Gasdynamics: Fundamentals and Applications
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Lecture 08
Quasi-1D Assumption

So we are considering the fluid flow equations for gas dynamic flows and previous classes we looked at general integral and differential forms of these equations. Now let us become more specific and look at particular assumption that is very often used to understand gas dynamic flows and it is quite useful which is the Quasi-1D assumption.

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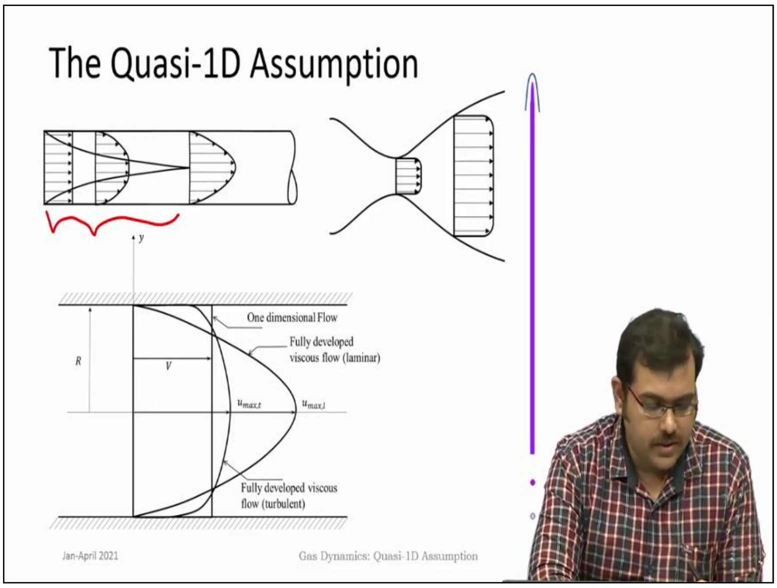

So what does this quasi-1D assumption imply? Let us take for example flows in ducts or in long pipes. If you look at the distribution of the velocities they have if you just go away from the entrance length which is over here which is where there is changes of the profile but after the entrance length the velocity profile is more or less frozen into this particular shape this is for a laminar profile.

While for a turbulent case the profile will be more sharp and then having a more rounded off shape in the centre. So now there can we can come up with an equivalent sort of profile or equivalent profile which has a constant velocity or the flow property is constant across the entire cross section of the duct. So, in order to do that the principles that; we use are still the same that all conservation should be followed.

So this is a the one dimensional profile having a constant velocity $\vec{V}$. Now if you look at the shapes for the laminar profile and the turbulent profile you see that turbulent profile more or less approximates this constant property across the duct while for laminar profile the variation is quite significant. So this is a simplification that can be used in order to understand how the flow behaves in many different scenarios.

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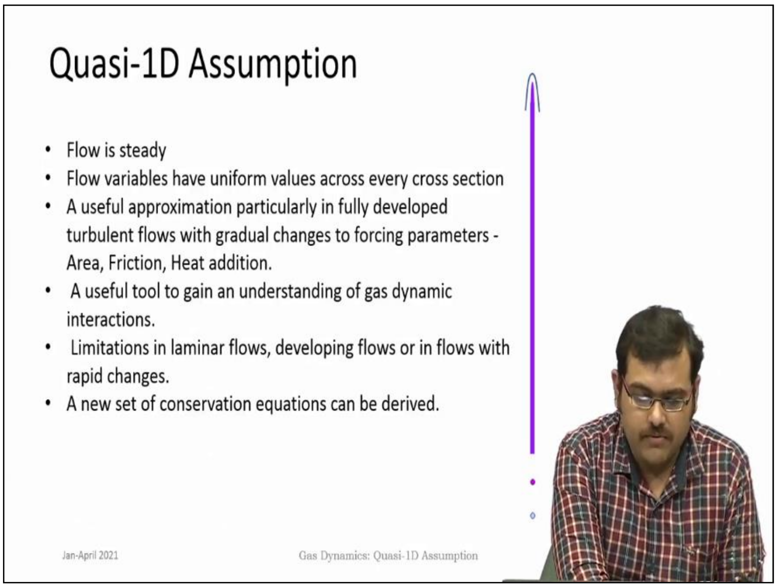

So there are many different things that can change in a compressible flow like you have changes in the duct area or you could have certain heat being added to the flow or you could have that there is a force of friction which is always present in these flows. So this quasi has a 1D assumption where you say that flow variables are uniform across every cross section. So if you consider a variable duct then across each cross section you are saying that that the flow variables like velocity pressure and temperature are uniform.

But they can change along the duct so along if this I consider as x-direction, along x-direction they can change but across this coordinate that is the transverse or y-direction or if it if you consider this as an axisymmetric this can be $\vec{r}$ the so in along those coordinates the flow variables are constant. This is a particular assumption which is the Quasi-1D assumption. And its quite reasonably close to turbulent flows because their real actual velocity profiles have similar nature.

And it is very useful tool in order to understand these different forcing due to area friction and the heat addition. So as taking this assumption one can derive a separate set of conservation equations. And bear in mind that these Quasi-1d assumption is applicable when there are only gradual changes to area friction and so on.

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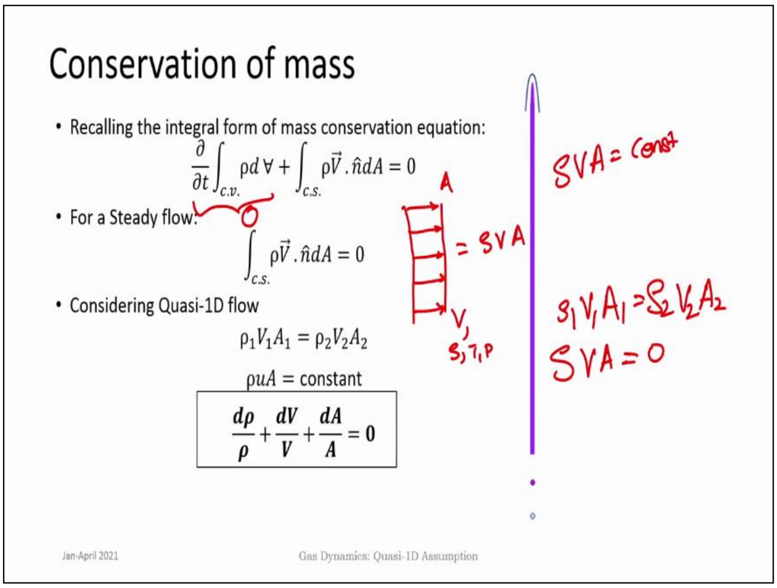

If sudden changes in area happen then it is not possible to use this approach to come with reasonable estimates. So let us take the integral form of the mass conservation equation we are considering only steady flows because of that this term is 0 and that is due to the unsteady term and we have only the flux term across the control surface at this $\rho V\ A$ and that is $\int_{cs}\rho\vec{V}\cdot\hat{n}\,dA$. And now the velocity is considered to be uniform across the cross section.

So if I consider a cross section here a and then the velocity profile is uniform at V. So then this is nothing but this integration directly gives us so is density not just velocity, density-temperature- pressure they are considered to be uniform across the cross section. So this will be $\rho V\ A$ that is the area of that duct or that particular cross section. So then if we consider 2 such cross sections then this statement of mass conservation implies $\rho_1V_1A_1\ =\ \rho_2V_2A_2$ or $\rho V\ A$ is constant.

Or $\rho_1V_1A_1\ =\ \rho_2V_2A_2$. Now V is just a variable it is just one value that is it is a uniform velocity here so given $\rho V\ A$ is constant, it can be written in the differential form also $\frac{d\rho}{\rho}+\frac{dV}{V}+\frac{dA}{A}=0$ this comes by just differentiating $d(\rho VA)=0$ equal to zero. So if you differentiate this you will get this form of the equation. So in the integral form it is ρVA is constant. And in differential form $\frac{d\rho}{\rho}+\frac{dV}{V}+\frac{dA}{A}=0$.

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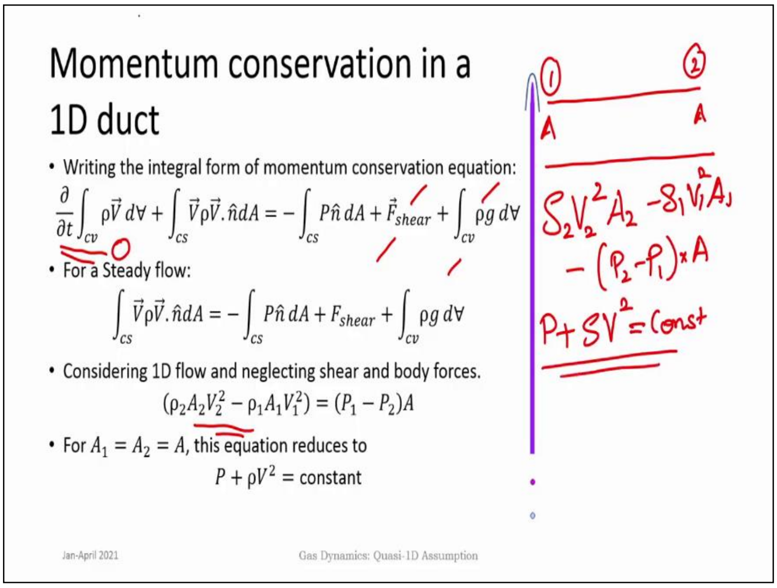

Now when we consider conservation of momentum there are two kinds one you can consider in a 1D duct that is the area cross-sectional area of the duct does not change so it remains constant so it is A everywhere. So, the other one is that you can consider a variable area duct which has a gradual change in area. The slight differences in the analysis though the principle is the same.

Again if you write the integral form of the momentum conservation and consider steady flows then the unsteady term is 0. Then you are left with the fluxes and the control surfaces. And your generally the shear forces are neglected so this term is neglected and so is the body force term. Body force term is also neglected and mainly because the forcing is due to pressure and so since the velocity and density have uniform profiles this integration turns out to be ρV^2 .

So, ρV^2 and you have two sections so one and two. So this will be $\rho_2 V_2^2$ - $\rho_1 V_1^2$ so which is the term that comes right here and then coming to the pressure forces. The pressure forces have negative signs. So this is actually minus $(P_2 - P_1)A$, where A is remaining constant so this is considered over here.

Now this if you do the rearrangement you come to the fact also using the fact that $A_1 = A_2$, you can write $P + \rho V_2^2 = constant$. Now this is valid for only one-dimensional ducts this integral form $P + \rho V_2^2 = constant$ for one dimensional duct.

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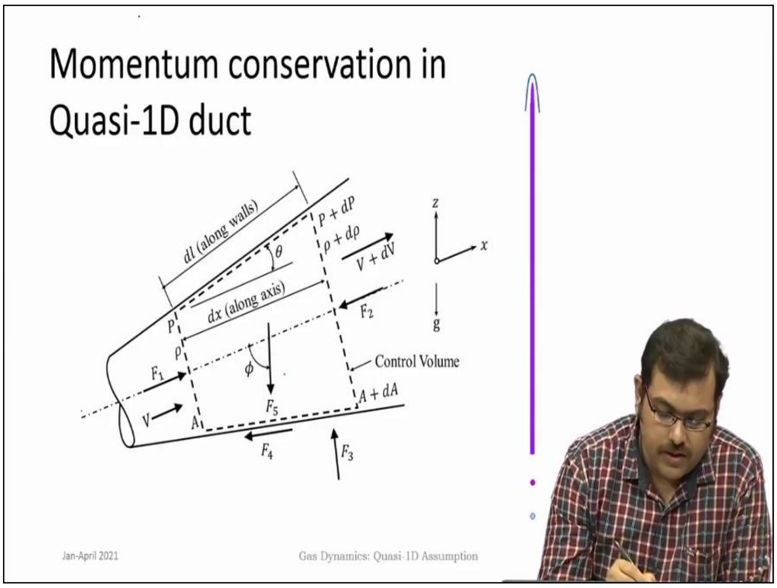

But when you come to ducts which are quasi-1d that means there is a gradual change in area then when you consider this momentum equation it has to be considered carefully by considering the area change also. Because now when you consider the pressure forces in the 1D case since pressure is uniform the integral of pressure over the lateral surface, lateral surface will turn out to be 0 because the of the cancellation of forces.

And only the pressure force at along the x direction will be affecting. But in variable area ducts which have gradual changes in area then you see that there is a component of the pressure force that varying pressure force along the wall or along the lateral surface that will appear along the x direction it is we can get that particular force also. So this has to be considered a little bit more carefully.

So here we have in this case a very gradually varying area with a certain divergence say θ and the forces the variables are P, ρ, V, A on the left hand side. And on the right hand side there is a small change in them $P + dP,\ \rho + d\rho,\ A + dA$ and so on. This force is due to gravity body force that is F_5.

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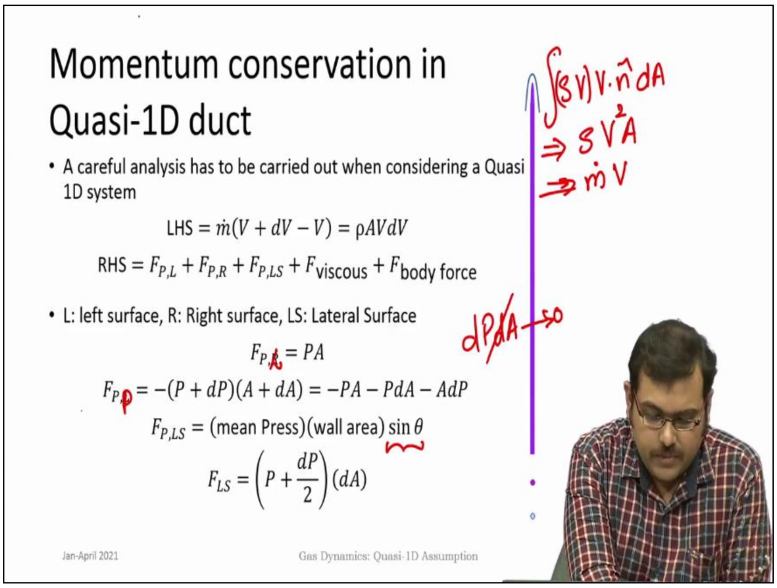

So now let us consider this so this has to be considered now. Now the integration can be performed because you are considering uniform variable. So you have on the left hand side this nothing but $\int(\rho\vec{V})\vec{V}\cdot\hat{n}\,dA$ so this is $\rho V^2 A$ which can also be written as $\dot{m}V$ that where $\dot{m}$ is constant. So that is how this term comes about $\dot{m}(V + dV - V)$ that is the left hand side which is $\rho A\,V\,dV$.

So now coming to the the right hand side of the equation these are the different force terms that is the forces due to; now all of them we are considering only along the x direction so that has to be borne in mind. So let us the force on the left surface the right surface the lateral surface, viscous force and the body force. Now pressure force on the pressure force on the left surface. So this is pressure force on the left surface which is PA.

So this has to be L and while on the pressure force on the right surface is $(P + dP)(A + dA)$ so this can be expanded and you are considering $dP\,dA$ is very small that can be neglected it can be neglected. So if you consider that condition then you can write this equation as $-PA - P\,dA - A\,dP$. Now you have to consider the pressure on the the force on the lateral surface due to variation of pressure.

Now and the component that appears along the x direction so this is the force and the component appearing so this is theta actually so that component appearing along the x direction is has this sine theta component. Now so the mean pressure so the force on the lateral surface area is the mean pressure multiplied by wall area multiplied by the sine of that component.

And if you do that the mean pressure is $\frac{P+dP}{2}$ and if you do that the sine component actually turns out to be dA that is the change in area. So you get $\frac{P+dP}{2}dA$ its mean pressure multiplied by 1.

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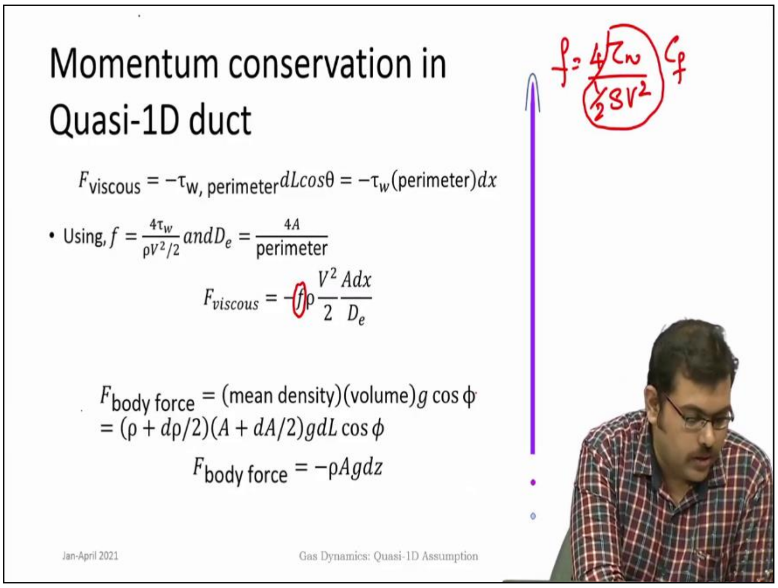

Now the viscous force is the shear stress along the wall multiplied by the perimeter area which is you can look at these and if this is dL this is $dL\cos\theta$ its along the x direction. So it is along the x direction and you can use the definition of the frictional factor that is or the coefficient of friction. So there are two kinds which can be used the way it is defined over here this is the friction factor which is f is $\frac{4\tau_{wc_f}}{\frac{1}{2}\rho V^2}$ while this term $\frac{\tau_w}{\frac{1}{2}\rho V^2}$ is the coefficient of friction.

So and thus the viscous term if this can be considered a mean friction factor then the viscous term is given by $f\rho\frac{V^2}{2}\frac{Adx}{D_e}$ and the body force is nothing but mean density multiplied by the volume and multiplied by the gravity constant $(\rho v g)$.

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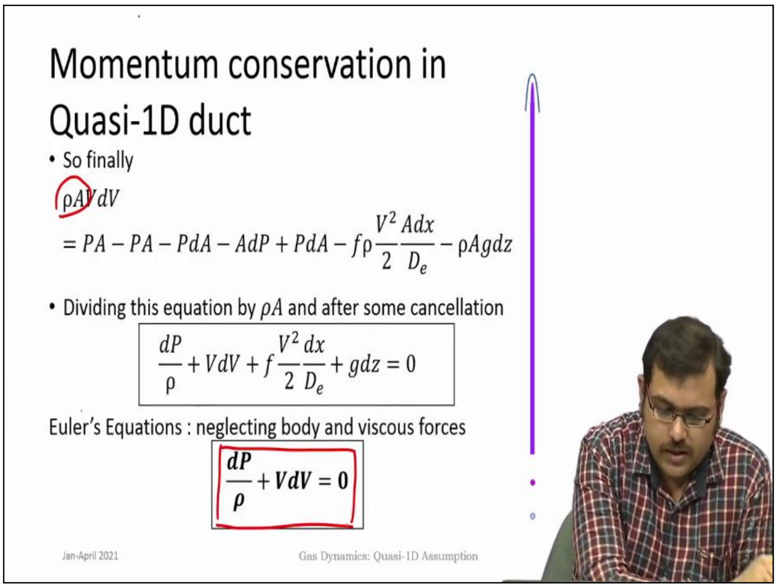

And the component that has to be considered along x-direction is the $\cos\phi$ term. So once that is done then you will get $gdL\cos\phi$ which is here, this is the component along the x direction and $dL\cos\phi$ is actually dz. So you get $\rho Agdz$. Now you can finally sum everything up so once you sum up everything then you are left with this equation. So and you can divide throughout by $\rho A\, V$.

So once you divide everything by ρA that is this term then you get $\frac{dP}{\rho} + VdV + f\frac{V^2}{2}\frac{dx}{D_e} + gdz = 0$. So if you neglect body and viscous forces then what we get is $\frac{dP}{\rho} + VdV = 0$ this is the Euler's equation. So now we see that for a Quasi-1D duct the final equation is of this form. And when you are considering Quasi-1D deducts you have to be really careful with the analysis and should consider the lateral forces also.

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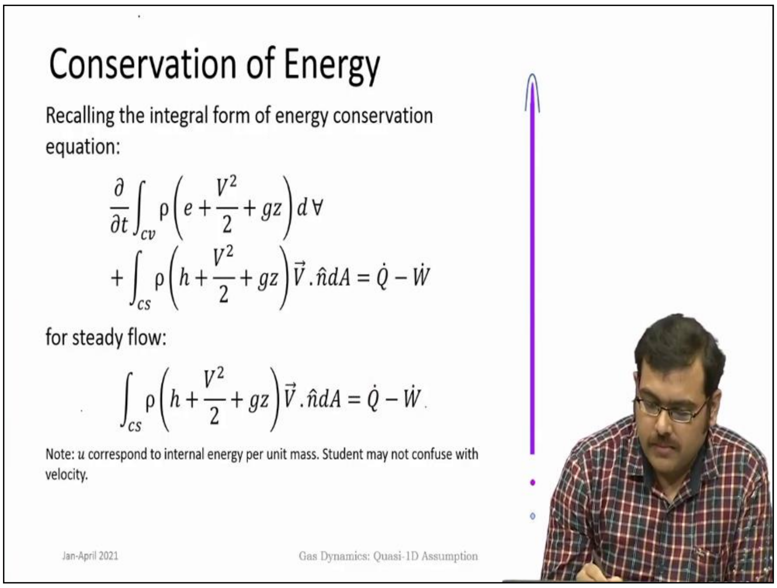

Now having considered mass and momentum now we come to the energy equation. Now when we look at this energy equation again we are considering steady flows. So for steady flow energy equation you have $h+\frac{V^2}{2}+gz$, h is the enthalpy and on the right hand side you have net heat added to the system and work done by the system.

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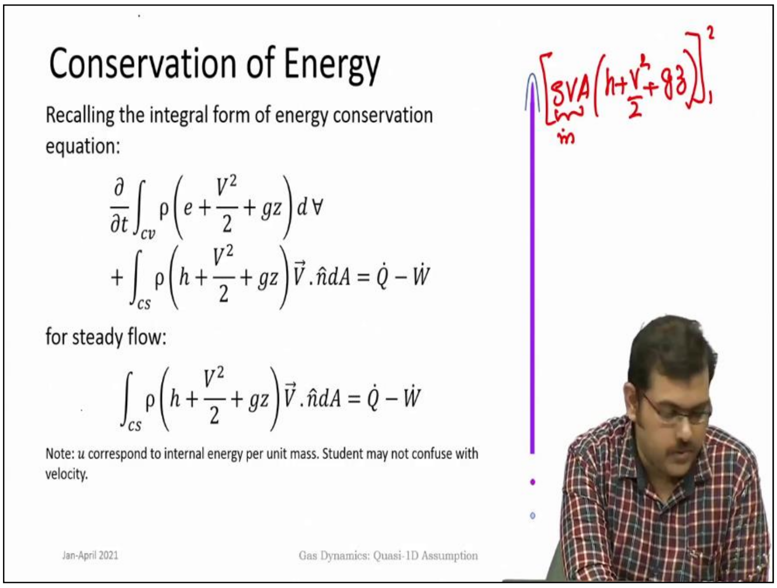

So now you can this is the total energy total energy is $h_1+\frac{V_1^2}{2}+dz$ that is now since all parameters are constant you get $\left[\rho VA\left(h+\frac{V^2}{2}+gz\right)\right]_1^2$ this will be the integral across two surfaces so going from one to two. So this is the integral so this is nothing but mass flow rate $\dot{m}$ and this is the total energy $h+\frac{V^2}{2}+gz$.

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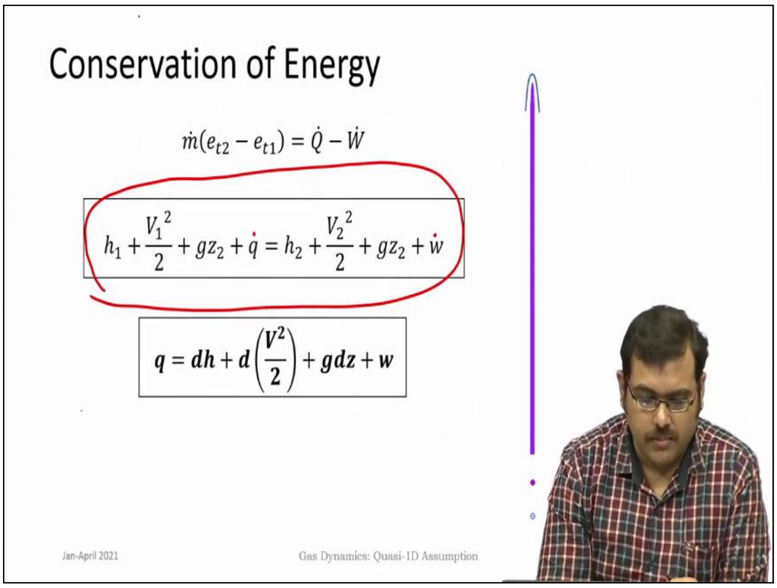

That is what is written over here $\dot{m}(e_{t2} - e_{t1})$, total energy is equal to $\dot{Q} - \dot{W}$. So now this has been expanded and you can write this in terms of enthalpy. So this is the final equation that you get after integrating $h_1 + \frac{V_1^2}{2} + dz_1 + \dot{q} = h_2 + \frac{V_2^2}{2} + dz_2 + \dot{w}$. This can be differentiated also and this is the rate at which the heat is added $\dot{q}$ is rate at which work is done $\dot{w}$ and $dh + d\left(\frac{V^2}{2}\right) + g\,dz$.

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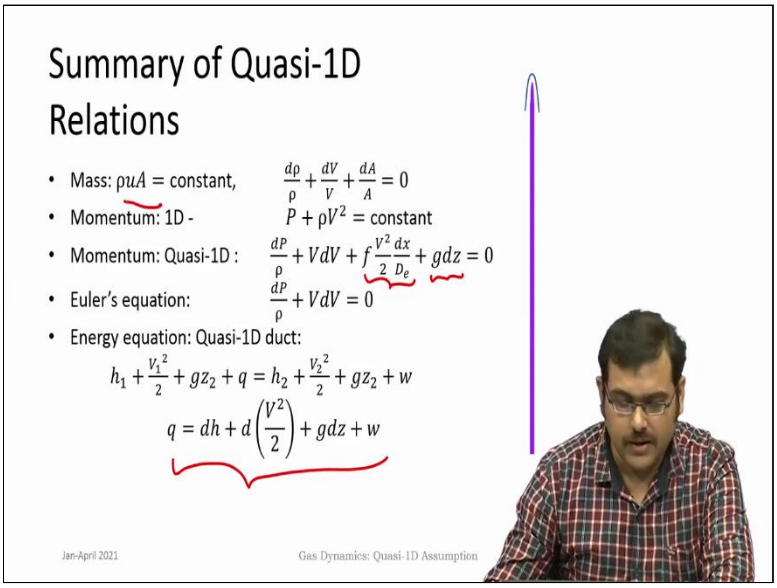

So if you look at all the quasi-1D relations you have in the integral form the mass flow rate, total mass flow rate through the system remains constant that is $\rho u\,A = constant$. If you differentiate it you get $\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$. Now if you consider the momentum in a 1D duct then it is $P + \rho V^2 = constant$. While in a quasi-1D duct that is the area is changing very gradually the relevant equation is $\frac{dP}{\rho} + VdV$ + friction term + body force term.

And if these are negligible $\frac{dP}{\rho} + VdV = 0$ for a quasi-1D duct the energy conservation is the total change of energy is related to heat added and work done and it can be written even in the differential form. So now having considered this Quasi-1D relations is a very useful assumption to make that the flow variables are constant across every cross section and then we can see how if there is a variation of area.

Or if there is a heat addition or if there is friction how it affects the various flow parameters and that is what we would be doing in further classes immediately. In the next class we will apply the 1D relation to look at how to see the speed of sound? How we can estimate the speed of sound it will be a sort of application of these material that we have just covered at the same time we will also get to know how we can estimate speed of sound.