Gasdynamics: Fundamentals and Applications
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Lecture 09
Speed of Sound

So in previous classes we have looked at thermodynamics fluid equations and then to a particular assumption Quasi-1D assumption where we consider that the properties of the flow like velocity pressure, density, temperature they remain constant across the cross-section of the flow. So that is a Quasi-1D assumption and then we had looked at conservation equations in the Quasi-1D framework.

Now let us apply some of these principles. So first look at a very important characteristic of compressible flows that is the speed of sound.

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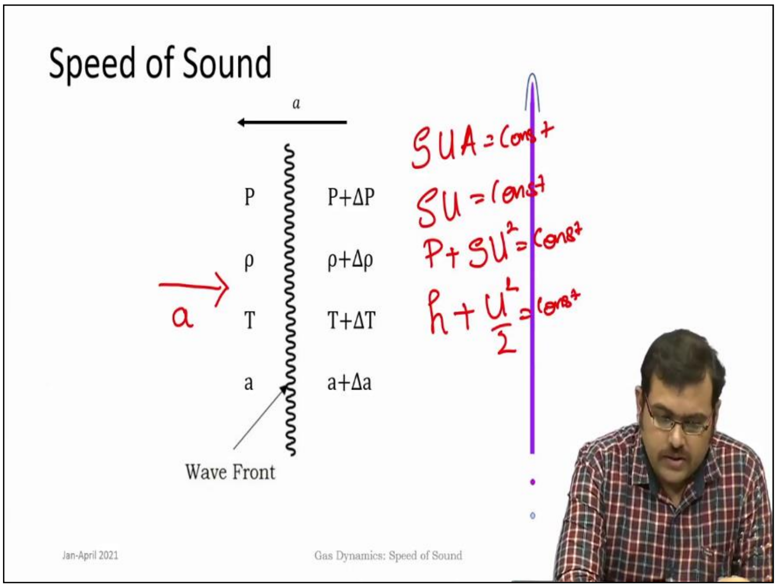

So sound waves are very important when discussing compressible fluid flow. As we had discussed in previous classes including the flow regimes where it is the main mode of information transfer among different parts of the flow. So you have when we have low speed flows or subsonic flows and then these waves can travel all around carrying the information but once the velocity of the flow becomes greater than velocity of sound the information propagates only along certain directions.

So now speed of sound being so important let us analyse the sound wave using 1D principles that we had just completed. So this wavy line represents the wave front of the sound as it is travelling from right to left so when as it is travelling to from right to left. Now in order to apply this we use the principle that if the body is moving or these waves are moving with uniform velocity it is having a certain velocity a then you can always transform the coordinates or transform and impose an opposite velocity in order to make the flow or make the wave front stationary.

So you can give a velocity a now that appears going from left to right. If you impose that then this wavy line front can be made stationary and then the steady flow in a equations can be applied on this wave front. So the equations for now this is a section which is a in one dimension. So you consider the one dimensional flow where you have $\rho u\ A$ is constant since A is constant you get ρu is constant, u is the uniform velocity and $P + \rho u^2\ is\ constant$ this is from momentum conservation.

There is no heat addition or work done in these processes so you get $h + \frac{u^2}{2} isconstant$ that is conservation of energy. So using these three principles let us look at how we can get to the speed of sound. Now as the wave passes over as sound it introduces a small change in the local static pressures, temperatures and densities as well as the speed. So that small change is represented by $\Delta P, \Delta\rho, \Delta T$ and the small change Δa.

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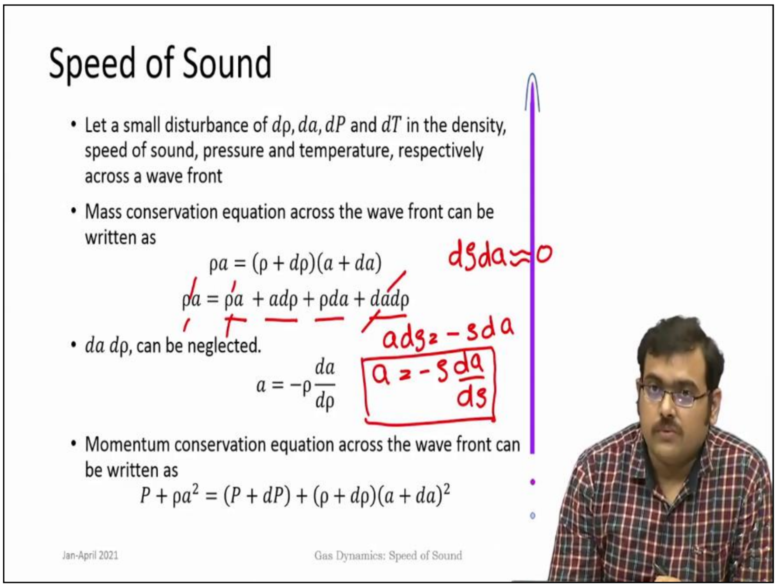

Now we apply these conservation equations across the disturbance. So if you write the mass conservation equation you get that $\rho a = (\rho + d\rho)(a + da)$ just the conservation of mass this can be expanded and you apply the condition that when you have two small quantities $d\rho$ and da getting multiplied these are approximately equal to 0, they are negligible. So this when expanded you get $\rho a + a\, d\rho + \rho da + \ da\, d\rho$.

Now this factor is negligibly small so it can be removed while these two get cancelled from here. So you get the equation a $d\rho = -\rho da$ or a can be written as $-\frac{\rho\, da}{d\rho}$. So we have from the mass conservation equation we get this particular form of the equation or this particular formulation this will be useful ahead. Now we consider the momentum conservation across the wave front.

So when you look at momentum consideration you have $P + \rho a^2 = (P + dP) + (\rho + d\rho)(a + da)^2$. So now here you have to expand this formulation of this right hand side and in that the terms $(da)^2$ square is very small similarly $(d\rho)(da)$ they are also very small and you can neglect them.

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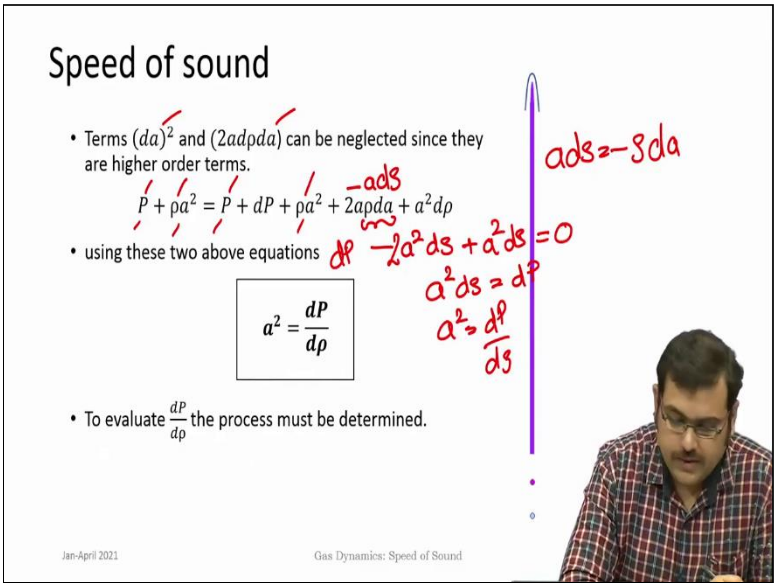

So you get $P + \rho a^2 = (P + dP) + \rho a^2 + 2a\rho da + a^2 d\rho$ and you can also so from these terms this gets cancelled. Similarly this gets cancelled and you can use the equation here that $a = -\frac{\rho\, da}{d\rho}$ and that can be substituted here and when you substitute that equation then you will get $a^2 = \frac{dP}{d\rho}$.

So when you combine these two equations what you get is that all these terms that is due to $(da)^2, 2a(d\rho)(da)$ they are all very small and they can be neglected and you can use the term that you got from the previous equation that $a\, d\rho = -\rho da$. So this is the equation or this is the term that we got from the previous equation and that can be applied over here there is a $(\rho)(da)$ term here and it can be replaced by $a\, d\rho$ and there will be a negative sign.

So that this equation then forms $-a^2 d\rho + 2a^2 d\rho + a^2 d$ and you have a dP term here so this is how you get that is equal to 0, so you get $a^2 d\rho = dP$ or $a^2 = \frac{dP}{d\rho}$. So once you get this so its this relates the speed of sound to the gradient or to the derivative of pressure $\frac{dP}{d\rho}$. Now in order to evaluate the speed of sound you should evaluate this derivative $\frac{dP}{d\rho}$ but we know that we should determine the process by which the sound is actually travelling or the thermodynamic process involved in the travel speed of sound.

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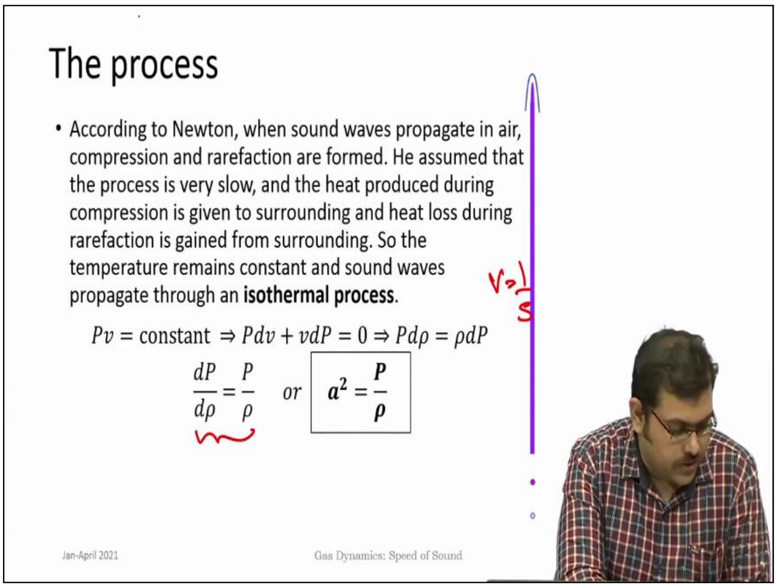

So now that one needs to determine the process when researchers were looking at it right from Newton he sort of said that sound wave propagates in air and when it propagates it does not change things to a great extent. And so he assumed that the *temperature remains constant* as the sound waves propagate through any medium. So he said that it is an isothermal process.

If you consider an isothermal process then the corresponding thermodynamic equation is $PV = constant$ this can be differentiated $Pdv + vdP = 0$. In terms of density it will be $\frac{dv}{v}$ is v is just $\frac{1}{\rho}$ you can substitute that and you can get $\frac{dP}{d\rho}$ is nothing but $\frac{P}{\rho}$ or you get $a^2 = \frac{P}{\rho}$. But it was seen that this was not correct according to experiments there was a discrepancy with experiments.

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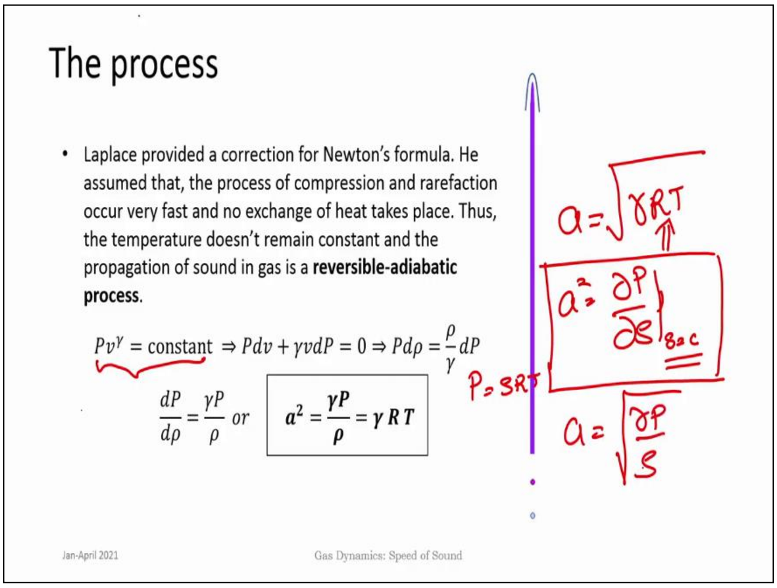

And correction was provided later in the sense that since these processes are extremely fast it moves quite fast there cannot be any exchange of heat in order to have an isothermal process you need to exchange heat and the heat exchange is generally a slow process. So but speed with which some travels is quite fast and there is no time for such heat exchange to take place.

So then the appropriate thermodynamic process is a reversible adiabatic process or that is an isentropic process. When you consider an isentropic process, then the relevant equation is $Pv^\gamma = constant$ and then if you differentiate this and get to $\frac{dP}{d\rho}$ it is $\gamma \frac{P}{\rho}$ or $a^2 = \frac{\gamma P}{\rho}$. And if you can also couple this with the equation of state $P = \rho RT$.

Then you get $a^2 = \gamma RT$ so now here we come to the equation $a = \sqrt{\gamma RT}$. Now remember in all these conversions we have assumed that the gas is a perfect gas and only in the case of perfect gases we can use these kinds of simplifications where we are saying gamma is $\frac{c_P}{c_v}$ and so on. But the most general equation is actually a^2 *is* $\left(\frac{\partial P}{\partial \rho}\right)_{s=c}$; entropy is equal to constant.

That is the correct thermodynamic process to be used and when you use that then this is the most general formulation for speed of sound. This is applicable in any scenario. While in specific to perfect gases you can simplify this to $\sqrt{\gamma RT}$ another equivalent formulation is using it is $\sqrt{\frac{\gamma P}{\rho}}$.

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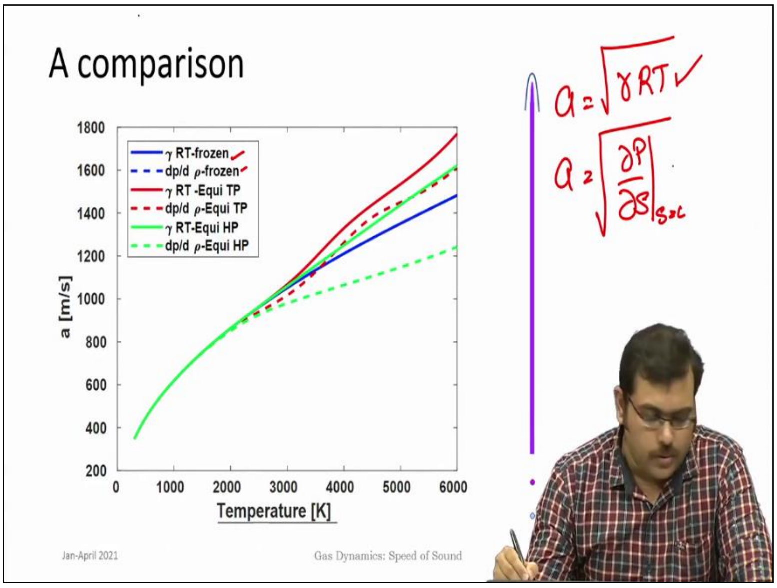

So we have done a bit of computation or a simple analysis to understand how the speed of sound varies with the temperature and what is a relevant equation to be used. This is; in a context of changes of c_P and c_v vs temperature, increases to higher values and you can and these are all plotted and there is a comparison between using the formula $a = \sqrt{\gamma RT}$ and determining a by square root of this generic formulation that is at entropy equal to constant.

This is done using computational tools and here you can see the plots and while for perfect gases which is represented by the blue line which is the perfect all through there is no difference between γRT and $\frac{dP}{d\rho}$. But when you bring in high temperature effects and there is other changes like chemical reactions happening then it can depart from such perfect behaviour and that departure is seen by these other lines there is a difference between γRT and $\frac{dP}{d\rho}$. .

So, depending on the situation and the problem under consideration while in majority of the cases that we do here we will be dealing with perfect gases and this formulation holds good. But if situations arise then we have to go back to the more fundamental equation which is this one which is the square root of $\left(\frac{\partial P}{\partial \rho}\right)_{s=c}$.

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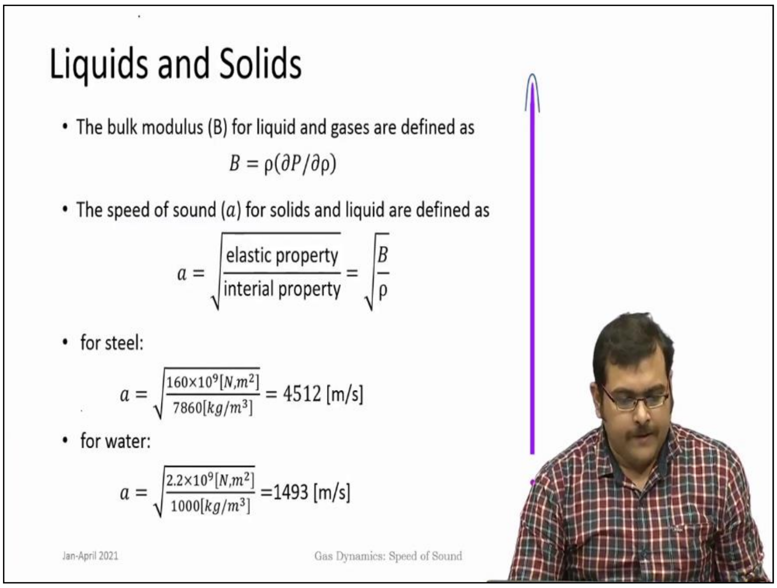

Now speed of sound is not just in gaseous medium it is also for solids and liquids and there the relevant property is the bulk modulus basically how changes of density with pressure that is what is the relevant property? So the speed of sound is related to the bulk modulus and the density so square root of bulk modulus by density. And you can see that for solids it will be very high the speed of sound can be very high for four thousand meters per second in steel while for water its about $1.5\ km/s\ or\ 1500\ close\ to\ 1500\ m/s$ in water. While for air at normal temperatures the speed of sound is around $347\ m/s$.

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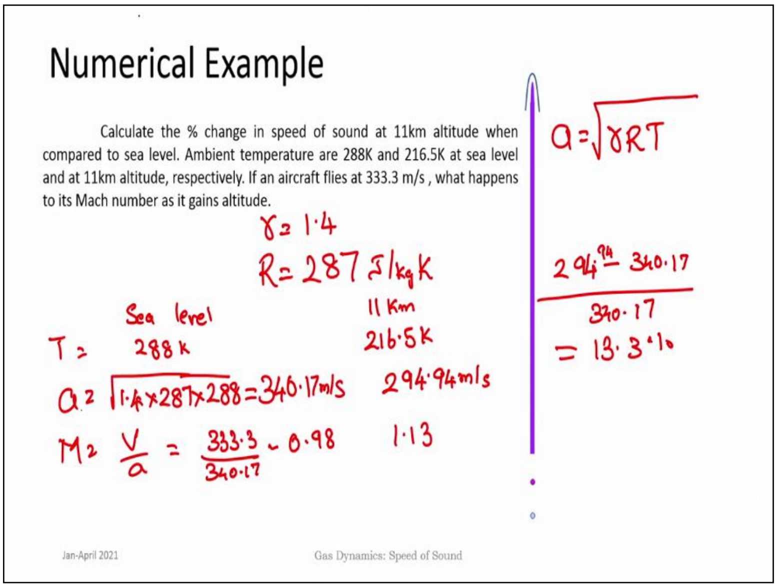

So let us just do a simple numerical based on the speed of sound. So here calculate the percentage change in speed of sound at $11\ km$ altitude when compared to sea level. Ambient temperatures are $288\ K\ and\ 216.5\ K$ at sea level and at $11\ km$ altitude respectively. So if an aircraft flies at $333.3\ m/s$ what happens to its mach number as the altitude changes as gains in altitude.

So let us look at this so what is given here so this is air so air has γ is 1.4 and generally R is taken as $287\ \frac{J}{kgK}$ and you have this at sea level and at $11\ km$ of the altitude temperature is given at sea level is 288 K here it is 216.5 K and the formula is $\sqrt{\gamma RT}$. So, one can find the speed of sound here. So a is in this case first case which is $288\ K$ this is $\sqrt{\{1.4 * 287 * 288)}$ is $340.17\ m/s$.

While at this case at $11\ km$ this is the same formula you have to substitute T as $216.5\ K$ and you will get $294.94\ m/s$. So what happens to mach number? Mach number is $\frac{V}{a}$ and now you can see that in this case the Mach number is $\frac{333.3}{340}$ which is close to 0.98. While on this side at $11\ km$ the mach number you can see that the speed with which the aircraft flies is greater than the speed of sound.

And that is the Mach number here is 1.13. Now the percentage change in speed of sound as the there is an increase in the altitude to 11 km is $\frac{294-340.17}{340.17}x100$ and this is change is about 13.3% there is a 13.3% decrease in the speed of sound. So as the flight moves to higher altitudes temperatures drop and there is a possibility that the mach number of the flight can change.

So with this we end the discussion on speed of sound we can now look at few numerical examples in the next class and that is the end of this class.