Computational Neuroscience
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Week – 08
Lecture – 38

Lecture 38: Single Cell Decoding - II: Using ROC Curves for discrimination

Welcome. So, we have been discussing discrimination based on rate responses of single neurons and we introduced you to the idea of hit and false alarm and how receiver operating characteristics kind of analysis is started. So, just to recall we have our hit as β which is a function of Z and Z is the threshold that we are using when we are discriminating the rate distributions. So, using use the same colour. So, this is the rate distribution for the plus stimulus this is the response rate distribution for the minus stimulus. So, this is plus and this is minus and we said that we will fix the threshold Z and based on that we will take the decision based on the rates every time I observe rate response of the neuron I decide whether it was the plus or minus stimulus based on whether it is greater than Z or less than Z.

And the Z can be varied from any from over the whole range of rates. So, this axis is our rate and this axis is the density or Pr. So, we said that our $\beta(Z)$ is nothing, but probability $r \geq Z$ given the plus stimulus and $\alpha(Z)$ is probability $r \geq Z$ given the minus stimulus. So, as we started out if we were to plot α and β as a function of Z that is what we will call the ROC curve.

So, this α will vary between 0 and 1 and β will vary also between 0 and 1. Now let us say we set Z equal 0. So, that is a very small value that is to the extreme left and let us ask ourselves for Z equal 0 how is our α and β which is going to define performance are going to be. So, when Z equals 0 that is the probability $r \geq Z$ given plus is obviously going to be 1 because the rate is going to be always greater than Z. Similarly $\alpha(0)$ similarly again it is always going to be 1.

So, at Z equals 0 we have the position of α and β at the top right corner of this plot. Now if we consider the Z to be extremely large or plus infinity then what we have is over here our $\alpha(Z)$ that is Z equals infinity. So, probability $r \geq Z$ is going to be 0 because no matter what the stimulus our rate is always going to be less than Z. So that means our α is going to be 0. So, we will be on this vertical axis on the β axis and exactly in the same way our β is also going to be 0 for Z of a very large value or plus infinity.

So, what we have is our α and β are going to be at these two points. Let us use a different color. This is Z equals 0 and here Z is infinity. So now let us remind ourselves the table for correct incorrect. Say here we have correct and here we have incorrect for a plus stimulus for a minus stimulus we have hit this is $\beta(Z)$ this is $\alpha(Z)$ that is false alarm this is $1 - \beta(Z)$ and here we have $1 - \alpha(Z)$.

So let us ask this question that can we have α and β such that the α β combination is below this diagonal line at this point. So what would this suggest? So first of all let us ask what is the probability of correct in this scenario in terms of α and β. So probability of correct means we simply have the average of these because we if we assume that plus and minus are occurring equally equal number of times then the probability of correct is $1 - \alpha(Z) + \beta(Z)$ divided by 2 that is probability correct. So in this case where we have these two only alternatives if you and the animal is supposed to always make a decision either plus stimulus or minus stimulus then the probability of correct cannot go below half on the long run. That is if we have probability of correct is less than half I mean not just due to chance because of the number of trials or something but on the average if it is here we are treating it theoretically so on the average if it is less than half then there is a problem in the experiment in the sense that the monkey may be using some other cue to know in some way that okay the I mean who knows that is just simply unlikely and unlikely meaning it is not possible.

So that means our probability of correct must be greater than equal to half which means that our $1 - \alpha(Z) + \beta(Z)$ must be greater than equal to 1 which means if we cancel out 1 our $\beta(Z)$ must be greater than equal to $\alpha(Z)$. So that means all of the points of the receiver operating characteristics curve the ROC curve must be on the upper part of this diagonal on the upper side of this diagonal and this region is not I mean it cannot come into this lower region. So this is forbidden territory for the curve. So now if we think that the monkey is always performing at chance level for any Z then what we have is that $\beta(Z)$ must be equal to $\alpha(Z)$. So this is going to be the receiver operating ROC curve for the case when for any Z the monkey is performing just at chance level.

What is that case in our experiment if we remind ourselves we have the random dot patterns. The random dot patterns are made to move in a particular direction by using a different kinds of coherence levels. The coherence level is the highest then as we know the two distributions the red and green with the minus and plus stimulus are going to be separated fully and probably will be making the correct choice almost every time. And when the percentage coherence is zero then our monkey is always going to be making a 50

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So here is the idea that may be that the area under the curve could be a measure of the percentage correct. So as in the case of Britain et al with the different coherences the curves that they got turned out to be for a slightly higher coherence would go up and for the higher coherence would go up and for the 100

And this is percentage coherence. So now based on the neurons the rate responses using this analysis as we have here we will be getting a percentage correct based on the area under the curve if it would be if we can show that the area under the ROC is the percentage correct. So let us go on here and now consider this idea that we have we are observing the rate in response to a plus stimulus and a minus stimulus. And let us without loss of generality we assume that the first stimulus is plus the second stimulus is minus. So this is R_1 and this is R_2.

Now if we take this idea further that we have to make this decision based on these two rates and the rates so are basically R is equal to R_2 given minus and here R is equal to R_1 given plus. And now if we use the response rate to minus as the threshold that is since it is a two alternative force choice if the minus is coming next then this R_2 is can be made our Z that is if R_1 is larger than R_2 we will say plus stimulus if R_1 is less than R_2 we will say minus stimulus is I mean has occurred the next one. So if we take this Z as R_2 then we can write out the probability of correct in this manner that it is going to be so if at for a particular Z the probability let us say we have a particular value of Z. So let us draw this curve here this curve here we have a particular value of Z and we need to find out what is the probability of correct and we need to we see that first thing is this Z is our threshold which is the rate 2 which means that how often will we see that that is our probability of rate given sorry. Probability of rate less than equal to Z will have the probability that the rate is given minus integrate this over R and multiply this by $\beta(Z)$.

So why do we mean that what do we mean here that this is the density $P(R)$ given minus times dR is the probability that the rate is on Z and just around Z, Z plus δZ. So Z plus δZ that is our probability and times our $\beta(Z)$. So we can also by the way we can replace the R by Z and this is $P(z)$ that that same distribution and we can have Z varying from 0 to infinity. So we what we are doing is we are saying what is the probability that the rate observed given a minus stimulus is Z which is the integral this without the $\beta(Z)$. But then we have to also be correct in the sense that the next rate is greater than or the first rate has to be greater than Z which is the probability of hit and that is $\beta\ Z$.

So if you multiply this 2 and integrate we basically get the probability of correct. So what we have here is integral dZ or rather I mean we can write it either way $dZ\ P(z)$ given minus times $\beta(Z)$ where Z is varying from 0 to infinity. Now what is our $\alpha(Z)$? $\alpha(Z)$ is that false alarm that is again we have the rate response for the second stimulus R_2 that is the observation that it is Z and we need to integrate that we know that given the stimulus is minus we are saying that the stimulus is plus. So actual stimulus was minus then if this is our Z we essentially get the integral of the Z being 0 Z to infinity that is over this range if we integrate and say that the probability of the $P(Z)$ given minus dZ. So again the same way this is the probability that we are observing our rate second rate given the minus stimulus is going to be Z or around Z Z plus dZ and our the integral of the same thing over Z to infinity is going to be that is the Z for Z larger than this threshold value I am sorry this should be rate in this case we have a rate here rate given minus dR and the rate is going to be varying from Z to infinity that means our decision is plus stimulus.

So this is the probability of observing a particular rate given a minus stimulus and we are setting this R to be the Z. So now what we are saying is that our we will say it is a positive stimulus when the rate is varying from Z to infinity that is we are above that rate given a minus stimulus this is our false alarm that is $\alpha(Z)$. Now if we consider our $d\alpha(Z)/dZ$ by applying the rules for integral derivative of an integral where we have the function of the variable for which based on which we are taking the derivative if that is in the limits then this derivative turns out to be minus p of Z given minus. So now the probability of correct if we consider is minus $p(z)$ given minus times dZ is our $d\alpha$ from this equation. So our probability of correct here can be written as minus integral 0 to infinity $d\alpha$ change with the change of variable times $\beta(Z)$.

So now if so this is the case I am sorry the we have to change the limits here because we have changed the variable from Z to α. So we have $d\alpha$ here and so for Z equals 0 our α is 1 top right corner on the ROC and for Z equals infinity our α is 0. So this reverse direction of the variable over which we are integrating is countered by the minus sign in front and so what we get is percentage correct is integral 0 to 1 $d\alpha$ β as a function of α. So I think we should write this as only β because we have removed Z pardon me $d\alpha$ β and so obviously this is showing that if we consider the ROC as α and β as a function of α then the area under this curve β as a function of α is the percentage or probability of correct because this is representing this area. So based on this analysis using the rate responses for neurons what Britain et al showed that based on the responses of empty neurons in the different directions and different coherences the percentage correct performance based on the neuronal responses could well match the psychometric function of the monkeys that is their performance actual performance which means that for the different coherences if the monkey's performance was like this where this is around 10 percent this is around 1 percent then if this is 1 and this is 0.

5 then based on the neural neurometric functions that is in the for the neurons we get a similar sort of curve that matches with the performance. So this was only a correlational observation as we have discussed it is important to establish causation and this was one of the first studies in fact the first study in this kind of behavioral experiments where causation was shown as what Britain et al did and I mean their group did is they now stimulated neurons in a particular location in the empty and based on that they could alter the behavior of the monkey so in a predictable way. So that actually took the whole analysis to a step further and showed that indeed the rate responses are what are being actually used by the monkey to make the decision and is matching the actual performance based on the neural performances. Thank you.