Computational Neuroscience

Dr. Sharba Bandyopadhyay

Department of Electronics and Electrical Communication Engineering

Indian Institute of Technology Kharagpur

Week – 03

Lecture – 15

Lecture 15 : Properties of MLE

The Morris-Lecar equations are a system of nonlinear differential equations that describe the electrical characteristics of a neuron. The equations are given by:

$$C\frac{dV}{dt} = I_{\text{external}} - g_{\text{Ca}}m_{\infty}(V)(V - E_{\text{Ca}}) - g_K w(V - E_K) - g_L(V - E_{\text{Leak}}) \tag{1}$$

$$\frac{dw}{dt} = \frac{w_{\infty}(V) - w}{\tau_w(V)} \tag{2}$$

where V is the membrane potential, w represents a gating variable similar to those found in the Hodgkin-Huxley model, C is the membrane capacitance, g_{Ca}, g_K, and g_L are conductances for calcium, potassium, and leak channels, respectively, and E_{Ca}, E_K, and E_{Leak} are the respective reversal potentials.

A phase plane analysis of these equations involves plotting one state variable against the other, typically V against w, to analyze how the system evolves over time from a particular initial condition. This representation is known as the phase plane, where the axes represent the membrane potential (V) and the gating variable (w), which varies from 0 to 1 . The evolution of the system at any point (V_0, w_0) in this plane can be determined by the derivatives $\frac{dV}{dt}$ and $\frac{dw}{dt}$ at that point, indicating the direction and rate of change of each variable.

To visualize the dynamics, one might calculate these derivatives at a grid of points in the phase plane and plot vectors that represent the instantaneous velocity of the system at those points. This method helps in understanding the overall behavior of the system, such as identifying stable and unstable equilibria, trajectories, and other dynamic phenomena.

The uniqueness of the solution trajectory through any point in the phase plane is guaranteed by the existence and uniqueness theorem for differential equations, provided that the derivatives are continuous. This implies that trajectories do not intersect each other, reinforcing the deterministic nature of the system under continuous and well-defined conditions.

We continue our exploration of the phase plane in the context of the MorrisLecar equations by considering nullclines, which are crucial for understanding the system's dynamics. Nullclines are defined as the set of points in the phase plane where the rate of change of one of the state variables is zero. These are mathematically expressed as follows:

For the voltage variable V,

$$\frac{dV}{dt} = f_1(V, w) = 0$$

implies a relation between V and w which forms the V-nullcline in the phase plane.

Similarly, for the gating variable w,

$$\frac{dw}{dt} = f_2(V, w) = 0$$

defines the w-nullcline.

At any point on the V-nullcline, there is no net change in the membrane potential V, meaning if a trajectory crosses this curve, it must do so vertically (either up or down). On the w-nullcline, there is no net change in w, so trajectories must cross this curve horizontally.

Nullclines are pivotal for determining the system's behavior:

- The V-nullcline and the w-nullcline intersect at what are known as equilibrium points or fixed points, where $\frac{dV}{dt} = 0$ and $\frac{dw}{dt} = 0$. At these points, the system, if undisturbed, remains static indefinitely.
- These points are significant because they represent the neuron's potential resting states or any persistent activity modes.

To find the analytical forms of these nullclines, consider the Morris-Lecar equations slightly rearranged:

$$C\frac{dV}{dt} = I_{\text{external}} - g_{\text{Ca}} m_\infty(V)(V - E_{\text{Ca}}) - g_K w(V - E_K) - g_L(V - E_{\text{Leak}})$$
$$\frac{dw}{dt} = \frac{w_\infty(V) - w}{\tau_w(V)}$$

where $w_\infty(V) = \frac{1}{2}\left(1 + \tanh\left(\frac{V - V_3}{V_4}\right)\right)$ describes the steady-state value of w as a function of V. Setting $\frac{dw}{dt} = 0$ gives:

$$w = w_\infty(V)$$

By analyzing the intersection of these nullclines and their behavior in the phase plane, one can predict the dynamics of the neuron under various conditions. The stability of these fixed points can further be analyzed by considering the Jacobian matrix of the system at these points, leading to insights into the neuron's response to perturbations and external stimuli.

In our continued exploration of the Morris-Lecar model within a phase plane analysis, we focus on the behavior near equilibrium points and the system's response to perturbations. Consider the setup where:

- The v-axis represents the membrane potential.
- The w-axis represents the gating variable.

When the system is perturbed near the resting potential (denoted as v_{rest}), interesting dynamics emerge. Assume we initially perturb the membrane potential slightly to the right of v_{rest}, keeping w unchanged. Observing the system's trajectory in the phase plane, it typically follows a path that crosses the w nullcline horizontally and the v-nullcline vertically, returning to the equilibrium point. This behavior illustrates the neuron's response as:

- Initially increasing the potential slightly.
- Then decreasing below the resting potential.
- Finally returning to v_{rest}.

This trajectory can be visualized in a time-series plot of v and w, showing how the variables evolve over time from the perturbed initial condition. The key observation here is the sensitivity of the system to small changes in initial conditions, a characteristic of nonlinear systems, leading to significant variations in behavior, including action potentials or spikes.

If $v(t)$ is the membrane potential and $w(t)$ is the gating variable, then:

$v(t)$ starts near v_{rest}, rises, then falls below v_{rest}, and stabilizes back at v_{rest}.

This is represented as a spike or action potential.

The described behavior highlights the nonlinear dynamics of the MorrisLecar model and emphasizes the system's dependence on initial conditions. Additionally, this model simplifies some aspects by assuming immediate responses in some gating variables (e.g., m), which would typically provide faster dynamics, such as thresholds in a more detailed model.

As we progress, we will explore how these dynamics can lead to other phenomena like oscillations in subsequent discussions. This exploration aids in understanding not just the behavior of individual neurons but also their collective dynamics in neural networks.