Gas dynamics: Fundamentals And Applications
Prof. Srisha Rao M V
Aerospace Engineering
Indian Institute of Science – Bangalore

Lecture 58
Shock Boundary Layer Interaction- I

So, we are looking at some interesting flow features in high Mach number flows some discussions were had on hypersonic flows which are at very high Mach numbers where some special physical features become important. We also had a discussion on interactions of oblique shocks of different kinds they also lead to interesting flow features and there are some important design considerations also that figure when you consider shock-shock interactions.

Now we will see what happens when shocks come close to the body and they interact with the layer where viscous these are the effect of viscosity is really important which is the boundary layer. So, we look at shock boundary layer interactions. In majority of the discussions, we had in this course we had not touched upon viscous flows we were largely inviscid.

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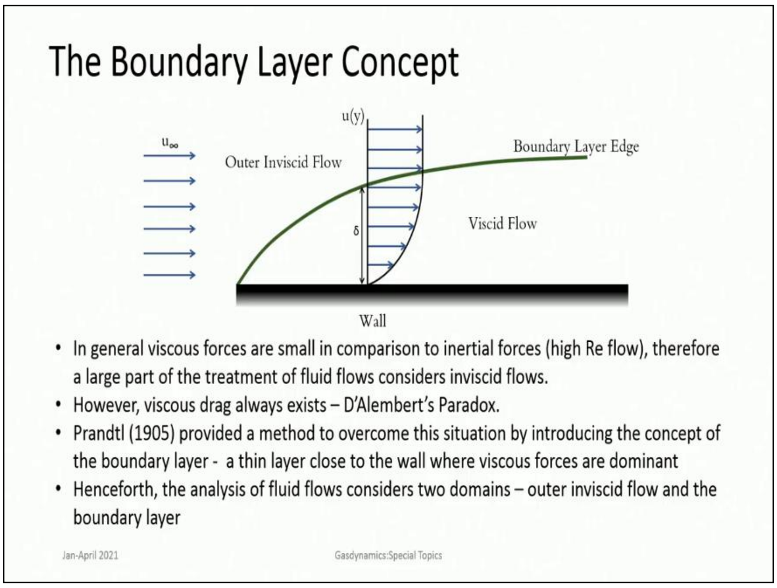

That is how generally the flow fields over bodies are described because in general the viscous forces are relatively small in comparison to inertial forces which is typical for any high Reynolds number flow. Therefore, in majority of the flow it we can consider that the flow is inviscid. But this consideration becomes a problem because always there is a drag associated with any, body in flow. But if you do not consider viscosity at all then if you consider a flat plate, it should not have any drag at all.

But that is not the case you always have drag this was the famous DL Birds paradox this was a consequence of neglecting viscous forces. It was Prandtl who provided a method to overcome this situation by introducing the concept of a boundary layer. A layer which is close to the wall where viscous forces are important viscosity plays a dominant role. Then from that point onwards all analysis of fluid flows has typically taken place by considering 2 regions.

Outer inviscid flow that is over here high Reynolds number flow outer flow we can essentially consider to be inviscid, and we have all the inviscid methods to be applied over here. While at the wall very close to the wall what is shown here is like a zoomed inversion near the wall there is a boundary layer that develops where viscous forces are important. At the wall it is a no slip condition that is velocity is 0 at the wall relative to the wall.

So, V = 0 at wall, and slowly the velocity increases gradually and then finally reaches the inviscid velocity at the outer edge of the boundary layer. If you consider just a flat plate and a uniform flow occurring over it then this velocity goes all the way from 0 to the uniform inviscid velocity u infinity which is very close to the edge of the boundary layer. So, you consider such bifurcation such 2 regions outer invisible flow inner viscous flow the boundary layer is where we consider viscous flow.

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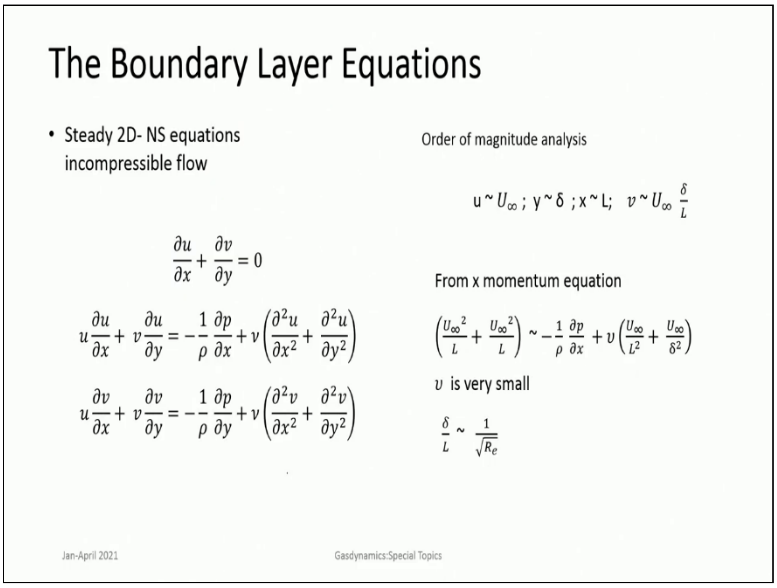

So, what is the advantage of this kind of an approximation it allows us to simplify the Navier-Stokes's equations great deal. If you consider we are considering in order to understand what a boundary layer has let us just consider the incompressible flow and 2D Navier-Stokes's equation for an example this is the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

and these 2 are the u momentum and v momentum, or x momentum and y momentum equations in x and y directions, respectively.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Now here if we had considered that there was no viscosity at all then all these terms would just go away, and we would be just left with the inviscid part. But that would not give us any drag at all therefore we have to consider viscosity. But what is it that is important this is what we would like to know when we consider a boundary layer kind of approach? To do this we will just do an order of magnitude analysis to know what terms in this.

These terms are just the convective acceleration terms this is the pressure term this is due to viscous forces. Now we should know which of these terms are important to understand this we say that the velocity 'u' goes scales or goes according to U_∞, u ~ U_∞, that is the outer stream u ~ U_∞. While we now say that there is a very thin layer which is just above the surface of the plate a thin layer develops which is the boundary layer.

So, we consider that in the y direction the important length scale is the boundary layer thickness delta while along x direction the length scale is just L. Now if you consider the continuity equation then here if you do the order of magnitude analysis you find that u goes as U infinity while x ~ L. Well, if you consider, v, let us say there is a proper scale for v which we can say is some v star by delta because y goes as delta.

So, from the continuity equation we find that we should have a scale that there is a v velocity in the boundary layer which has a scale which has a scale $v \sim U_\infty \frac{\delta}{L}$ and $'\delta'$ that is the boundary layer thickness is small compared to L. So, there is a very small v velocity outside here. In the free stream it was just a uniform flow parallel to the wall and now once the boundary layer forms, we find that there is a v velocity that is produced therefore the streamlines will shift away from the wall and this boundary layer thickness grows.

Now if you consider the x momentum equation then you similar scaling you can do and you find that the first term goes as $\frac{U_\infty{}^2}{L}$, second term goes as $\frac{U_\infty{}^2}{L}$, there is the $\frac{\partial p}{\partial x}$ term you have another term $\frac{U_\infty}{L^2}$ here another term $\frac{U_\infty}{\delta^2}$ in the y direction which corresponds to $\frac{\partial^2 u}{\partial y^2}$ and they are multiplied by nu which is the kinematic viscosity.

$$\left(\frac{U_\infty{}^2}{L} + \frac{U_\infty{}^2}{L}\right) \sim -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{U_\infty}{L^2} + \frac{U_\infty}{\delta^2}\right)$$

Now υ is very small. So, if we consider this multiplication b if you multiply such a small number with $\frac{U_\infty}{L^2}$ this becomes a very small number. So, this can be neglected in relation to other terms. But if you consider new multiplying by $\frac{U_\infty}{\delta^2}$, here δ is also small so you find that this term u nu u infinity by delta square is significant and it will be of the same order as $\frac{U_\infty{}^2}{L}$.

Therefore, you can get the relation that $\frac{\delta}{L} \sim \frac{1}{\sqrt{R_e}}$. So, when you go to high Reynolds number delta by L is very small or you have small boundary layer thickness.

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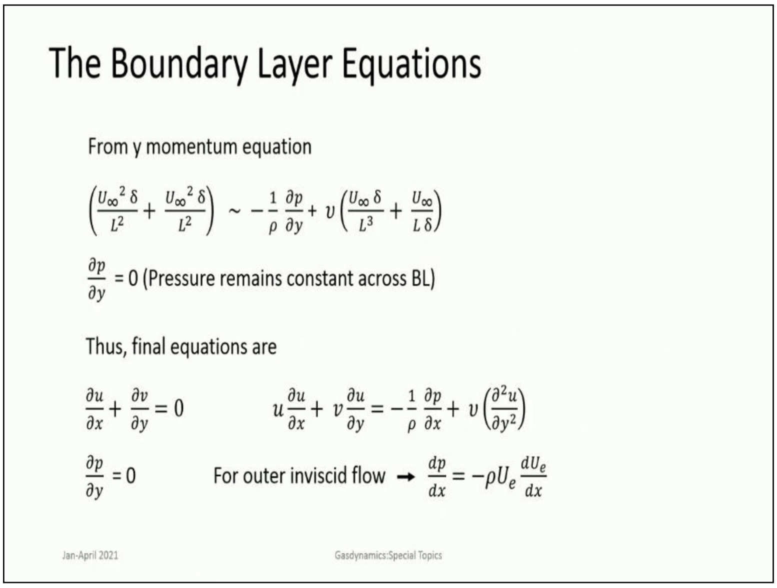

Now if you consider the y momentum equations then what you find from such an order of magnitude analysis is all velocity related terms are very small, they have delta being multiplied everywhere. Therefore, these terms are very small therefore we get that $\frac{\partial p}{\partial y}$ that is pressure variation across the boundary layer in y direction pressure is constant. Therefore, this leads to the set of boundary layer equation which are much simpler compared to the full Navier stokes equation which is continuity equation x momentum equation.

$$\left(\frac{U_\infty{}^2\,\delta}{L^2} + \frac{U_\infty{}^2\,\delta}{L^2}\right) \sim -\frac{1}{\rho}\,\frac{\partial p}{\partial y} + \upsilon\left(\frac{U_\infty\,\delta}{L^3} + \frac{U_\infty}{L\,\delta}\right)$$

In x momentum equation the shear stress term can consist of only variation of u in y. So, τ_{wall} is $\nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ at the wall. So, the shear stress, the important term is $\frac{\partial^2 u}{\partial y^2}$and across the particular across the y direction pressure is constant that is the third equation therefore the pressure gradient $\frac{\partial p}{\partial x}$. Now p is then a function of x only, it is not function of y.

Therefore, $\frac{\partial p}{\partial x}$ can be written as dp by dx which is imposed from the outer inviscid flow where you can evaluate it using Euler's equation or any inviscid formulation you can get this pressure variation. So, this is the boundary layer equations. So, and they are much simplified compared to Navier stokes equation and this is what is normally used you consider an outer inviscid flow from which pressure applied on the boundary layer and hence the body can be calculated.

While the viscous forces that appear on the body can be calculated by using the boundary layer equations.

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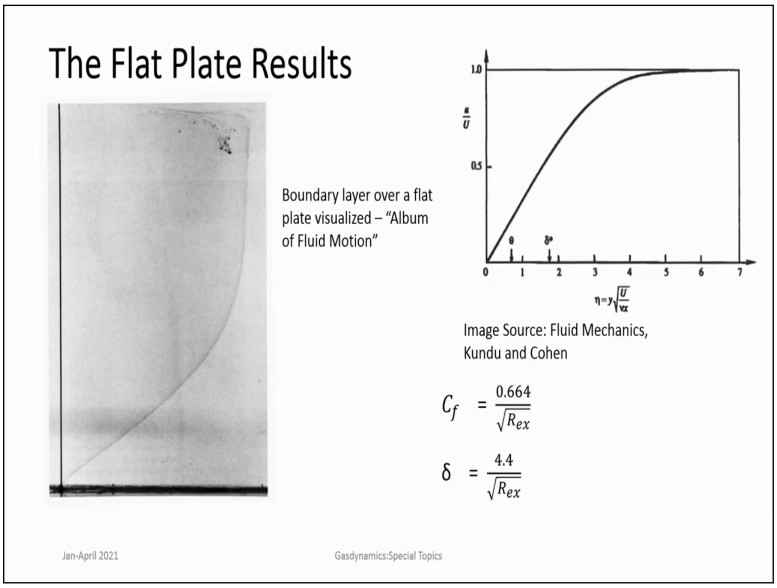

So, here is the picture of the boundary layer visualized on a flat plate this is taken from album of fluid motion and you can see a nice development of the boundary layer over here which is of this kind. The same kind of profile can be obtained by solving the boundary layer equations for a flat plate. This represents the solution for a flat plate this is u/U in non-dimensional terms the y coordinate written in non-dimensional terms.

So, for a laminar flow one can evaluate what is the coefficient of friction it becomes a function only of Reynolds number along x direction.

$$C_f = \frac{0.664}{\sqrt{R_{ex}}}$$

The boundary layer thickness it goes as

$$\delta = \frac{4.4}{\sqrt{R_{ex}}}$$

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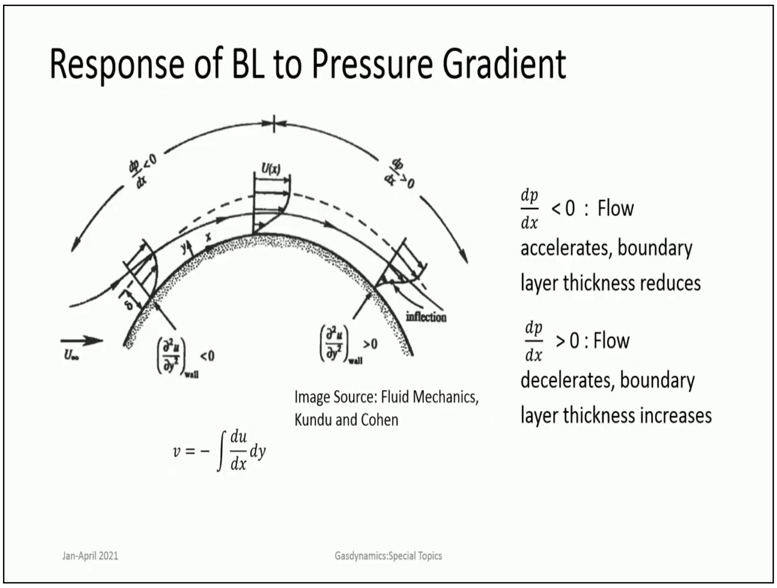

Why did we have all these discussions because an important phenomenon occurs around bodies when there is pressure gradient in case of a flat plate there is no pressure gradient it is just a flat plate. But other bodies for example airfoils or spheres cylinders here initially the flow accelerates for a flow which is coming in this direction. Initially flow accelerates and thereafter in the rearward portion the flow decelerates.

Then the boundary layer is affected due to these changes in velocities the change in velocity produces changes in pressure when flow accelerates pressure decreases. So, you have a $\frac{dp}{dx} < 0$ and when flow decelerates pressure increases $\frac{dp}{dx} > 0$.

So, this term is 0. So, basically it is a balance between pressure forces and the shear forces the viscous forces at the wall. If you look at it if there is an accelerating flow that is called as a Favourable pressure gradient $\frac{dp}{dx} < 0$ you find that $\frac{\partial^2 u}{\partial y^2} < 0$. But as if $\frac{dp}{dx} > 0$ then $\frac{\partial^2 u}{\partial y^2} > 0$.

Now $\frac{\partial^2 u}{\partial y^2}$ is a derivative of $\frac{\partial u}{\partial y}$, which is shear stress is proportional to $\frac{\partial u}{\partial y}$. If $\frac{\partial^2 u}{\partial y^2}$ is increasing, then the slope at the wall should keep increasing it should keep increasing. So, in accelerating flow the boundary layer thickness reduces. You can also see it because of what happens to v velocity.

Because v, velocity is negative du by dx dy is just from the continuity equation. So, if $\frac{\partial u}{\partial x}$ increases that is an accelerating flow v decreases but if $\frac{\partial u}{\partial x}$ decreases which is a decelerating flow v increase. Increase in v means increase in boundary layer thickness now we find that if pressure gradient is increasing or its adverse pressure gradient.

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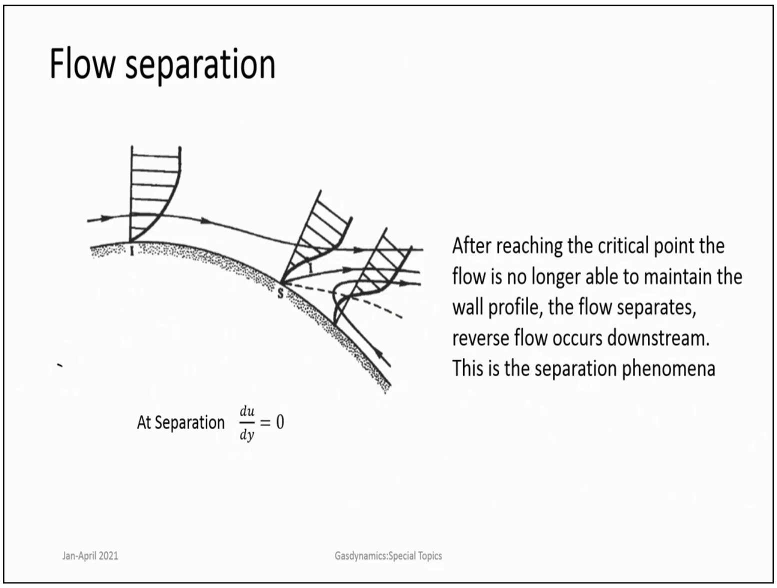

Then $\frac{du}{dy}$ is increasing. So, this slope keeps increasing and it can at the max reach a limit where $\frac{du}{dy} = 0$ and further if there is an increase in pressure the flow cannot maintain the tangency condition and it just separates from the wall. The flow separates from the wall and there is a reverse flow that occurs. This kind of flow phenomena is known as Flow separation.

This is very important in fluid dynamics because it gives rise to large pressure due to pressure drag due to separation.

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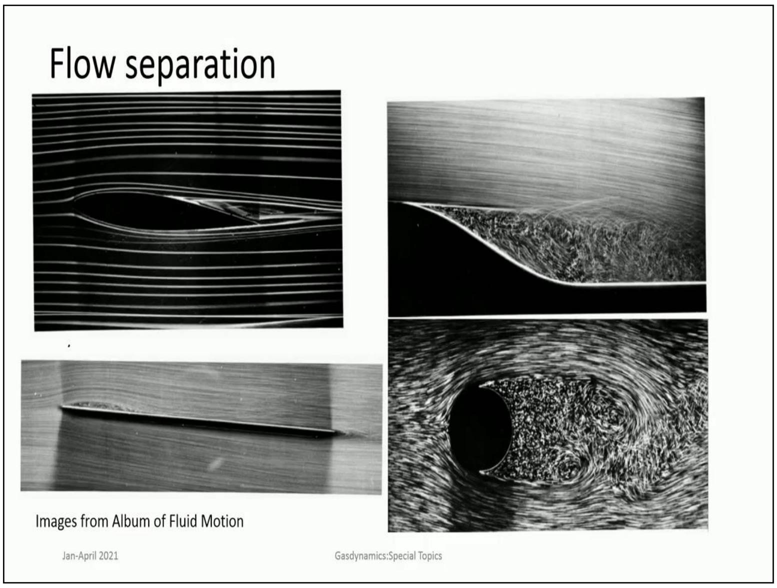

There are several instances all these again from album of fluid motion. For example, here you have an Airfoil in uniform flow and the streamlines are made visible using a certain dice. Here you find that the flow separates at this point. It is no longer as uniform as its over here and here the flow separates. Here there is a wall this is a wall which has a certain contour, and the flow separates from this point.

There is a region where there are lot of re-circulating vertices. Similarly, a cylindrical flow this is very familiar many of you should have come across this and here flow separates and there is large region of recirculation. And the separation leads to large increase in drag. Similarly, you find on a flat plate.

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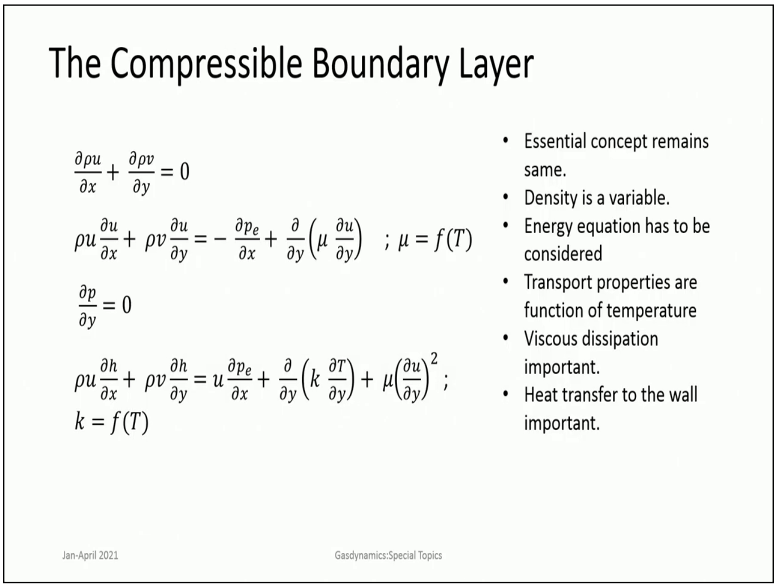

Why we did all this discussion is because this phenomenon is important in the context of compressible boundary layers and shocks and how they interact with boundary layers when we come to compressible boundary layers. So, here density is not a constant otherwise the essential concept is all the same it is remaining the same. But density is a variable also we must consider the energy equation.

So, if you consider the energy equation which is over here another term which becomes important is the viscous dissipation that is conversion of kinetic energy into heat energy or internal energy due to viscous effects due to dissipation. So, this term becomes important consequently temperature increase in the boundary layer inside a compressible boundary layer therefore the wall heat transfer that is $k\frac{\partial T}{\partial y}$.

So, an appropriate wall heat transfer becomes important. So, when you consider a compressible boundary layer is not only the momentum equation you have to consider the energy equation and variation of density. Also, the transport coefficients which is viscosity and thermal conductivity they become strong functions of temperature. So, all these additional features must be considered.

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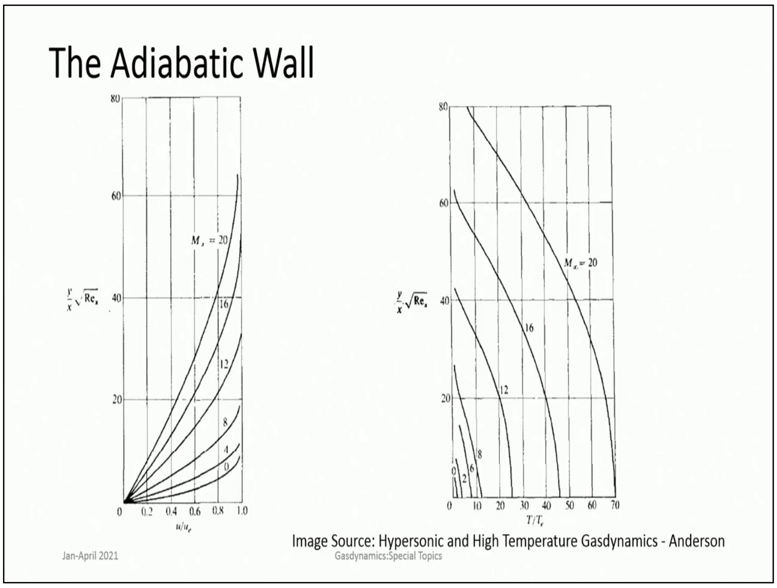

Now if you these are calculations of the boundary layer in a compressible domain and if you see these calculations. Here we must use an appropriate boundary condition for temperature at the wall or heat transfer at the wall this is for an adiabatic wall. Then you see that as Mach numbers increases the boundary layer thickness is increasing significantly. So, this is Mach number 0 this is the thickness for M ach number 0 delta 0 while this is Mach number 20.

So, you can see that Mach number 20 is so much larger than Mach number for 0 and you need to consider an appropriate boundary condition. So, thermal boundary layers are also important, and they also increase with increase in Mach number. So, boundary layer thickness increases significantly with Mach number.

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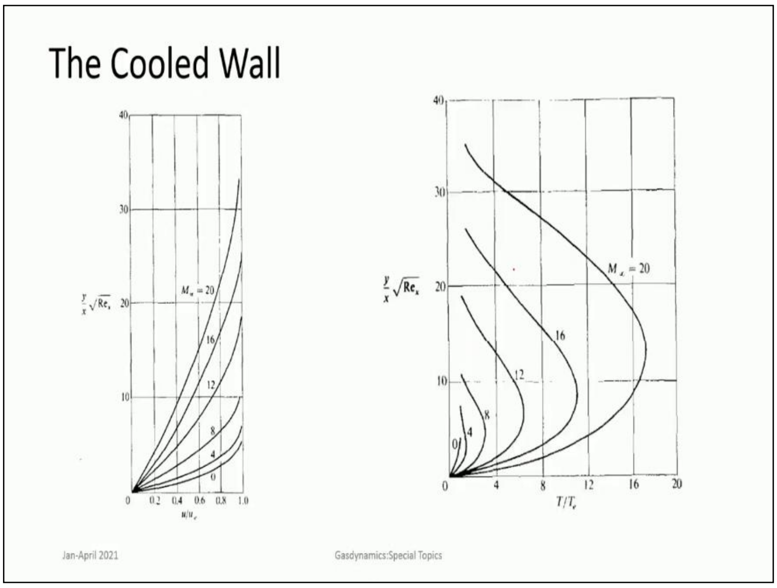

Now if you consider an adiabatic wall a cooled wall that is a heat transfer is a ring where temperature heat is being removed. Then what happens is there is viscous dissipation that is happening in the boundary layer. So, temperature increases in the boundary layer. But at the same time heat is being removed at the wall. So, a maximum temperature occur somewhere in between within the thickness of the boundary layer.

But the essential fact that boundary layer thickness increases with Mach number remains the same. But now you see that the Mach number thickness the boundary layer thickness is lower in comparison to adiabatic wall.

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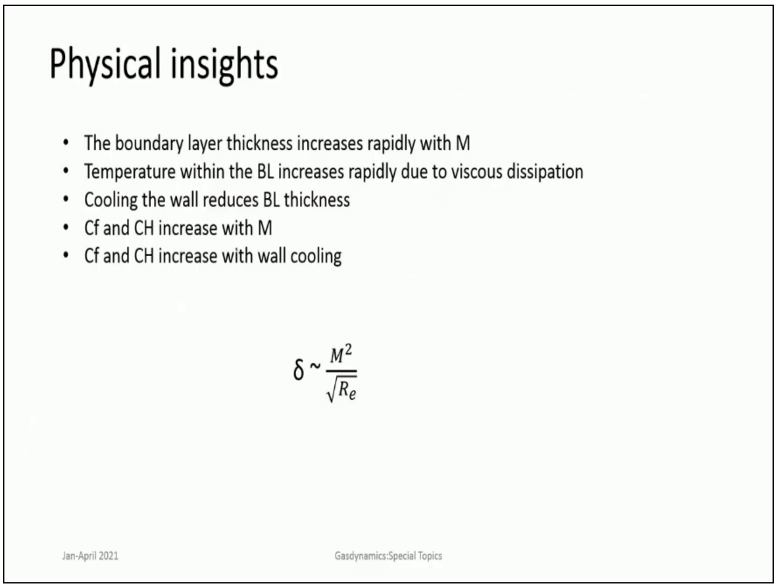

So, the physical insights that we gain from these discussions is in the context of compressible boundary layers. The boundary layer thickness increases rapidly with Mach number temperature increases in the boundary layer due to viscous dissipation. Then if you cool the wall boundary layer thickness reduces the same kind of effects happen to both the skin friction coefficient as well as the heat transfer which you see here corresponds to Stanton number.

So, in general the kind of increase that is found is boundary layer thickness goes as Mach number square by square root of Reynolds number. Now in the next class having understood a little bit about boundary layers, which is a total topic. Let us see what happens when a shock comes and interacts with the boundary layer in high Mach number flows in the next class, thank you.