Gasdynamics: Fundamentals and Applications
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Lecture 06
Flow equations – Integral Form

Let us begin the lecture today we look at flow equations. Until now we have been looking at thermodynamics in the context of gas dynamics or compressible flow. Again please go through thermodynamics if you are not comfortable with it and thermodynamics is essential to the study of gas dynamics. Along with thermodynamics come the fluid dynamic equations that the equations for fluid flow.

So today we look at the integral forms of these equations. There are different ways in which fluid flows can be analyzed. This particular approach is known as the control volume approach.

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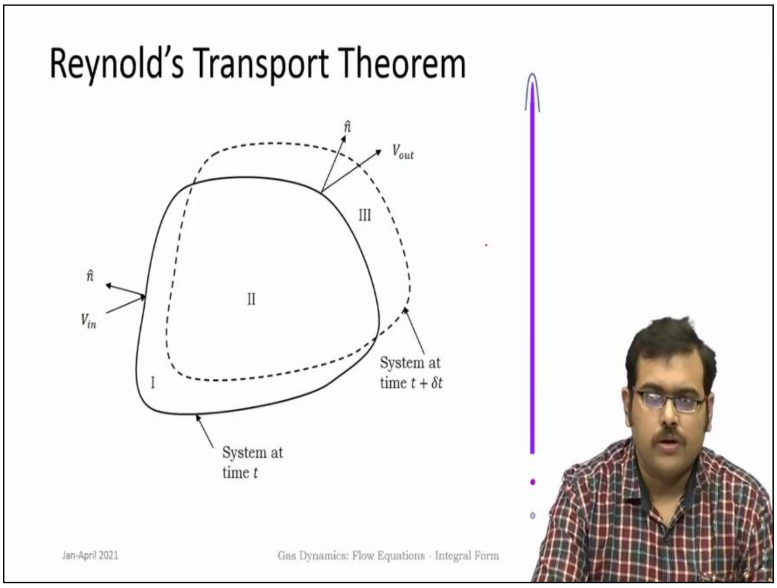

So for control volume approach the essential sort of methodology approach is the Reynolds transport theorem where an arbitrary control volume is sort of constructed around the fluid flow that we want to analyze. And we look at the conservation of different quantities like the mass the momentum and the energy within the control volume. There are several advantages with this kind of approach.

We do not specifically need to know all the details of the flow that go inside in such a control volume and you would be more interested in looking at what happens at the control surfaces the fluxes in and out and if there is any change to the parameters within the system. This is distinct from a kind of in differential analysis which needs complete information at every point in the flow.

So this quite powerfully used for many, many applications to come up with relations very easily that can be used for parametric analysis and so on. So how does this; so once the control volume is defined the system changes with both time so you are looking at this control volume and looking at how does the system change with respect to time.

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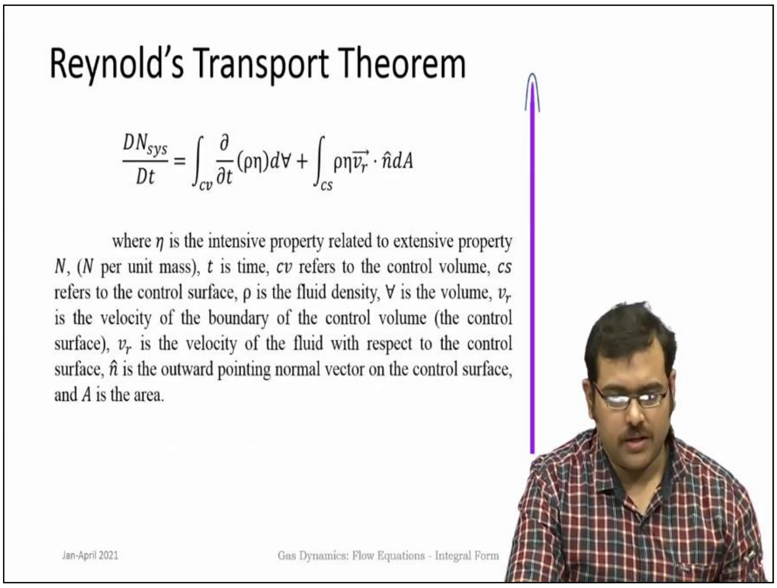

So N is any property of the system and that is the total property while that is N over here while η what is represented here, is per unit mass. So for a certain volume of fluid within a control volume the mass is given by the density multiplied by the control volume. So in this case we are taking a small volume that is dv. So this is the mass of the system. So, multiplied by any intensive property.

So the rate of change of that property with time and the total change that is called the change within the system within time happens due to two things. One is due to the change of the property within the control volume itself or due to the inflow and outflow of these properties as the flow passes through the control surfaces. So that is the flux of the variable and that part is coming over here.

So the total change of the system or what is known as the material derivative or looking at the system by following the particle or the system along its motion that change is related to two quantities that is change within the control volume and the fluxes which come into and go out of the control volume. So here $\hat{n}$ is the normal vector to the control surfaces.

So this is a very generic sort of expression which can be applied to many, many variables it relates the rate of change of that property with respect to time.

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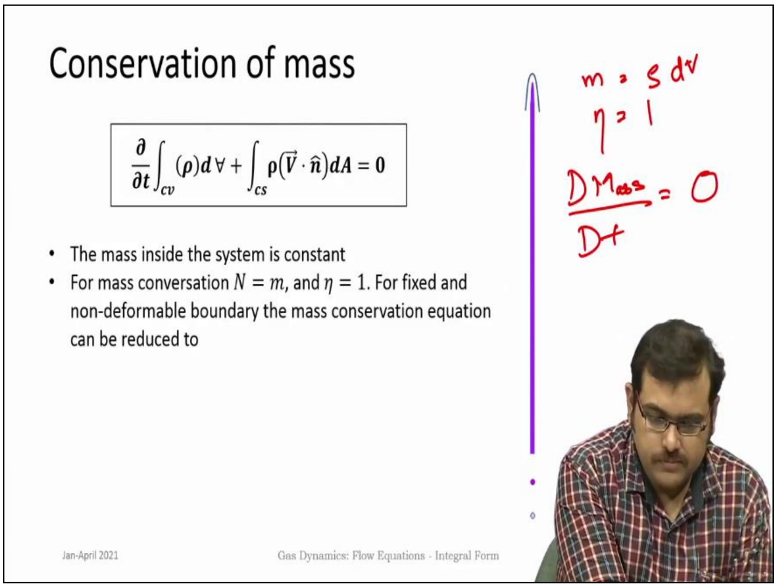

So now let us look at different conservation laws for the fluid flow systems. First one we will look at is the conservation of mass. So the total mass within the system gets is conserved, mass does not change. So what we need to look at is, look at the Reynolds Transport Theorem and find out what is that intensive quantity. In this case the mass of the system is given by density multiplied by volume itself.

So because of that the value of that intensive property is 1. So if you look at the Reynolds transport theorem and put $\eta = 1$ you will get the equations for conservation of mass in the system. So, mass does not change. So because of this on the right hand side there is no $\left(\frac{Dm}{Dt}\right)_{system} = 0$. So because of that on the right hand side you have zero.

And $\rho\vec{V}\cdot\hat{n}$ is the flux of mass out of the system into and out of the system. So we are considering in this case, a fixed control volume that the boundaries do not change in time if they undergo motion then the relative velocity has to be considered. But for all of our practical purposes we consider a fixed control volume and because the control volume and control surfaces are fixed then the derivative of time can be taken out of the integral otherwise it has to be within the integral.

So for a non-deformable boundary this statement that is $\int_{cv} \rho\ dv$ that is mass within the control volume plus changes due to the fluxes which come in and out of the control volume through the control surfaces is going to be zero. So this statement is straight forward and it is the conservation of mass.

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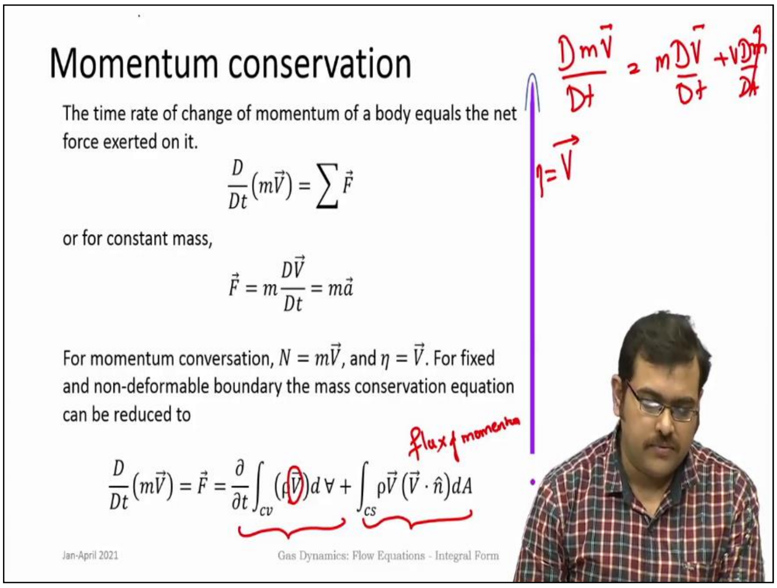

Now let us look at the momentum conservation. So now momentum conservation or the momentum equation is nothing but the statement of the second law of motion. Newton second law of motion; written with the fluid dynamics perspective. So you are looking at rate of change of momentum which is $\frac{D(m\vec{V})}{Dt}$. Now here you should pay more attention because momentum is a vector quantity.

So the; directions of these each component of momentum has to be considered carefully. Now already we know that mass is getting conserved. So you can write this can be written as following the principles of $\frac{m(D\vec{V})}{Dt} + \frac{\vec{V}(Dm)}{Dt}$ but from mass conservation this is anyway it is not there it is zero. So, it is $\frac{m(D\vec{V})}{Dt}$ and what is the net change of momentum equal to it is equal to the net force that is applied on the control volume that is being considered over here.

So $\frac{m(D\vec{V})}{Dt}$ so m multiplied by total derivative of $velocity, \vec{V}$ that is nothing but acceleration that is equal to the net force applied on the control volume. So now if we expand that now the intensive property here is the $\vec{V}$ and so we can $\eta = \vec{V}$ and that is how it is getting substituted over here. And for a non-deformable control volume again we can write by the Reynolds transport theorem.

The rate of change of momentum is equal to the sum of rate of change of momentum within the control volume with respect to time and change due to the flux of momentum which is the this is flux of momentum that comes from the different control surfaces around the control volume. So what is this now equal to so this has to be equal to the net force being applied on the control surface and that has to be considered carefully. So what are the different forces that come on to the control surface.

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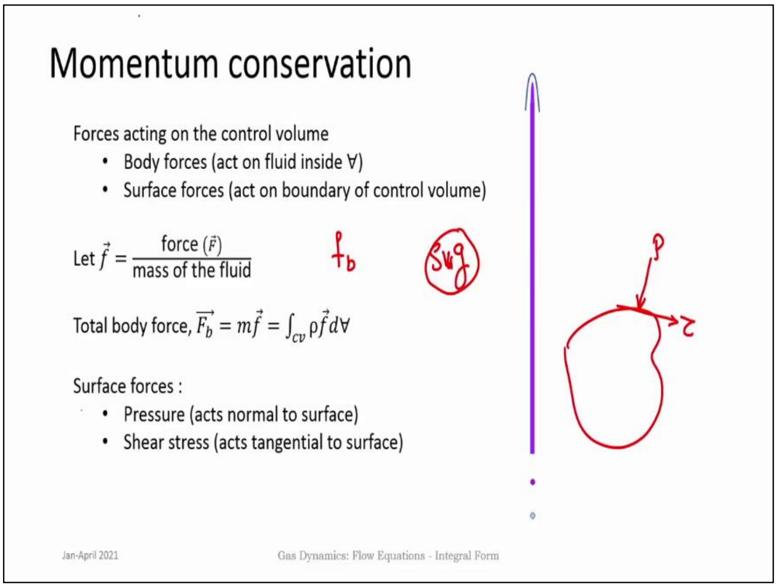

So the different control volume the different forces that come onto the control volume are body forces which act on all the fluid inside the control volume and the surface forces which act only on the boundary of the control volume. So are on the control surfaces and $\vec{f}$, a typical $\vec{f}$ parameter is the force per unit mass of the fluid. So for body forces this can be a $\overrightarrow{F_b}$ that is you can say it is the body force.

And typically the body force that will be a considered is the weight of the mass that is present within the control volume which is just nothing but the gravity force that is it will be $\rho v\vec{g}$. So this is the body force that is commonly considered in these kinds of flows. Now the different surface forces that come onto the control surfaces are the pressure forces and the shear stress force or shear stresses.

And we know from their characters that pressure always acts normal to the surface and shear stress which is usually represented by $\vec{\tau}$. So this is pressure, shear stress acts tangential to the surface. So when we consider these pressure forces and shear stress forces we have to take appropriate components when we are doing the analysis. So details of how we will go about doing this for particular cases like the approach taken for a one dimensional system will be dealt in detail in coming classes but you have to understand the principle.

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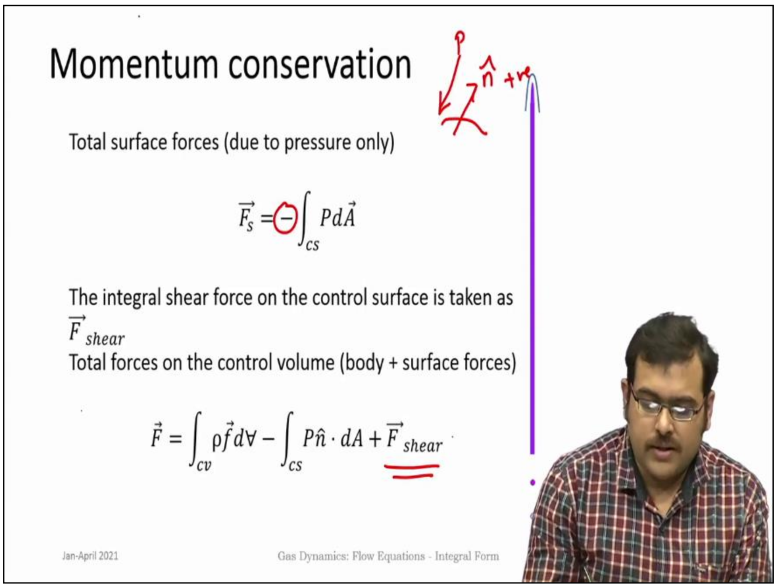

So now the pressure force always acts inward to the control surface so it will have a "-ve" sign so the force is acting inward. The sign convention is outward normal is positive. So this is positive and pressure acts in a direction which is opposite to that so you get negative pressure. So the integral pressure force over the control surface is $-\int_{CS} P \; d\vec{A}$ over the control surface.

Now shear stresses the overall the complete integral is taken as a lumped parameter $\overrightarrow{F_{shear}}$ force of shear. We will generally these forces are quite small compared to the pressure forces and they are considered in even most of the analysis they are considered as very small and negligible in the inviscid kind of analysis. But there are certain cases when we consider the shear forces also.

So when we come to that we will go into the detail of how to include shear forces otherwise the integral can be lumped into $\overrightarrow{F_{shear}}$.

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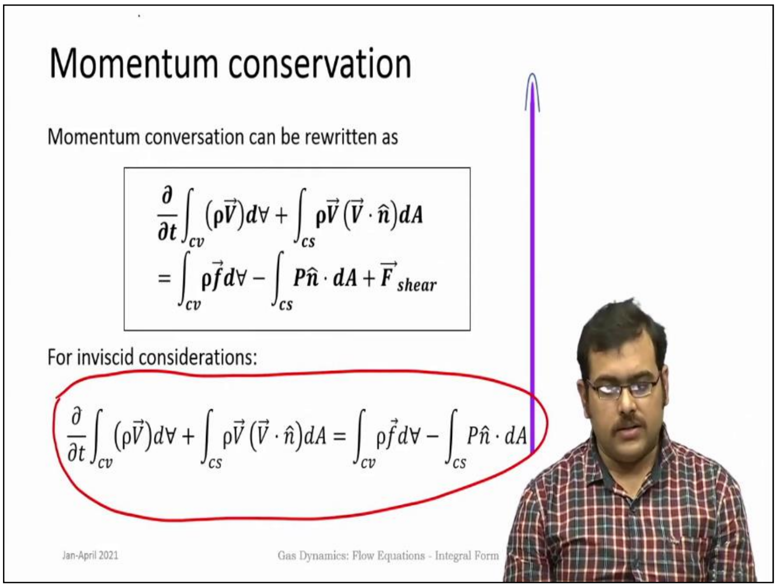

So now we are in a position to write the complete momentum conservation equation or the second law of Newton second law for the fluid flows which is a rate of change of mass within the control volume plus the fluxes across the control surfaces is equal to the sum of body forces and the pressure force and the shear force. So this is the momentum conservation equation.

Now in as I was discussing in majority of cases, we first neglect the shear forces because it is considerably smaller. So, we do not consider shear forces for the first cut analysis and then later on we can add if it is important. So inviscid considerations we do not consider $\overrightarrow{F_{shear}}$ and the equation that comes about is given here. So, we have done conservation of mass where mass is conserved.

The second is momentum conservation then the next that comes into picture is the energy conservation i.e., conservation of energy.

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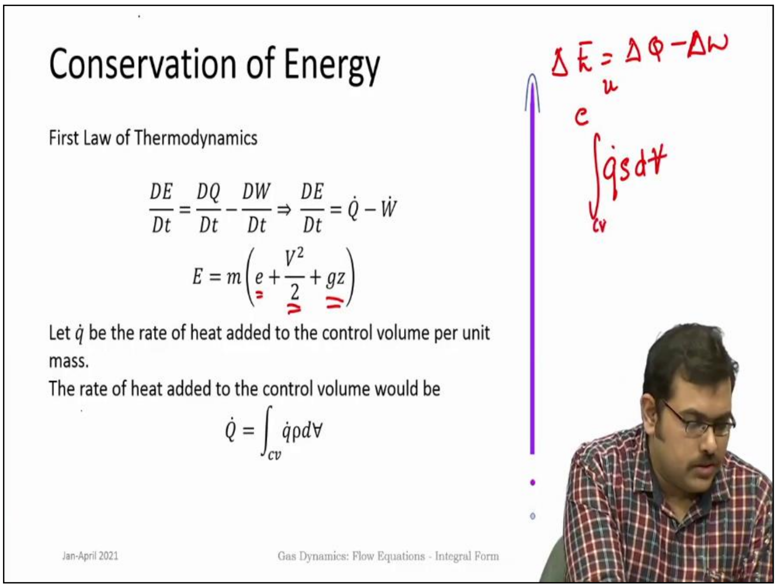

So conservation of energy is nothing but the statement of the first law of thermodynamics. Again we use the Reynolds transport theorem which relates the total derivative of a quantity to the changes within the system and the fluxes across it. Now what we consider is the total energy here and we know that from the statement of the first law of thermodynamics this is equal to change in energy is equal to rate of heat added to the system and the work done.

So if this is taken as negative so, that means, work done by the system so $-\Delta W$. So now in this case from in the previous discussions of the thermodynamics we considered the internal energy to be a variable u, but specific internal energy. But now if we look into fluid flow equations particularly in Cartesian coordinates then u is generally used for the velocity in the x direction.

So in order to avoid confusions in these variables we will go because we will be coupling this with fluid flow. We will go with the variable e for the specific internal energy. So the total energy of the system is a combination of or sum of internal energy, kinetic energy and potential energy which is given here. So this given here and as stated the total energy is mass times this summation of all the energies $m\left(e + \frac{V^2}{2} + gz\right)$.

And $\dot{q}$ can be the specific rate of heat added to the control volume per unit mass. So, it is a specific quantity and similarly one can represent $\dot{W}$ also. So the total heat that is added to the control volume can be represented as $\int_{CV} \dot{q}\, \rho\, dv$ over the control volume. So, total amount of heat that is added to the control volume. Now so from this the intensive property for energy is this parameter $\left(e+\frac{V^2}{2}+gz\right)$.

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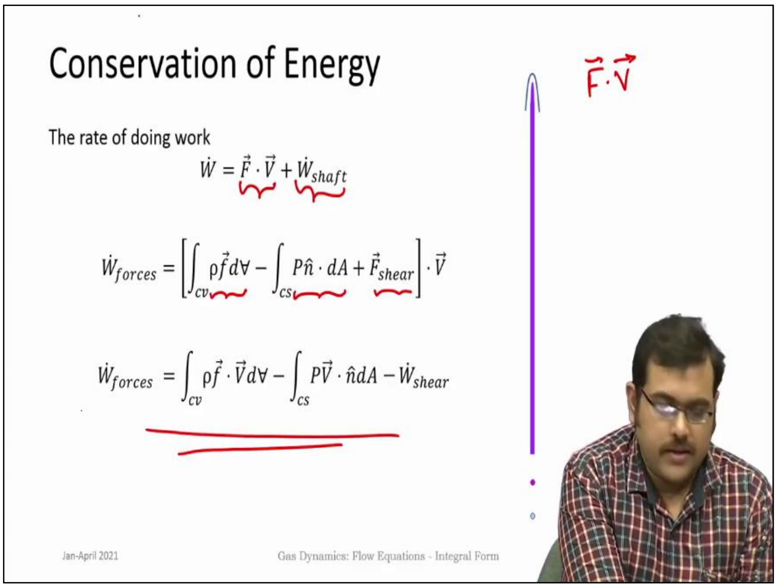

The rate of doing work now work has to be considered carefully. Just in now in the momentum equation we saw that various forces act on the control volume and the control surfaces. So if there are forces and there is motion then there is work being carried out and we can calculate the work that is rate of doing work. Rate of doing work is nothing but force velocity where force dot product of force into velocity which is given over here.

These are all the forces that come due to the fluid on to the fluid flow but you can also have other external forces some work is taken out through the shaft. So as is done when you are using fluid flows for converting in energy and so on. So once that work is additional which is shaft work. So, the different work due to forces will be work done carried out due to body forces and the pressure forces and the shear forces.

So there are the three components and their dot product with the velocity will give you the rate of work done for these forces. While so that is what is represented over here where this velocity is getting inside the integral.

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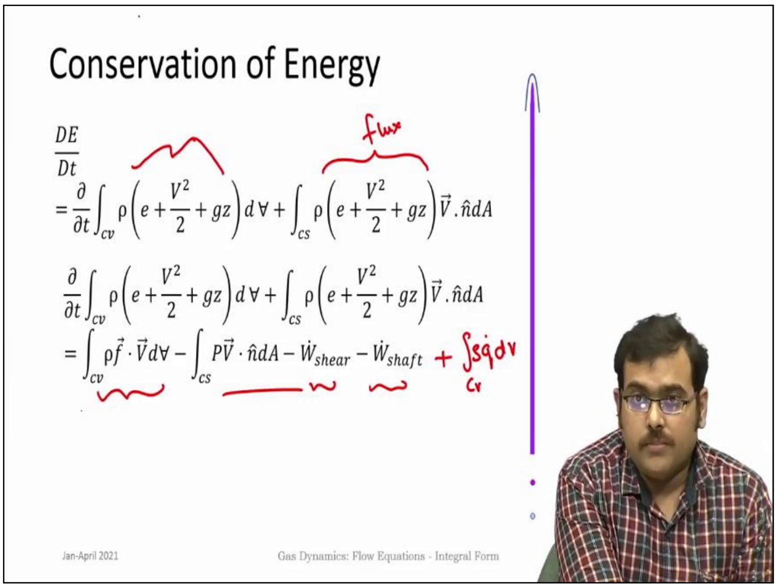

Now if you sum that up what it says is the first law of the thermodynamics where this is the rate of change of energy within the control volume. So this part and this is the flux of energy that total flux of energy that comes across the control surfaces and that is equal to work done by different forces and also you have the work done by shear as well as that of the shaft. And also there is the term due to the heat so that is $\int_{CV} \dot{q}\, \rho\, dv$ that is control volume.

So that is change in rate of change of energy is equal to the total amount of heat that is added to the system and work done on the system.

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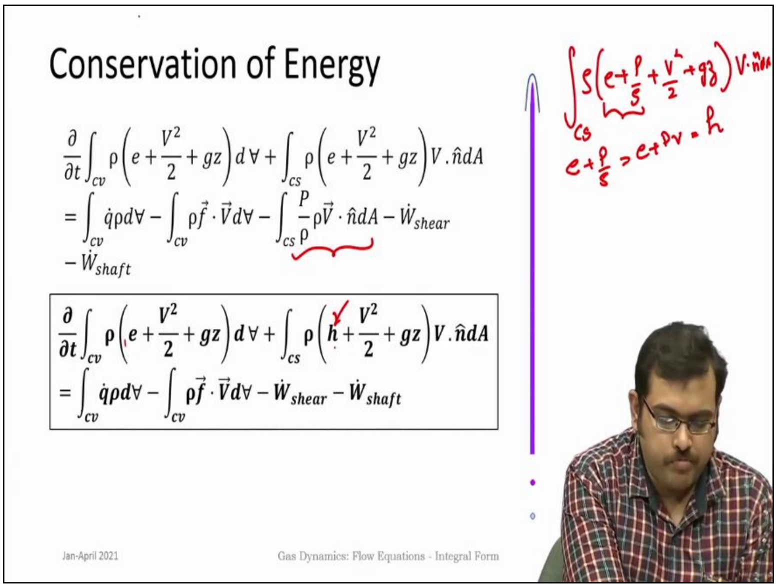

So if you put that and put everything in place then you can give get the final equations. So now we can also carry out a small transformation because you have this variable $-\int_{CS} \frac{P}{\rho} \vec{V} \cdot \hat{n}\, dA$ p.

So this variable is there so that can be included within this is over the control surfaces while this also integration is over control surfaces. So this can be moved to the left hand side of the equation and you will get the term within the control surface as $\rho\left(e+\frac{P}{\rho}+\frac{V^2}{2}\right)$.

So this is what you get so here this the term $e+\frac{P}{\rho}$ is nothing but $e+Pv,\ v$ is specific volume this is enthalpy, h. So you look at this equation you have the term enthalpy coming into picture here at this place. Please note that this is coming in the fluxes and it is not there in the rate of change within the system. So if this also needs to be considered additional terms will come on the right hand side.

So this is rate of change within the system here it is E which is the internal energy but for the fluxes you can consider the enthalpy.

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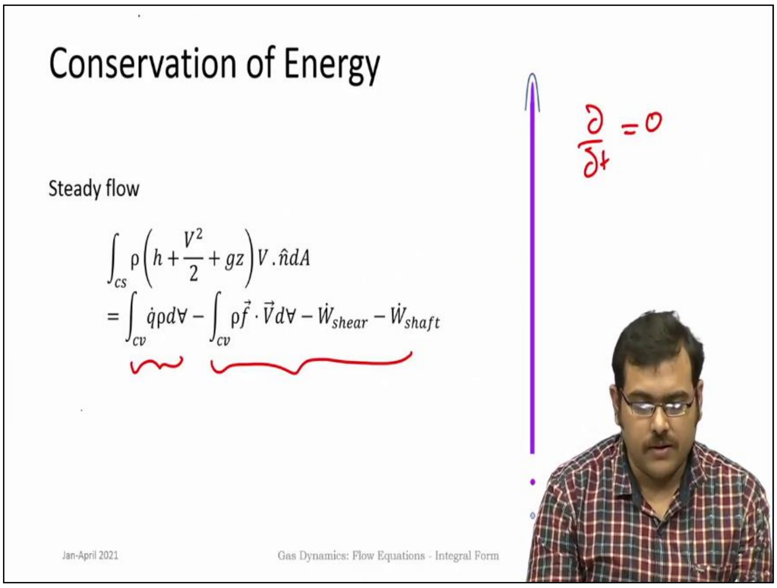

So finally when we come to a steady flow that means the system is not changing with the rate of change with time the current within the control volume is zero so $\frac{\partial E}{\partial t}=0$ as the equation. Then you are left with only the fluxes across the control surfaces and that is over here this $h+\frac{V^2}{2}+gz$, $\int_{CS}\rho\left(h+\frac{V^2}{2}+gz\right)\vec{V}\cdot\hat{n}\ dA$ is equal to the heat added and work done by the system.

So, this is the final conservation of energy. So now we have the three conservation principles all of them put together.

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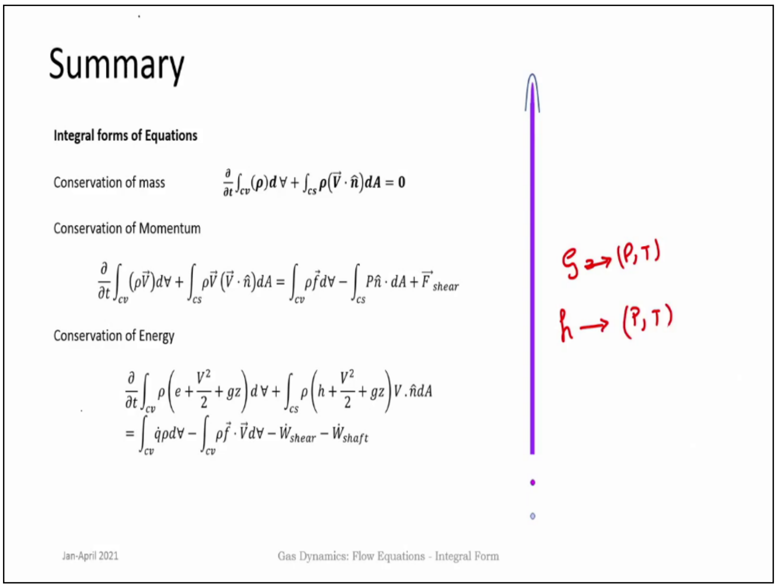

So, any analysis of fluid flows we should include all the three equations conservation of mass, conservation of momentum and conservation of energy. So now we need principles which will tell us how h is related to other variables like h how is it related to pressure and temperature. These information comes from thermodynamics. So and how density is related to pressure and temperature.

So these relations come through the thermodynamics that is how thermodynamics enters into the picture. So this is a coupled system because ρ is not a constant here and all these three equations along with the equation of state have to be used to solve any particular problem. Very soon we will see how this can be applied to get certain simplified forms of these conservation equations in the context of say one dimensional systems. And then we can look into more depth of the application of these principles.

So next class what we will do is look at another form of the fluid flow equations which is the differential form. In differential form it looks at the fluid field itself so its looks at every particular point and the neighbourhood of that point and how the fluid behaves or how it changes in the neighbourhood of such point. So that is a differential approach that is applied over there. We will see that one in the next class.