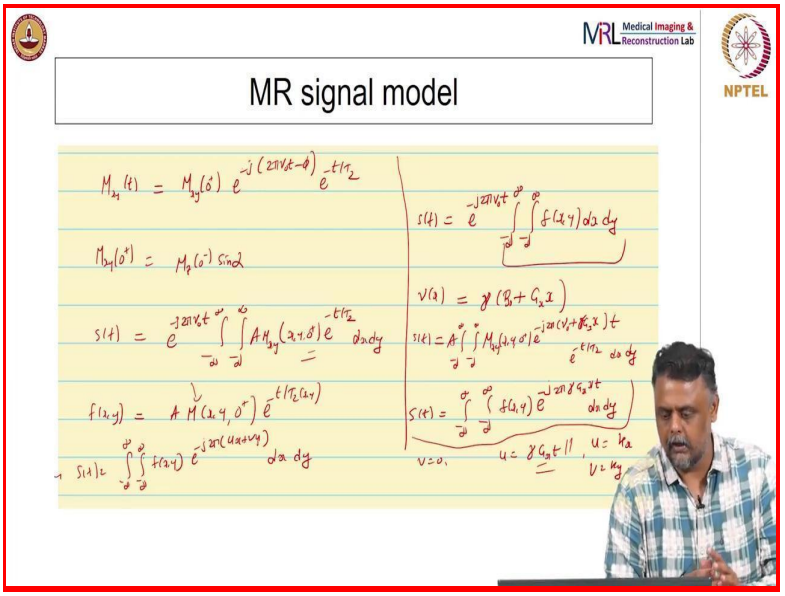

So, what is the signal model for this, we will just do that quickly, before we go any further. So, this is the signal in the plane, the transverse magnetization after being after an alpha excitation that is after the films have been flipped by angle alpha. So, the $M_{xy}(t) = M_{xy}(0^{+})e^{-j(2\pi\nu_0 t-\phi)}e^{-t/T_2}$whereas symbols have their usual meaning and this is M_{xy} and this other term just after is basically just before $M_{xy}(0^{+}) = M_Z(0^{-})sin\alpha$

So, we have this spins in the excited slice and we want to model the spatial distribution of the proton density T 1 and T 2 and we assume that the slice is fairly thin so that there is no variation of course it is not entirely true there is z variation, but we assume that it is fairly thin but there will be some spatial variation of the transverse magnetization immediately after the excitation. And the received signal then we can model as a signal that we get you to that M x y and that relationship is typically written in this form.

$$S(t) = e^{-j2\pi\nu_0 t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} AM_{xy}(x + O^{+})e^{-t/T_2}dxdy$$

the origin of the signal that we are all working with, so the signal that is acquired that is that is received by the coils is actually the function of the magnetization that is been flipped into the plane, but since there is a variation because of the variable there is some variability in the magnetic field etc., there is a variation in M_{xy}.

So that is why we have to integrate M_{xy} and that A is some amplitude if all the other constants have been absorbed in A. Now, this is the general signal model that we will be working with. Now, we can once again we can make it further simpler by using this expression $f(x,y) = AM(x,y,o^{+})e^{-t/T_2(xy)}$ So, then your signal becomes $S(t) = e^{-j2\pi\nu_0 t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy$

So, basically the received signal if you de-modulate it because the $e^{-j2\pi\nu_0 t}$ a very highly oscillatory rate ν_0 is a very high frequency oscillation, but you demodulate it. So, if you demodulate this signal, it is basically just this integral, just the integral or the spins or spin density or T2 relaxation times etc. in that particular slice.

So, then we still need something to make sure that we can localize x and y. So, the way to do that would be to apply a gradient along the x axis. So, what happens if you apply a gradient along the x axis? So, gradient along the x axis would correspond to your frequency, Larmor position being changed the following way $\nu(x) = \gamma(B_0 + G_x x)$ plus G_x is the strength of the gradient along x.

So, of course, you can always compare it with if we when we did the slice selection gradient when we apply another gradient, we applied the G_Z changing it appropriately. Now, once we remove that G_Z, so everything reverts back to 0. So, then now if you apply the x gradient again, $\gamma(B_0 + G_x x)$

Now, once we have this, then we have to modify the expression we have written. I write here appropriately, because it is no longer $\gamma\nu_0 t$. ν_0 has been modified because of the presence of the x gradient. Now when we do that, then all we have to do is let me just write this one expression so that we understand where you are going from.

So, the signal we are talking about, then becomes

$S(t) = A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu_{xy}(x,y,O^{+})e^{-j2\pi(\nu_0+\gamma G_x x)t} e^{-t/T_2} dxdy$. So, the part that is changed is when you can still take out the $e^{-j2\pi(\nu_0+\gamma G_x x)t}$. So, then what you will get is that signal that you measure is basically this. $S(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi\gamma G_x xt} dxdy$

So, this is the important concept in MR imaging this particular equation. So, you can now identify, so we have or we have what you have done whenever you identify the position with the frequency. So, we can interpret this expression as a Fourier transform with one of the frequencies in the case frequency corresponding to y direction being set to 0. So, how do we do that, so, you can set $v = 0$ and we can also set $u = \gamma G_x t$

So, then if we plug it back in there, and you can see that this the expression we have is the form of a Fourier transform, where instead of gamma $\gamma G_x t$, we just put U and V you set to 0. so, if you write this as some function $S(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(u_x+v_y)} dxdy$ .So, this is basically in the form of a Fourier transform, or in this case, it is a Fourier transform where we are trying to integrate over your spatial variables. So, we have the spatial frequency corresponding to U and V being defined by here. So, this is the way we encode positions using the gradients. So, this is one of the fundamental equations for this, the fundamental equation for understanding MRI, where we interpret the signal as the Fourier or the Fourier transform of the quantity you are trying to measure.

So, this f(x,y) encompasses everything, it tells you about the magnetization as a function of x, y, it also tells you about the relaxation, the spin properties T1 and T2 times, so, in this case T2 as a function of x and y, this is already encapsulated in f(x, y). And the application of the gradient along x captures makes it in the form of Fourier transform.

And tip and one of the in the terminology that in an MRI study is called k space. So, you will call $u = kx$ and $v = ky$, so that this is referred to as k space, so you would have to measure all of k space so in order to figure out the Fourier transform, and then you do the inverted, so that is where that is why that is the acquisition strategy in MRI. So, this is the first step towards casting it in the form of a Fourier transform.

And the way to actually measure would be to scan the so called k space by taking on a range of frequencies and how do you adjust that by changing your gradient, so that is one way of doing it is to change the gradient or the most effective way of doing it is changing the gradient, by changing the gradient you will capture a variety of k x and k y and therefore, that will give rise to your Fourier transform that is k space acquisition and using that you can reconstruct your image, that is interpretation of our image acquisition in a MR signal in an MR system.

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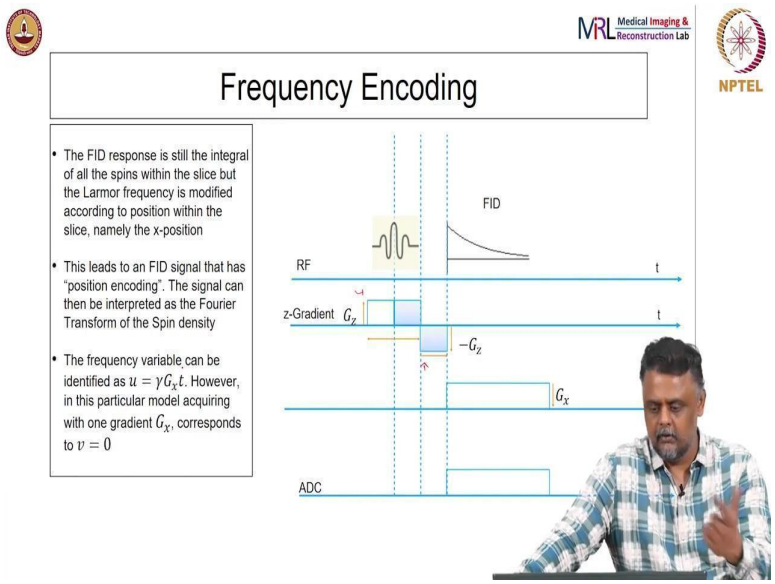

So how does it work in the for-frequency encoding? So, as you saw the FID is still the integral of all the spins within the slice, but the Larmor frequency is with what we tried to do is to modify it according to position within the slice namely along x. So, because we apply the gradient along x. Now, this leads to a FID signal that has position encoding, we saw then this signal can be interpreted as the Fourier transform of the spin density. In this case, it is M and of course, it also is a function of t 2.

The frequency variable we identified as $u = \gamma G_X t$ in this particular acquisition mode with one gradient it corresponds to v=0. So, this is the upper sequence diagram RF turned on the gradient along z also turned on for slice selection, the refocusing gradient for the application of G x it is turned on, in this along the time axis it is always on during which we actually do the ADC, this leads to our image being which is the scanning k space, but you see that the this here it only corresponds to a certain V equal to 0, so we are stepping along u, but v is still 0.

So, ADC of course samples these signals in time, U is given $u = \gamma G_X t$. So, that tells you gives you the whole range of u's and of course you have to change v in order for us to sample the entire k space. So, how do we change V. That is another question we will answer in next slide.

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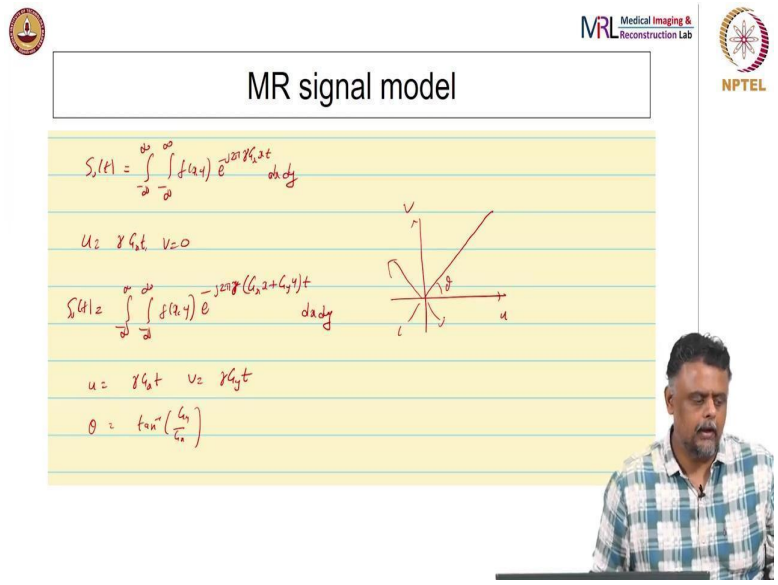

So, the MR signal model we have already seen just to recap I will write this down as you saw the application of the gradient leads to the following signal. $S(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi\gamma G_x xt}dxdy$

then you made the identification that $u = \gamma G_X t$ and $v = 0$. Now, how do we do this scanning, now, we saw V equal to 0 then how do we change I have changed this. So, the way to do that is also to apply another gradient along x, it was along y. So, then your signal model would be $S(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi\gamma(G_x x+G_y y)t}dxdy$

And so from here we identified the Fourier domain frequencies as $u = \gamma G_X t$ and $v = \gamma G_y t$. And so, what is the Fourier trajectory that we are sampling here? The trajectory we are sampling here is obviously, we are going in this direction $\theta = tan^{-1}(\frac{G_y}{G_X})$ or if you want to plot it, so, if you are looking at the k space diagram, so to speak, so, U, V by changing G x and G y in appropriate steps, you are basically sampling along a certain direction like this that is a theta.

So, by sampling radially by strain changing the strength of G x and G y you will end up scanning along arbitrary directions and then once you scan the entire U, V, this is called polar

scanning. So, once you scan the entire U, V space of course, you can do interpolation on the Cartesian grid and do the reconstruction. This is the polar scanning protocol.

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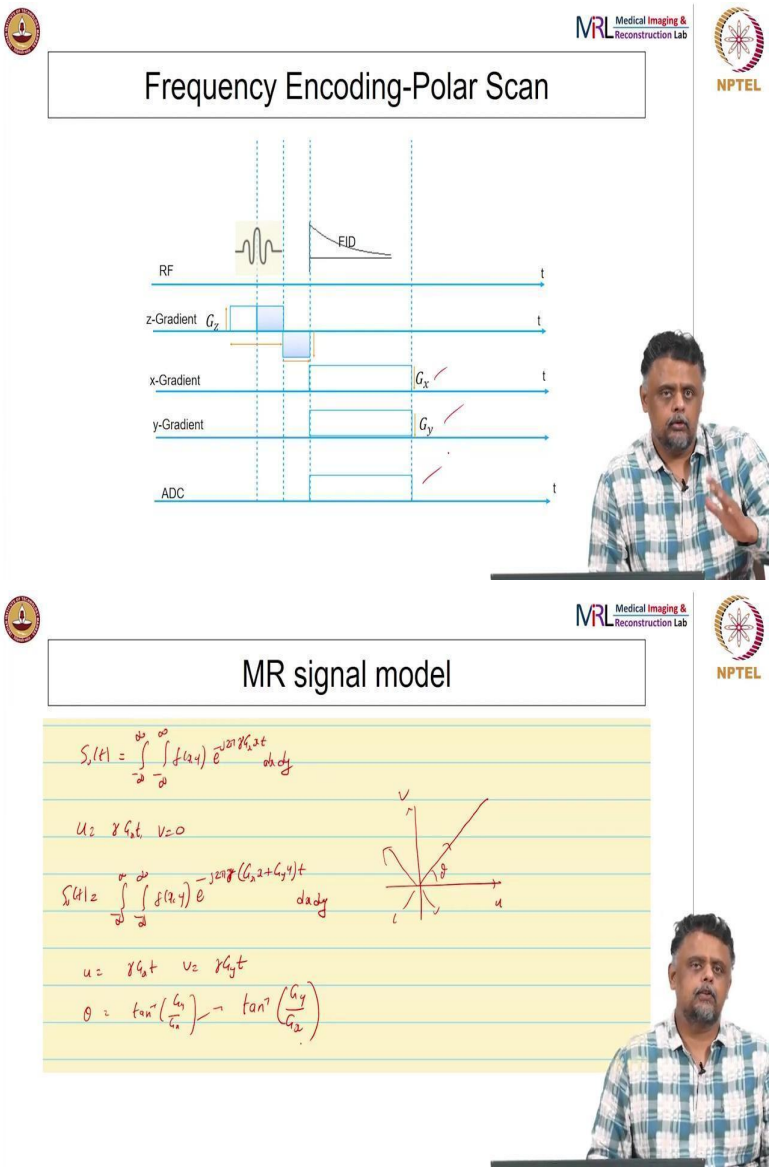

So, here just to summarize again, scanning the Fourier space requires the repetition of pulse sequences. Like we saw because one we only have V equals 0, so we have to scan for different values of U, V in order for us to get to the appropriate positions in k space.

So, this is accomplished by an additional gradient along y direction is called a G y gradient and the application of gradient along x direction with zero gradient along y made the radial direction x. And of course, the directions can be changed, in this case, we apply both the x and y gradient simultaneously.

And the pulse sequence has to be repeated with different gradients to cover the different caspase trajectories. So, if you want different angles, then you change the strength gradient,

you get a different angle, and then you sample along that direction, so that is the idea behind the polar scan kind of pulse sequences.

What is shown here, everything else remains the same, the x and y gradients are turned on simultaneously and of course you do the ADC acquisition as a function of time. So, this once again, and of course, we keep changing the strength of G x and G y to get different angles, so that is what we saw. So, the theta the angle at which you scan is given by the strength of G x and G y.

So, by changing G y by G x you can get different angles and then of course, you sample along that direction, so that gives you the case-based trajectory. So, this is for a polar scan again, once again, this is probably not this is just for the concept, there are different ways of traversing, traversing k space.

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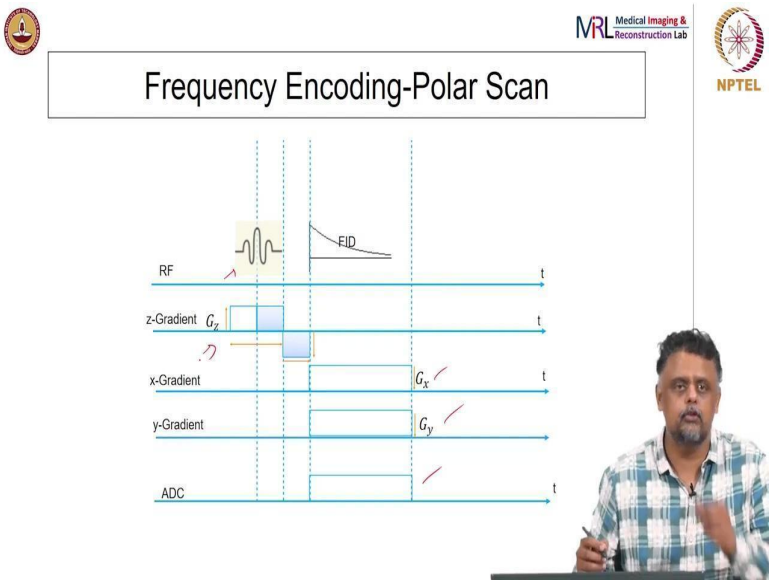

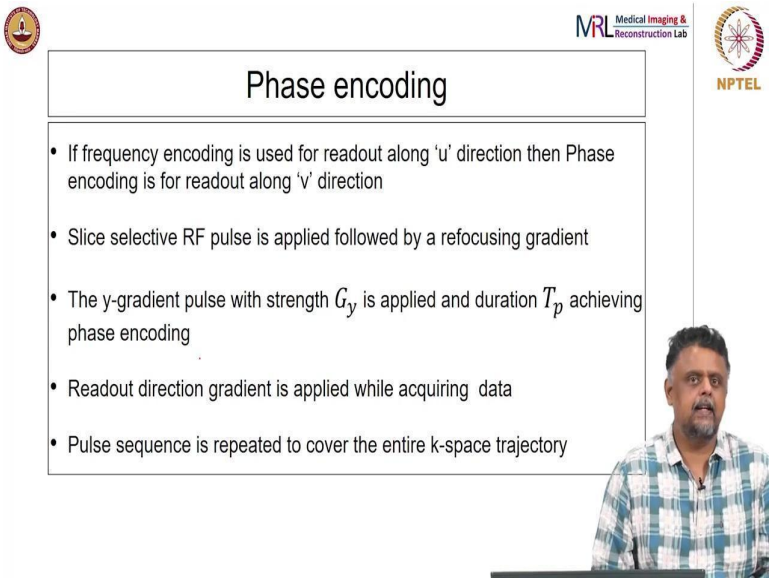

The other way is the phase encoding. So, here polar scan is one technique to traverse k space in 2D. I do not think that is a very common thing to do, but that is I think one of the earliest things that was done. But phase encoding is what is done more commonly. So, the frequency encoding is used for readout along U, the U direction in the k space. And the phase encoding is for readout along the V direction.

So, the way it works is we do a slice selective RF pulse, followed by a refocusing gradient, you saw that the usual, this is the one we are talking about. This is the RF pulse slice selection, this is usually standard you will see this everywhere. And then you apply the Y gradient with strength G y for a certain duration achieving phase encoding. Readout direction

is then applied while acquiring data. Then the pulse sequences, again, repeated with different strengths to cover the entire k space trajectory.

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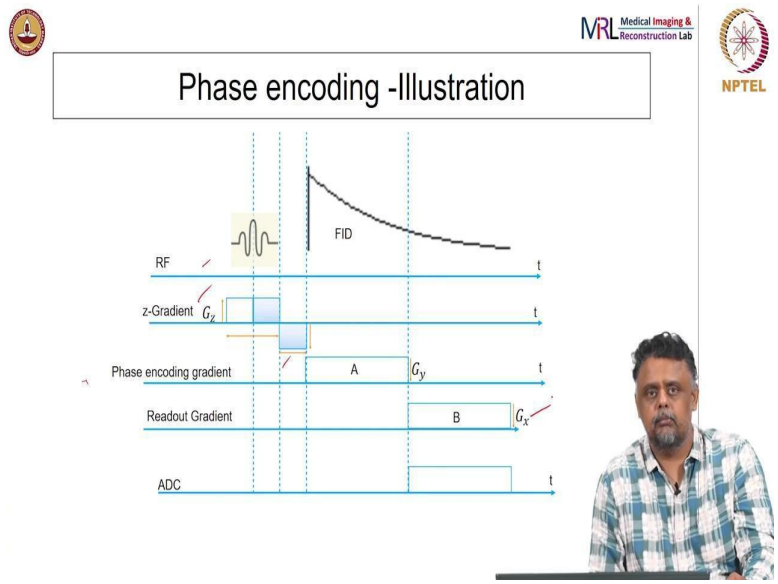

Now, we might wonder, so first, we will look at this diagram first. So, what we do is we have the usual RF pulse applied along with their slice selection gradient, refocusing pulse, FID and usually the diagrams that come after I have just put it so that in time it looks great, nice. So, you apply the phase encoding gradient for a specific period of time and then you switch it off and then you apply the readout gradient right now, and you acquire the signal at the same time. So, what does this accomplish?

So, typically what happens is like so, when you have the RF selection, slice selection gradient on then all the spins in that particular slice have been tipped into the field of view into the plane and into the x y plane and they are all processing with the same frequency, which is basically corresponding to whatever gradient plus the B 0 field is.

See applying the phase encoding gradient and switching it off, once you switch it off they all go back to the previous B 0. So, what is applying the phase encoding gradient accomplish? The phase encoding gradient imparts a phase change. So, they might be rotating in the same frequency but they will be off by a phase.

So, face it, so every, it gives you a position dependent phase to the rotation if procession to the position and that is preserved even after the removal of the gradient G y. So, the phase encoding gradient only does that it gives you a position dividend phase value and then when

you do the readout gradient when you have your gradient on, as you are acquiring the data, then there is a push and dependent frequency also added into the mix.

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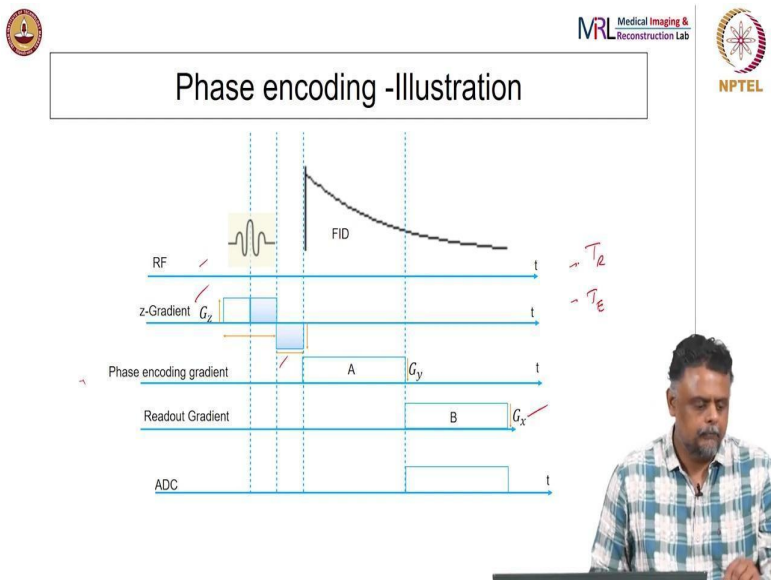

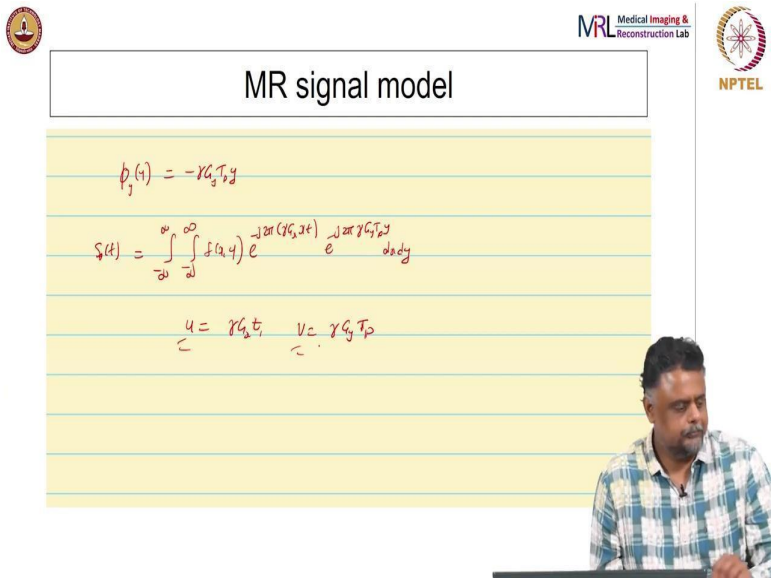

So, the way that model works let us just, there are some we will look at the model first, but before we go any further, I have been kind of very superficial about this time axis here, I am just indicating some arbitrary times I have not indicated what these times are, these times are actually specific time.

So, there is something called time to echo that is when you measure, there is a time to repeat because all of these have to be repeated for different values of G x and G y. So, when we do that, you cannot do it right away, so there is a time gap between these repetition that is called

repetition time or time to repetition, T R and T E time. Time to echo is when Nu as in the middle of your measurement time window, so this T E.

So, all these are actually very crucial, these timings are crucial and I am not actually mentioning them, I am just giving you an overall idea of how these measurements are done so you understand how it is done. So, let us just go back to the signal model, if you have the phase encoding gradient. So, the phase encoding gradient, what does it do? It is not too different, because now, the one thing you have to again remember is that we are applying the phase encoding gradient for a fixed amount of time for a fixed TP.

And so the signal model so, we will write down the single model, but before that, we got to see what is the face accumulated due to this? The face accumulation like mentioned before is a function of position which is $\phi_y(y) = -\gamma G_y T_0 y$ So, your signal model then becomes,

$$S(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(\gamma G_x x t)} e^{-j2\pi(\gamma G_y T_p y)} dxdy$$

so, now, your $u = \gamma G_x t$ and $v = \gamma G_y T_P$ so, this T p is fixed.

So, this is your baseband, so called baseband signal, of course, modulated by that $e^{-j2\pi v_0 t}$ which I have not indicated here, this is the demodulated, this 0 here is the demodulated signal and this is how you scan the k space. So, you change by changing the gradient descent, B by changing the gradient so on and so forth, so that is how U, different levels of U, you can change by changing the gradient.

Similar things you can do for V or for what you do is for different values of the gradient and at the same time, you will get a different V, and then you keep V, U constant, and you can sample that way also. So that is typically how it is shown in all these images and all these pulse sequence pictures, which is I have not shown them here, but maybe at a later time, I can actually show you how it is done.

Because typically, the range of these gradients are shown because the rate gradient can go from some minus max to a plus max. And that is I have not shown here in any of these diagrams, but that is how we typically have to interpret it. We will look at some more of these more practical image acquisition pulse sequences. This is just to understand the model for your acquisition, this is the model of the signal.

And how you can encode your position using these gradients is the fundamental concept that I would like for you to understand. That is how it works. And at f (x y), that function that you are measuring, as a function of position is basically that has both the term that has all the relaxation time terms, as well as the magnetization density in it and that is it, it is just a complicated function that you measure.

And so, your images that you measure in MR, are both functions of so-called spin density, which is basically the magnetization as well as the t1 and t2, relaxation times, these are basically the physical quantities that you are actually trying to measure by using your MR images. So, this is in contrast to CT image, where the quantity you measure is actually a function of electron density. It is μ you are measuring for a CT we saw, we are measuring Mu, Mu is the attenuation coefficient, which itself is a function of energy and atomic number, but it is actually more a function of density.

So, what you are measuring is basically the density, the actual physical density of the object that you are measuring. In this case, you are looking at magnetization density, as well as some relaxation time properties. So, this is more of a nuclear property that you are measuring, this is the property of the nucleus, you are measuring this specifically water. So, in CT, you are actually measuring electronic properties, so it is kind of similar, but kind of different that way, the mechanisms of course, are also different.

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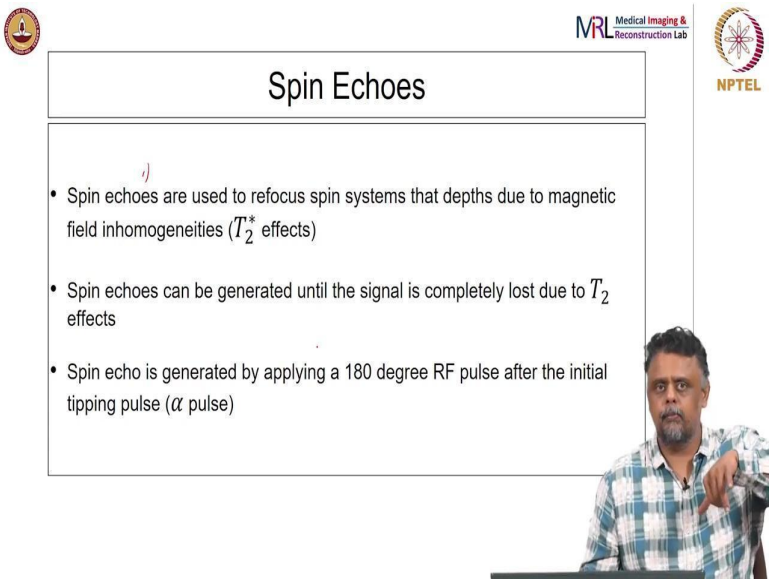

So, one of the commonly used or say the practical should I say I will not say commonly used, so, the way the FID decays rapidly do two different effects. One is the magnetic field inhomogeneities. So, the spin echoes are basically that you can generate the spinner course, till your signal is completely lost due to t 2 effects.

How do you generate the spin echoes? So, that is what you want to get at. Before we go there, we just have to understand that the echoes that the FID generates are decades due to the t 2 effects, as well as the t 2 star effects. And this is a very rapid decay, so measuring that is slightly difficult. But if you get a spin echo then you can measure that much easier because this FID is generated instantaneously as soon as you flip this spin into the plane, FID starts generating and it is just a day away.

Because the defacing effects are the defocusing effects of magnetic field inhomogeneities and the spin-spin interaction is very fast, these are milliseconds or less. So, in order to get some more time for measurement, instantaneous measurements are not possible, so it gets better and also to get a better amplified signal you measure spin echoes.

The spin echo is basically generated by applying a 180-degree RF pulse after the initial tipping pulse. So, you tip the spins into the plane or at some angle alpha using the RF pulse and then you apply one more pulse which will flip it under 180 degrees. So, it basically reverses it by fitting it off as a 90 degree flip. It flips into the plane and then if you do the 180 degree, it will just flip out in the opposite direction.

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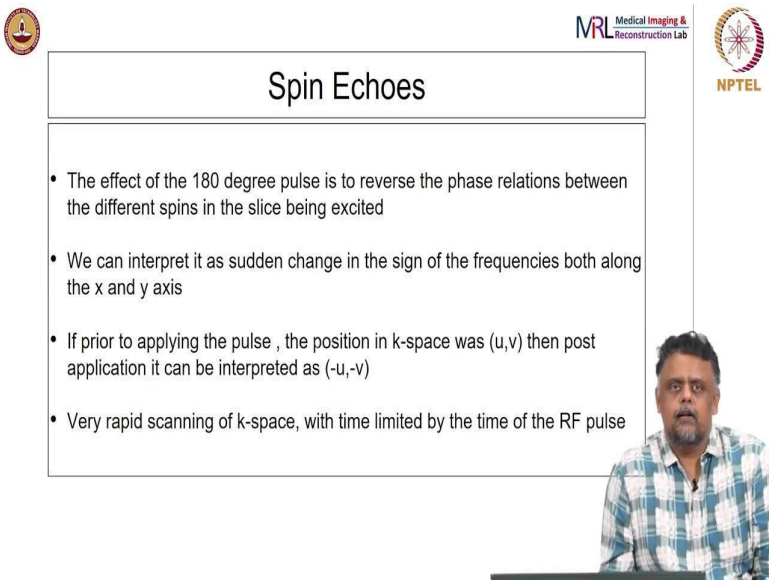

So, the order is the 180-degree pulse do, 180-degree pulse reverses the face relationships. So, if there is rapidly defacing, as soon as the spins are flipped into the plane, it starts to deface. So, once you flip one more 180 degree the direction of defacing is the same so they will actually come together again to give you another echo, so another signal. So, how do we interpret this 180-degree pulse? We can interpret it as a sudden change in the sign of the frequencies both along the x and y axis.

So, if prior to applying the pulse k position was u, v, then post application, this can be interpreted as (-u, -v). So, this enables a lot of what they call image acquisition tricks, we will not go into details, but the 180 degree pulse the Spin echo pulse is what is typically measured.

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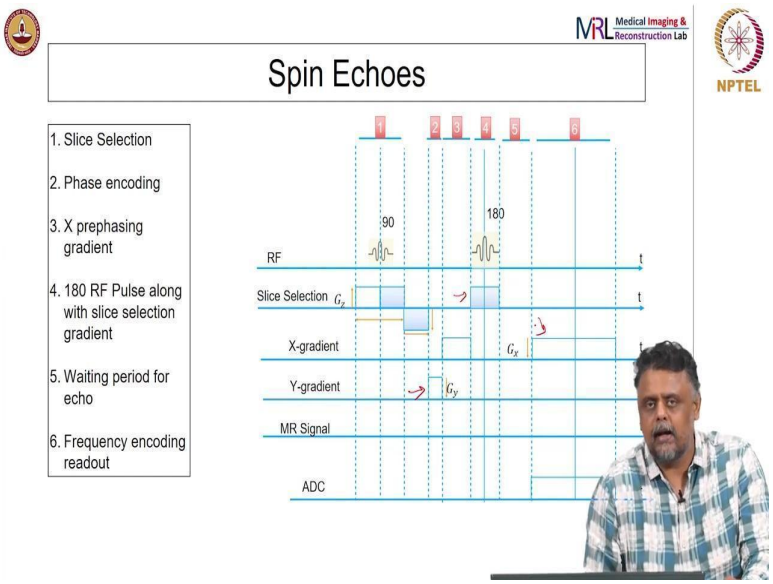

Let us look at the sequence. So, slice selection one, on top you see that numbers have written two is the phase encoding step, I think that is the G y gradient being applied following that, you have the 180 degree RF pulse along with the slice selection. So, you get one more slice selection gradient and then you wait for a particular period of time, this time two echo and along with that you apply the readout direction.

So, if you are not sure here is the phase encoding this is the other 180 degree pulse with the you have to do the 180 degree pulse with the gradient slice selection because you want to do the spin flip in that particular section, once that is done after waiting period, you apply the

readout direction pulse and you get an echo. I have not drawn the echo here but that is, this is the general spin echo sequence, but just kind of the most practical sequence.

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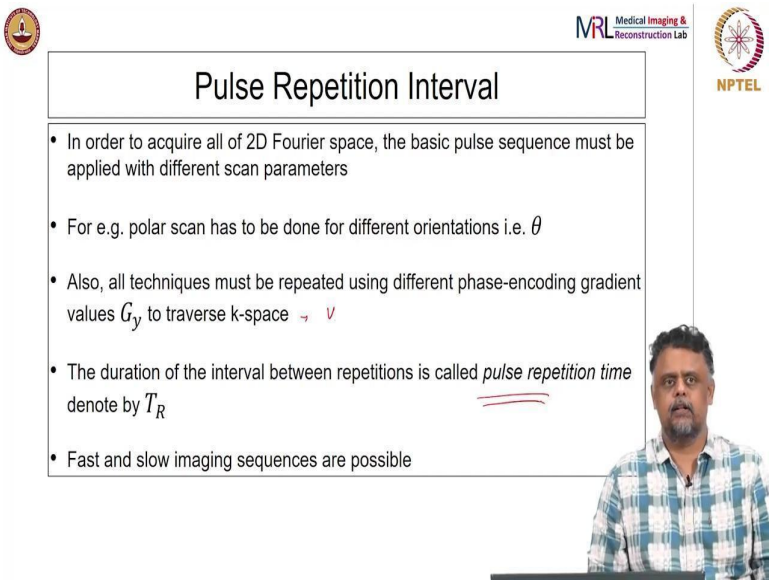

So, but the pulse repetition interval that I spoke briefly earlier. So, in order to acquire all of 2D space, so we are really looking at now if you look, we just looked at one value of G x and G y So, in order to acquire all of 2D Fourier space the basic pulse sequence must be applied with different scan parameters. So, for what I mean by scan parameters.

So, for polar scan it has to be done at different orientations and then all techniques will be repeated using different phase encoding gradient values, you apply you change the G y then you get a different v, so, this gives you different v's.

And so, every time you have one you apply one G x one G y, sample this signal you wait, and that waiting period is the pulse repetition, you wait a little bit because you want everything to get back to its stationary state, and then you repeat the experiment. So, in this fast and slow or possibly slow meaning you have to wait a considerable amount of time before you do the next repetition.

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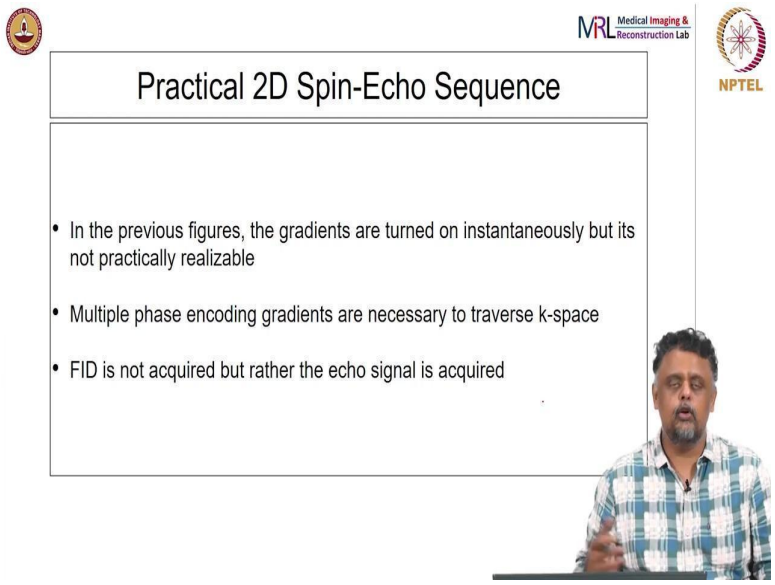

And fast is you do not really wait, but you continuously acquire for there are and then each of them has its advantages and disadvantages. There are some other things also that I have not shown, so in all the figures I have shown the gradients to be turned on instantaneously that never happened, there is a slope to it, so it is not instantaneous.

And multiple phase encoding gradients, I have not shown that at all, but it is typically that is required to traverse k space. FID is not acquired. I was told the only echo signal is acquired. That is the only possible thing to acquire at least that is the understanding. So, the echo signal is a better signal to acquire, not the FID.

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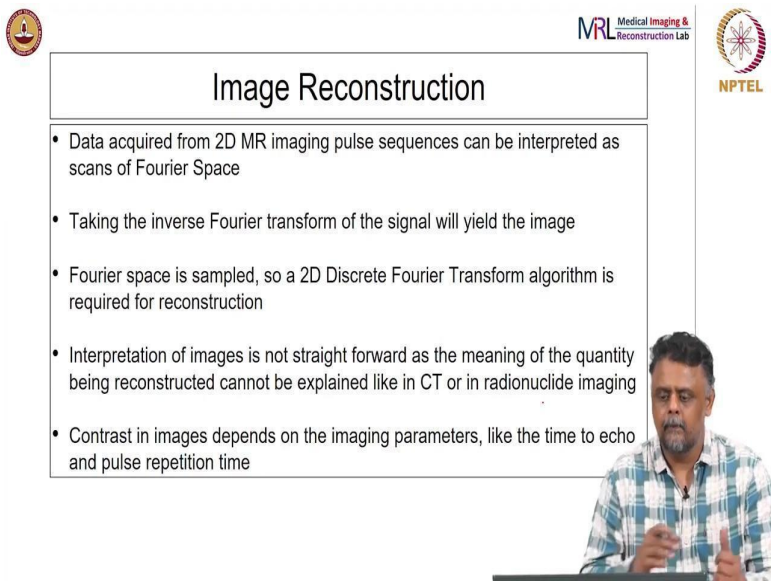

So, what do you do with the images? So, data acquired from 2D MR imaging with what you are seeing are called 2D MR imaging pulses and we saw that when we write down the signal model, it can be interpreted as a scan of Fourier space. So, taking the inverse Fourier transform will yield the image, by the way it is not a simple inverse Fourier transform. I had to do a discrete Fourier transform.

So then, the third problem that we also talked about is that the interpretation is not very straightforward, because the meaning of what you are reconstructing cannot be explained very simply like CT I said, it is a attenuation coefficient. It is a function of density, electron density or physical density of the object or radionuclide imaging, wherein what you reconstruct is basically the radiotracer concentration.

And so, but then the problem is here, it is not so straightforward, but we do know that f of x y we saw has multiple quantities in it depending on how you choose the time to image, forensic imaging parameters like time taken and pulse repetition time, they can answer and encapsulate multiple properties.

So, for instance, one property is the spin density which is proportional to the magnetization, the other property is a t 2 relaxation time, one more property is the t 1 relaxation time, these are the three important properties that are captured by an MR image. That is also the quantities that give rise to contrast in these images like for instance, some tissue appears bright, some tissue appears not so bright or darker; that depends on how you tweak this time to echo and pulse repetition times.