**Gasdynamics:**
**Fundamentals and Applications**
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**Lecture-05**
**Thermodynamics-Numerical**

So, we have looked till now on certain basic definitions of compressible flow and mach number and speed of sound and then looked at thermodynamics because it is integral to the description of gas dynamics. So, let us look at a few numerical, so as to familiarize with these concepts and how to quantitatively look at these numbers?

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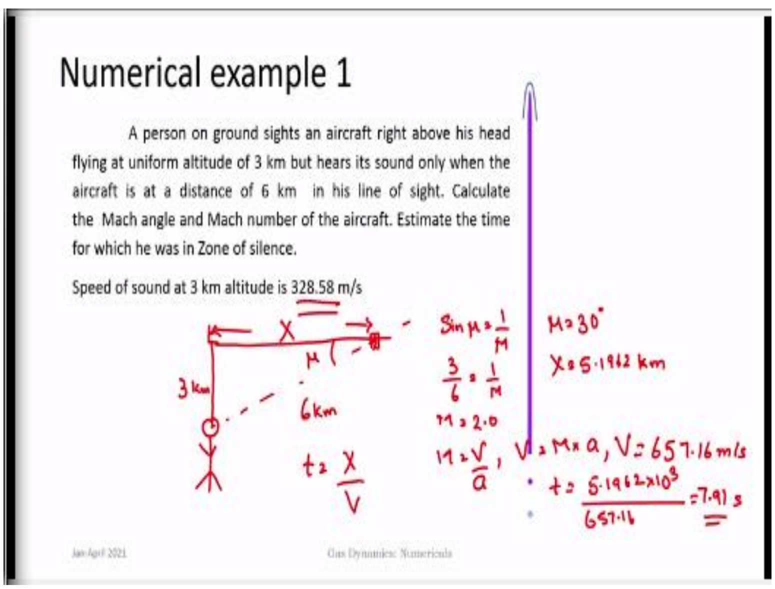


So, in this example 1, so a person on ground sights an aircraft right above his head flying at uniform altitude of 3 km. But here it would sound only when the aircraft is at a distance of 6 km in his line of sight. Calculate the mach angle and mach number of the aircraft, estimate the time for which he was in zone of silence. So, it is always useful to sort of look at it in a diagrammatic fashion.

So, now let us look at what has been given here it is good to represent it diagrammatically. So, if you consider a person is standing here and he observes an aircraft that is going right above his head. But at that time he did not hear anything but along his line of sight, so this is his line of sight.

And when the distance is 6 km, so this is 3 km, he is able to observe the sound or he is able to hear the sound of this aircraft.

So, that means at that point of time the mach wave just passed over that particular point where the person was standing. So, that means this is the mach angle over here $\mu$ and then we know that mach angle is related to mach number $\sin(\mu) = \frac{1}{M}$. So, directly from the trigonometric relation over here $\sin(\mu) = \frac{3}{6} = \frac{1}{M}$ or you can get that mach number(M) is 2.0, ok.

Now if you considered this, then the second point over here is that estimate the time for which he was in the zone of silence, ok. So, that is the second part to calculate the zone of silence we need to find out the time. Now the distance that is actually moved on ground during this time is this one which is X so by the aircraft.

So, the time taken is during this time is t is basically X by velocity of the aircraft $\left(\frac{X}{V}\right)$. Now the velocity can be calculated by the definition of mach number itself, so mach number (M) is $\frac{V}{a}$. So, you get V is M multiplied by the speed of sound ($M\ a$), need to know the speed of sound and the speed of sound is given here 328.58 m/s. So, you will get the velocity from that velocity is 657.16 m/s.

So, now you need to find this X you can do it by trigonometric relations, ok. So, you know the angle here, angle is if you know sine, sine is $\frac{1}{2}$ that means angle is $30^{o}$ and then you can calculate X. So, X is of 5.1962 km here, so then time then is t will be 5.1962 x $10^3$ m divided by the velocity 657.16 m/s which is you get about 7.91 s.

So, this is related to the concept of mach angles and the concept that in supersonic flows you cannot know that an object is passing right at the moment it is sighted or it is at that position, but at a later time when the mach wave passes over that particular point. So, there are definite directions of propagation of information in a supersonic flow, so this is the concept that is important over here.

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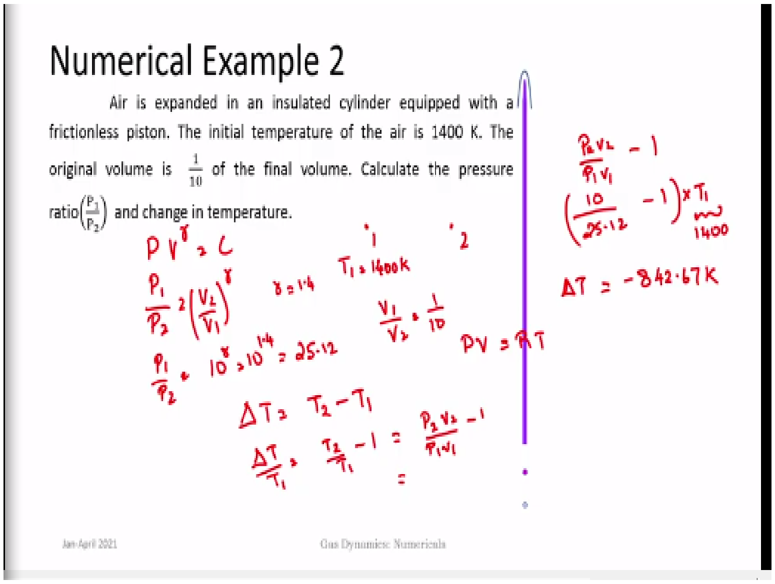


Now let us look at the second example it is related to thermodynamics, ok. So, here the question is air is expanded in an insulated cylinder equipped with a frictionless piston, the initial temperature of air is 1400 K, the original volume is $(1/10)^{th}$ of the final volume. So, if I take two states say 1 and 2, one state 1 and state 2 initial temperature is given it is $T_1$ is 1400 K and the original volume is $(1/10)^{th}$ of final volume.

So, what is given is $\frac{v_1}{v_2}$ is $\frac{1}{10}$ of the final volume, calculate the pressure ratio and change in temperature. So, how do we go about doing this? So, this involves the concepts of the ideal gas and also the concept that it is an insulated cylinder equipped with a frictionless piston. Now this is the case of an adiabatic flow without any friction or so any irreversibility. So, it is a reversible adiabatic process and that kind of a process is the isentropic process.

So, it is an isentropic process that is given over here. So, we know the ratio of the volumes. So, the corresponding thing is $pv^{\gamma}$ is constant that is equation that you would use here. So, what you would get is $p_1$, so we need to find out the ratio $\frac{p_1}{p_2}$ is actually $\left(\frac{v_2}{v_1}\right)^{\gamma}$. This is air, so for air $\gamma$ is 1.4 and $\frac{v_2}{v_1}$ is 10, so you get $\frac{p_1}{p_2}$ is $(10)^{\gamma}$ which is $(10)^{1.4}$ which is 25.12 .

So, the pressure in the initial case is higher and it is 25.12 times the pressure at the final point, ok. So, now the second question is that calculate the change in temperature, what is the change in temperature? That is you need to know what is ΔT which is $T_2 - T_1$ that if I divide this by $T_1$, I will get $\frac{\Delta T}{T_1}$ is $\left(\frac{T_2}{T_1} - 1\right)$. Now we know the process that is happening over here, it is an isentropic process and the isentropic process we know the relationships between the different quantities.

So, now also you can use the $pv = RT$ which is the ideal gas law. And that means here you can use $\left(\frac{p_2 v_2}{p_1 v_1} - 1\right)$ because $R$ is a constant in the case, so you can use this. So, this you can write it in terms of so you have already found out what is $\frac{p_1}{p_2}$ *and* $\frac{v_1}{v_2}$. So, $\frac{v_2}{v_1}$ is 10 and $\frac{p_1}{p_2}$ is 25.12 ok, so this comes out to be a negative number.

So, and this gets multiplied by the $T_1$ which is, so this is this number is 1400 K, so you get a ΔT that is change in temperature is equal to - 842.67 Kelvin, ok. So, we have addressed the two things, so the two important concepts that were used here is that fact that the process involving an adiabatic reversible process is an isentropic process using that we were able to calculate the pressure ratio given the volume ratios, ok. And the concept of the ideal gas law to calculate the change in temperature ok. So, this is an example where these two concepts were involved.

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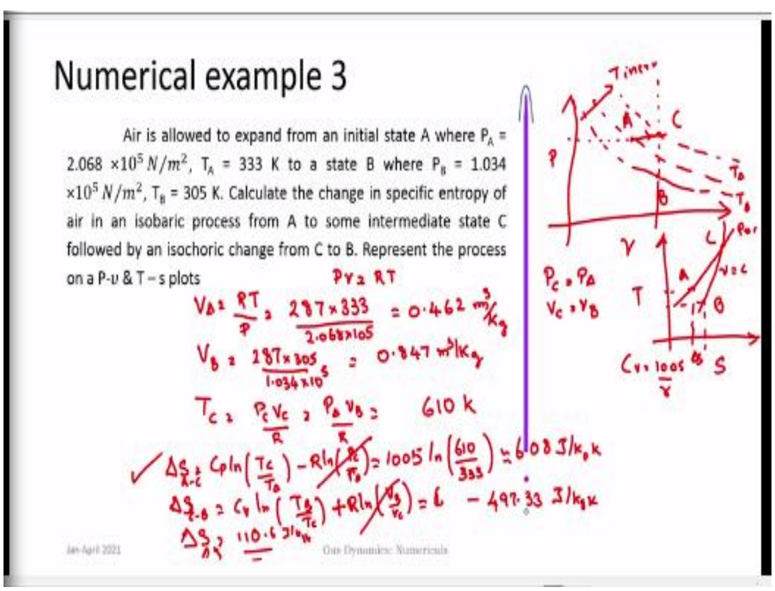

Now let us go to the third numerical example. So, in this air is allowed to expand from an initial state A where $p_A$= 2.068*$10^5$ Pa. And temperature is 333 K to a state B where pressure at B as 1.034*$10^5$ Pa and at temperature at B is 305 K. Now calculate change in specific entropy of air in an isobaric process from A to some intermediate state C followed by an isochoric process from C to B.

So, the state movement from state A to B is achieved via an intermediate state C. And the relationship is that from A it goes to state C which is through an isobaric process and then from C it is followed by an isochoric change, isochoric process ok. So, this is the point, so let us and then represent these processes on a $pv$ and $Ts$ diagrams ok. So, quickly if we look at the $pv$ diagram because the pressure.

And so if you look at that, so this is $pv$ diagram, so $p, v$ is given. And we know that temperature actually increases when you go in this direction, so T increases. So, you can draw to constant temperature isotherms on a constant temperature. So, this can be the lower temperature which is $T_B$ and this is $T_A$. And it is known that this is say point A which is at higher pressure and this is at point B which is at lower pressure, ok.

And so the isochoric process is here is a vertical line and a isobaric process is a horizontal line passing through A. So, immediately from this description we see that C is lying here which is that means it has an increase in temperature. So, here as it goes from this state to this state there is an increase in temperature and further followed by a reduction in temperature and pressure. So, this is the advantage of using these charts that help us immediately without qualitatively understanding the flow field or the thermodynamic processes.

So, now if we do calculate it numerically then you can do it the concepts involved here are what is an isobaric process? What is an isochoric process? So, since you know that $p_B$ equal to this is known $p_C = p_A$ and $v_C = v_B$ ok. Now when both pressure and temperature are given, we can calculate $v_A$ is using the ideal gas relation $pv = RT$. So, $v_A$ is $\frac{RT}{p}$ you can do the substitutions here this is air.

So, we can take that and $\frac{333*287}{2.068*10^5}$ which is equal to 0.462 $\frac{m^3}{kg}$ ok. So, similarly $v_B$ you can calculate this will be $\frac{287*305}{1.034*10^5}$ and this turns out to be 0.847 $\frac{m^3}{kg}$. So, directly we know $v_C = v_B$ so we know the value for $v_C$ here. So, we can calculate $T_C$ can be calculated from here that is $\frac{p_C v_C}{R}$ which is PA $\frac{p_A v_B}{R}$ given the relations.

So, here if you do plug in these numbers you will get this is 610 K, this is about 610 K. Now if you want to look at the entropy, then change in entropy from A to C that is what you are interested in, so this is an isobaric process. So, $\Delta s$ you can use the relation $\left[c_p \ln\left(\frac{T_C}{T_A}\right) - R\ln\left(\frac{p_C}{p_A}\right)\right]$ but $p_C = p_A$. So, this term goes off, you would directly get the $c_p$ as, $c_p \;=\; 1005 \;\ln\left(\frac{T_C}{T_A}\right)$, $T_C$ is known here, 610 K, $T_A$ is 333 K.

So, this you should get a value close to 608 $\frac{J}{kgK}$ specific change and change for the process from C to B. This is A to C for this is an isochoric process, so you could use a $\left[c_v \ln\left(\frac{T_B}{T_C}\right) + R\ln\left(\frac{v_B}{v_C}\right)\right]$. And here it is a isochoric process, so this goes off, so you get you have to use a value of $c_v$ which is 1005 by $\gamma$, ok.

So, this value if you plug it in, then you will get this as a negative number which is - 497.33 $\frac{J}{kgK}$. So, you can calculate the net change of entropy and that is you can you have to add these two. Next change in entropy and you will get that $\Delta s$ is from A to B is about 110.6 $\frac{J}{kgK}$. To cross verify you can always go back and just take because the entropy is going to be a state variable is a state variable.

You just have to you know pressures and temperatures, you can use this relation and the cross check that you get if you substitute $T_B\ and\ p_B$ then you should get the same number that you get over here ok. To now represent this on a $Ts$ diagram, so this is a $Ts$ diagram and the constant pressure process is a curve. So, starting from a value over here which is A it is a curve which goes up to value over here because temperature increases.

And then isochoric process is has a slight higher slope, so you come to a point here which has a slightly lower temperature. But you observe that as we have seen delta S is positive. So, here this is pressure equal to constant, this is volume equal to constant, ok. So, this particular example involves calculation of entropies, ideal gas laws and the thermodynamic charts. So, I think this is a good sort of exercise to be familiar with these concepts. So, that when we go on to do them in gas dynamics, you become comfortable with these kinds of computations. So, the next class we will start looking at fluid flow equations.