Gasdynamics: Fundamentals And Applications
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Lecture 56
Hypersonic Flows – II

So, in the previous class we looked at a brief introduction on hypersonic flows and what are their characteristics. Hypersonic flows are high velocity flows they are high Mach number flows, and they have some typical characteristics. One thing is that the flow kinetic energy is a significant fraction of total enthalpy therefore whenever there is a change in velocity produces large changes in temperature also.

So, near stagnation points of bodies very high temperatures are encountered. So, high temperature effects are important in hypersonic flow. The other aspect is that as Mach number increases. Then the shocks come closer and closer to the body. So, at such high speeds as a hypersonic flow the shock is really very close to the body. So, we get what are known as thin shock layers it is. So, close to the body.

Then the other sorts of characteristics are with respect to thick boundary layers boundary layers become very thick and therefore they may affect the inviscid flow also. If you typically because of the high temperature effects and high heat transfer in hypersonic flow the way to minimize it is to go about having blunt shapes almost all hypersonic vehicles have blunt shapes.

But blunt shape produces a bow shock a bow shock of this nature it is not attached. But a detached shock and this detached shock has a variation, or a curvature of the shock produces entropy gradients along the streams or streamlines over here. So, from Croco’s theorem we know that entropy gradients produce vorticity therefore you get rotational flows, and they form an entropy layer, and this affects the flow over the body.

So, the entropy layer can go and wash over the body. So, we have various such physical effects like very high temperatures therefore you may have chemical reactions. Air as a simple mixture of nitrogen oxygen and other gases may not remain the same it may undergo chemical reactions even ionization. So, when these physical features become very important then we call the flow as hypersonic which typically occurs for Mach numbers greater than 5.

So, a good sort of ballpark number to call something as a hypersonic flow is Mach number greater than 5. But really these effects must become dominant. Then we are in the hypersonic regime. Those I have in the previous class we discussed in detail I just gave a brief review of that. Now in this class what we would do is we have seen how complex the hypersonic flow or physical features in hypersonic flow are.

But at the same time certain aspects like thin shock layers shocks coming very close to the body they produce certain and high Mach numbers.

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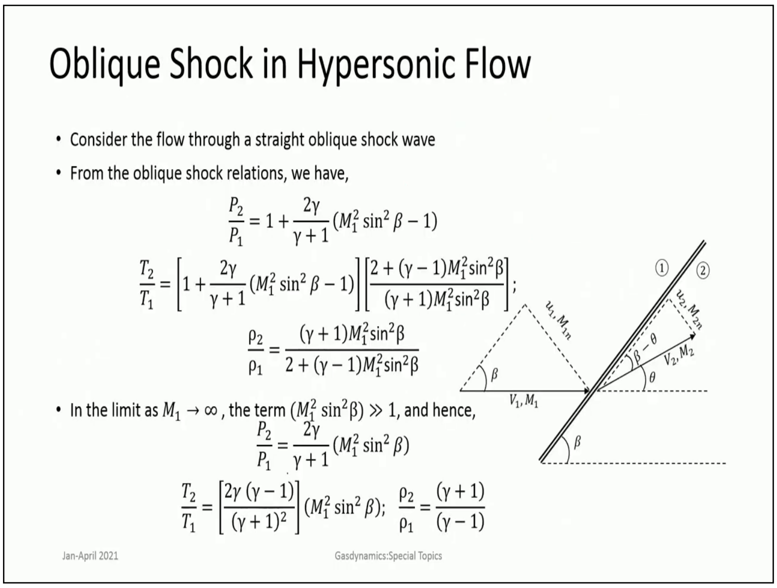

They produce certain simplifications which also allow us to say extract estimates of the forces around bodies in hypersonic flow using relatively simpler theories rather than very complex theories. To get an understanding of this let us just take the oblique shock in hypersonic flow we know the oblique shock relation. So, you have the incoming flow, and we know the shock turns the flow towards itself and it turns by an angle theta creates a shock of angle beta and this is something we have discussed in detail.

If you look at the Oblique shock relations

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^2 \sin^2 \beta - 1)$$

Similarly,

$$\frac{T_2}{T_1} = \left[1 + \frac{2\gamma}{\gamma + 1}(M_1^2 \sin^2 \beta - 1)\right]\left[\frac{2 + (\gamma - 1)M_1^2 \sin^2 \beta}{(\gamma + 1)M_1^2 \sin^2 \beta}\right];$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_1^2 \sin^2 \beta}{2 + (\gamma - 1)M_1^2 \sin^2 \beta}$$

Now let us take the limit this is we are considering perfect gases to understand certain features of the flow. So, let us try to understand what happens when Mach numbers become very high. So let us take this equation and let Mach number go large. So, in that sense when M_1^2 goes large very high.

Then the term 1 has very little significance. So, this has very little significance. So, you can consider this to be $\frac{2\gamma}{\gamma+1}(M_1^2 \sin^2 \beta - 1)$ even this one is very small. On the other hand, if you take $\frac{\rho_2}{\rho_1}$ and apply the condition that M_1^2 goes very large we have 2 factors here both in the numerator and the denominator they become very large therefore you get the equation that

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)}{(\gamma - 1)}$$

So, the density ratio across an oblique shock goes towards finite values as you increase Mach numbers. So, density ratio approaches finite value $\frac{(\gamma+1)}{(\gamma-1)}$, because of this

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \Big/ \frac{\rho_2}{\rho_1}$$

So, you can substitute that here and as much simplified expression for $\frac{T_2}{T_1}$ can be obtained. So, as we put the limit that Mach numbers go to large values. Then we can produce simplifications to oblique shock equations.

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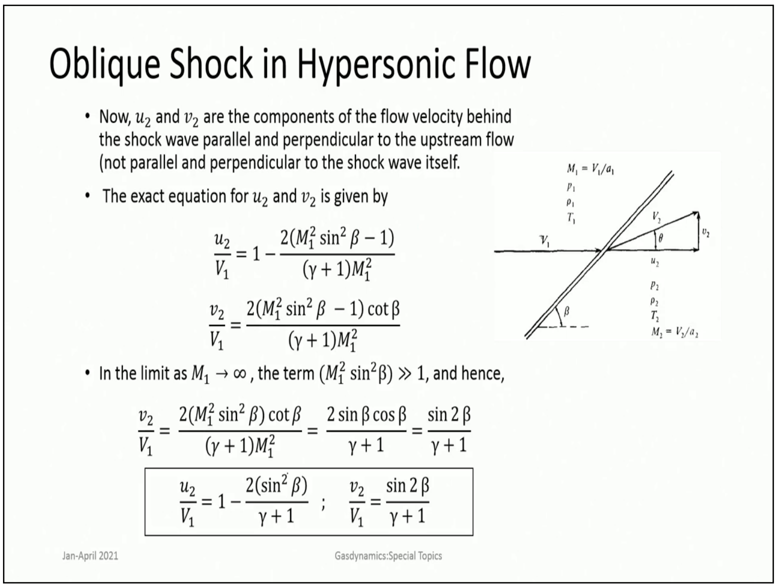

So, you can also consider the components of flow velocities parallel and perpendicular to the upstream flow. This is slightly different from the usual discussions where we consider parallel and perpendicular to the normal shock. Why this is done is most of the calculations for forces are with respect to free stream therefore it is good to know what happens to velocity is parallel and perpendicular to the free stream after the shock.

This is just a transformation of the velocity vector into another coordinate system. So, this can be accomplished and again we apply the limit that Mach numbers go to very large values, $M_1 \to \infty$. So, then we find that, $\frac{u_2}{V_1}$ that is the parallel component for the shock it goes as

$$\frac{u_2}{V_1} = 1 - \frac{2(M_1^2 \sin^2\beta - 1)}{(\gamma + 1)M_1^2}$$

So, the good thing over here is that you are there is that explicit dependence on Mach number is getting removed of course beta depends on Mach number. But soon we can see that as the Mach numbers increase clearly large. Then even beta becomes somewhat independent of Mach number. So, what we see here is that we are getting quantities which are becoming independent explicit dependence of Mach number is getting removed.

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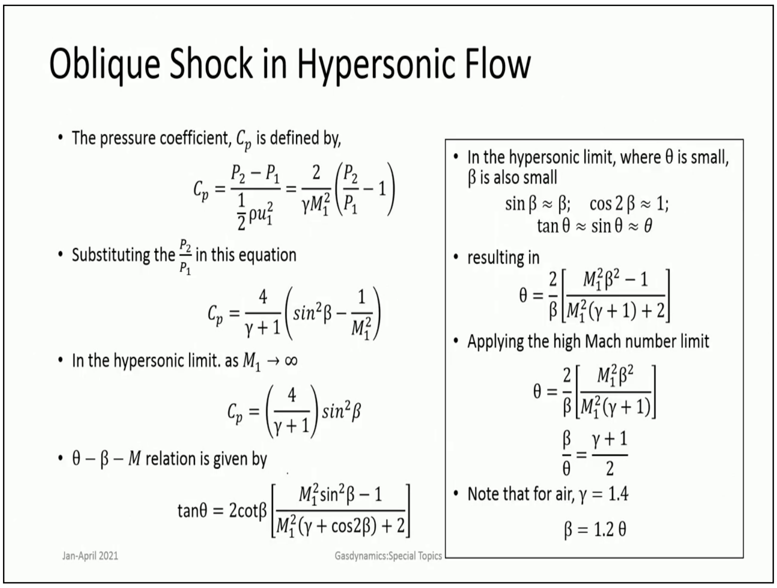

So, always we are interested in finding out what is the pressure distribution over bodies. So, we want to know what is, pressure coefficient and pressure coefficient is determined as

$$C_p = \frac{P_2 - P_1}{\frac{1}{2}\rho u_1^2}$$

where P1 can be P∞, which is the free stream conditions $\frac{1}{2}\rho u_1^2$. So, we can make this, this should now be standard transformation from $\frac{2}{\gamma M_1^2}\left(\frac{P_2}{P_1} - 1\right)$.

You can do this by taking P1 out and. Then multiplying and dividing by gamma $\frac{P_2}{P_1} - 1$. So, in this equation we can apply $\frac{P_2}{P_1}$ that we just obtained from the previous considerations. Then we get the equation

$$C_p = \frac{4}{\gamma + 1}\left(sin^2\beta - \frac{1}{M_1^2}\right)$$

So, again you can see that for coefficient of pressure also becomes explicit dependence on Mach number is removed here.

But further if we consider the $\theta - \beta - M$ relations for Oblique shocks written here,

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right]$$

So, this expression if you consider this and consider that we are having a hypersonic body in Hypersonic flow in the limit of large Mach numbers also that it is a slender body when we say slender body the angles of the body are small. So, theta is small.

So, if theta is small, we know as a consequent beta will also be small. Therefore, we can make these approximations $\sin \beta \approx \beta, \cos 2\beta \approx 1, \tan \theta \approx \sin \theta \approx \theta,$ both approximate as $\tan \theta \approx \sin \theta \approx \theta$. If you put those approximations into this equation, you get this particular result here and then combine it with the fact that a Mach number is going to very high values M 1 tends to infinity, we get

$$\frac{\beta}{\theta} = \frac{\gamma + 1}{2}$$

or if you put gamma as 1.4, this becomes $\beta = 1.2\,\theta$, that means beta is hardly 20% greater than theta.

So, this is very much consistent with what we had discussed about thin shock layer. So, what we find here is that beta should be very close to theta. So, even if you look at this final discussion over here at high Mach numbers beta also becomes independent of Mach number explicit dependence is not there.

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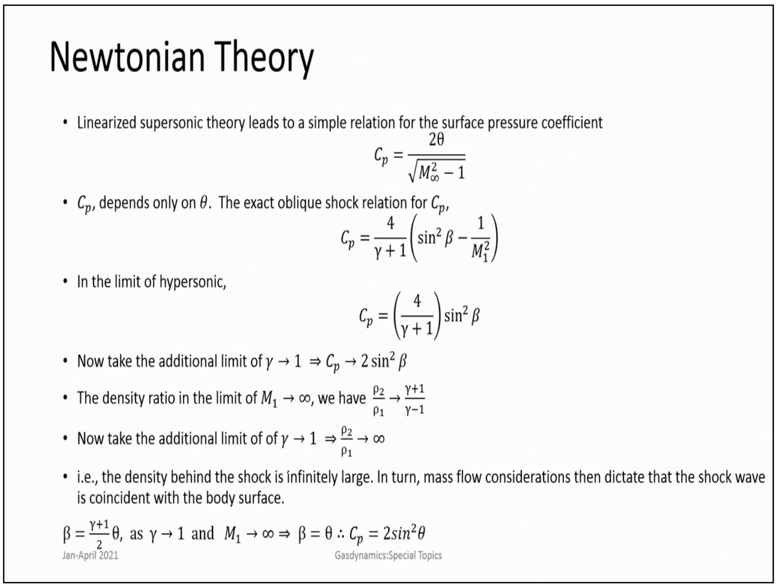

So, what happens because of this? So, because of this we find that in hypersonic flows certain theories that were developed very early on even by Isaac Newton they are impact based theories for forces over bodies in fluid flow where Newton considered that that if you have a stream of fluid coming in velocity with the velocity u infinity and it impacts a body let us take a flat plate kept at an angle θ

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Then the flow is suddenly turning to become tangential to this stream or particular wall and therefore if we do a control volume analysis around this body assuming that all the velocity normal to this or rather all the velocity just gets goes along the tangential direction and the normal velocity just gets cancelled off if we do such a calculation. Then we can find out what is C_P? The C_P is then $2 \sin^2 \theta$.

This was the expression that was given by Newton very long time ago. But this did not consider details of fluid flow and now we know we are able in a position to plot the streamlines and find out what are the pressures around such bodies but in hypersonic flow for many reasons including the fact that the sharks they are very close to the body. So, if you consider what happens over here on a thin wedge placed in hypersonic flow the hypersonic flow velocity u infinity goes across the shock and immediately it becomes tangential to the body.

This thin shock is occurring very close to the body. Therefore, we approach the assumptions that were initially taken by Newton quite some time ago therefore we expect that C_P equal to 2 $\sin^2 \theta$ will be a good approximation for the bodies in hypersonic flow. In fact, we can do it more rigorously also in supersonic flow small perturbation theory we saw

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

which simply stated that C_P depends on local surface inclination.

The Newtonian theory also states that C_P depends only on the surface inclination in the hypersonic limit if we look at C_P, we find that it is depending on sine square beta which we have just now done. We also saw that at large Mach numbers of β approaches theta it is very close to theta. Explicitly if we take the limit that gamma goes to 1.

Then $\beta = \theta$, and $\frac{4}{\gamma+1} = 2$. So, even if we consider a thin shock layer, we can see that whatever Newtonian was Newton was predicting in hypersonic flows in the limit of large Mach numbers we are very close to it. So, Newtonian methods to estimate quickly estimate the aerodynamic forces is used effectively in hypersonic flows it is an estimate to start with.

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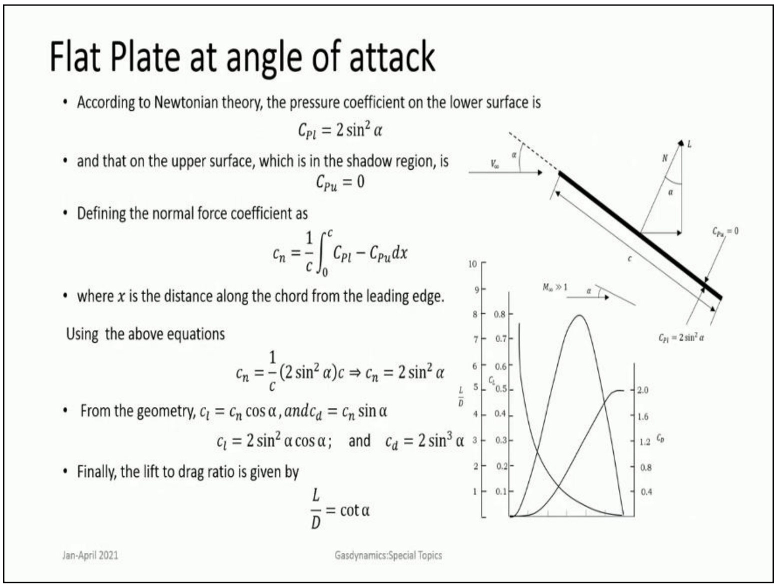

Later more refined theories can be used. For example, if you consider a flat plate at an angle of attack α. Then we can find what is the force on this plate. The region behind the flow that is the shadow region this is shadow region that has zero pressure that is what is taken the or rather Cp is very low it can be considered as 0. While only Cp is considered for the windward facing flow where the impact takes place where at this lower half it is $2\sin^2\alpha$.

Therefore, the normal coefficient of force can be easily calculated it is nothing but integration of difference in pressures over the chord. So, it becomes $c_n = 2\sin^2\alpha$ now lift and drag can be calculated they are components of this normal force the lift is C n cos alpha drag is C n sine alpha therefore you get lift and drag of this kind. Finally, L/ D ratio is also can be found it goes as cot alpha.

So, these are the plots of C_L, C_D and L/ D for the vehicle. So, now if it is completely placed at 90^o then all the force is just drag and we get C_D, and that case should be 2.0 which is what is coming over here. So, it comes out to be 2.0 while C L gradually decreases down to zero while L/ D occurs at a maximum at in between point over here.

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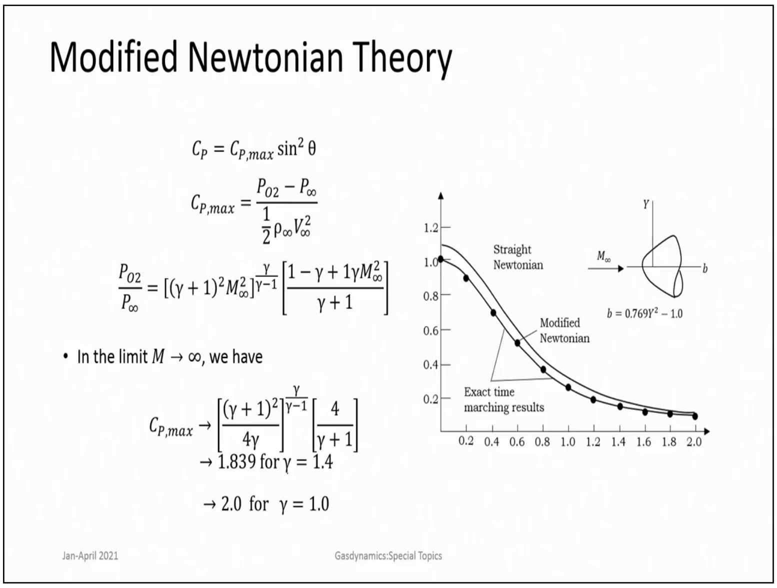

So, you can apply these Newtonian methods. Later there were corrections done to the Newtonian it was from approximate methods. So, the correction was that there is a stagnation pressure that occurs at any body. So, the reference pressure or reference Cp can be taken to be the stagnation pressure here at the nose which is Cp max therefore,

$$C_P = C_{P,max}\sin^2\alpha$$

So, this is plotted Cp,max can be found out from normal shock relations because at the stagnation point it is a normal shock. Therefore if 1 plot using Newtonian methods straight Newtonian without any modification is the upper curve while the modified Newtonian is much, much closer to more exact evaluations using numerical techniques of a flow over certain shape body.

So, modified Newtonian gives an improved estimate of the coefficient of pressure. So, with these discussions what one must understand this while physical flow features in hypersonic flow become quite complex. Very fact of high Mach numbers allows certain simplicity and to be done and surface inclination methods like Newtonian are quite useful in estimating the coefficient of pressure.

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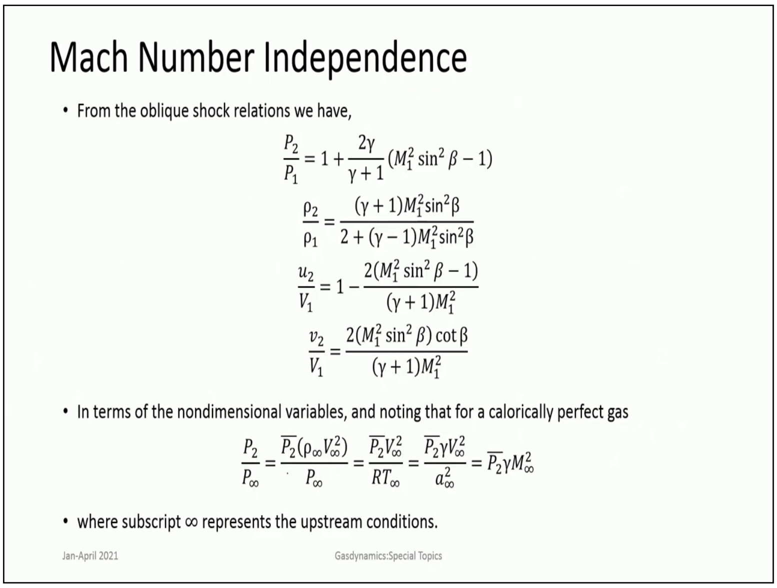

So, this is a fact that we have been while discussing the oblique shock relations also we have noted that if we non-dimensionalise the variables appropriately like coefficient of pressure C_L, C_D or $\frac{\rho_2}{\rho_1}$ or P2 appropriately non-dimensionalised by $\rho_\infty V_\infty^2$, they all become independent of Mach number. So,

$$\frac{P_2}{P_\infty} = \frac{\overline{P_2}(\rho_\infty V_\infty^2)}{P_\infty} = \frac{\overline{P_2}V_\infty^2}{RT_\infty} = \frac{\overline{P_2}\gamma V_\infty^2}{a_\infty^2} = \overline{P_2}\gamma M_\infty^2$$

$\overline{P_2}$ is a normalized pressure.

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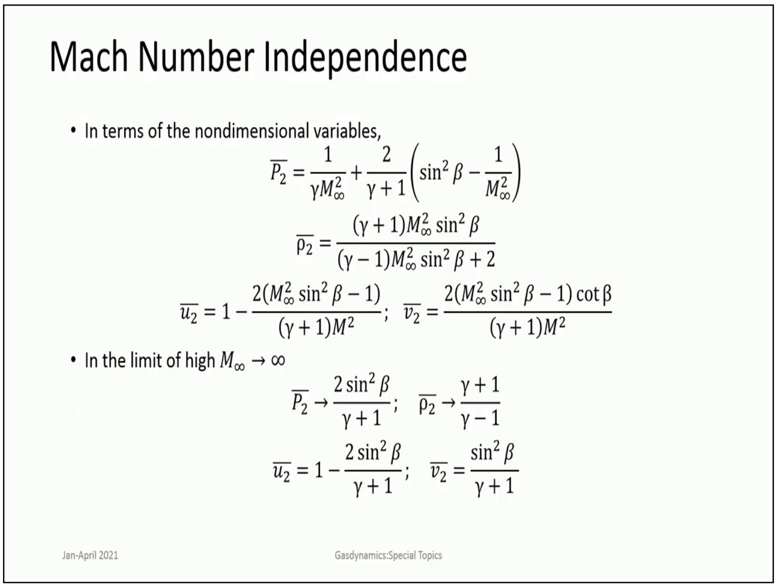

Similarly, $\overline{\rho_2}$ is normalized density, $\overline{u_2}$, $\overline{v_2}$ are normalized velocities. We find that as the Mach numbers go to large values normalized pressures densities and velocities, they become independent of Mach number this is known as Mach number independence principle. So, from this we have an idea that in hypersonic limit Mach number really does not play a major role. So, the flow becomes somewhat independent of Mach number.

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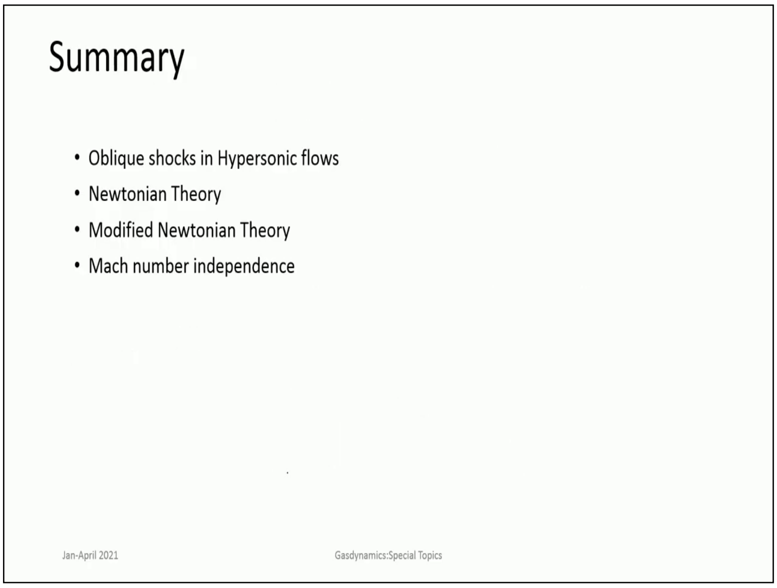

So, if you look at the hypersonic flows a brief introduction that has been done over here, we have considered an introduction into physical aspects of hypersonic flows. Then there we discussed the complexity that arises like high temperature effects, thick boundary layers and so on. Thin shock layers and as a result of say thin chalk layers high Mach numbers of certain simplifications are possible in the analysis of hypersonic flows.

First estimates of say aerodynamic coefficients can be evaluated based on simple local surface inclination theory like Newtonian theory or modified Newtonian theory if we apply the limit of high Mach numbers to oblique shock relations in hypersonic flows, we see that a certain independence of Mach number is achieved in normalized values. So, with this we the area of hypersonic flows itself is quite vast involving various aspects related to heat transfer effects viscous effects and high temperature effects and so on which is difficult to cover in such a short time.

So, but these short lectures were to introduce this topic. So, that if anyone is interested, they can further pursue them. With this we close on hypersonic flows. But we look at some other interesting aspects related to shock interactions and shock wave boundary layer interactions in the coming classes, thank you.