Gasdynamics: Fundamentals.. And Applications
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Lecture 47
1D Flows with Heat Addition Rayleigh Flows - Numericals

So, we are looking at Rayleigh flows we have finished discussions on the concepts of Rayleigh flows theoretical part and also the equations.

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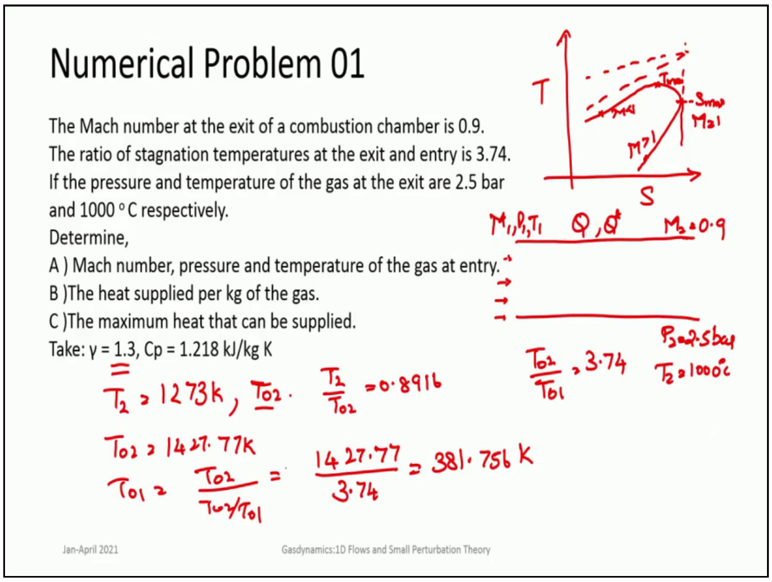

So, let us apply them always remember the Rayleigh curve it is very important. So, if you can keep this in mind that this is the qualitative description of how a Rayleigh flow, Rayleigh curve looks like these in here stagnation temperatures keep changing. So, one should look at stagnation points also and there is a maximum entropy point which is for Mach number equal to 1 and there is a point where there is maximum enthalpy or T is temperature is maximum.

Subsonic branch is at the top and supersonic branch is at the bottom and heat addition always drives the flow towards sonic conditions. Maximum amount of heat that can be added is at the sonic point which at the critical point. So, if you take any point and take the critical point the difference of stagnation temperatures will tell you the maximum heat that can be added. So this is the highlight.

Now let us look at some of the problems. This is a problem number one. the Mach number at the exit of a combustion chamber is 0.9. So, always useful to draw a schematic, a constant area duct and Mach number at exit M2 is 0.9 and ratio of stagnation temperatures at exit and entry so $\frac{T_{02}}{T_{01}}$ is given, $\frac{T_{02}}{T_{01}}$ = 3.74. If pressure and temperature of the gas at the exit is 2.5 bar and T_2 is 1000 degree centigrade respectively. Determine Mach number, pressure, temperature.

So, we need to know M_1, P_1, T_1, heat supplied what is q and the maximum heat that can be supplied and then q* corresponding to the entry conditions. So, γ is given γ =1.3 please make a note of that and Cp is given 1.218 kJ/kgK. So, for this what do we look at. So, we know $\frac{T_{02}}{T_{01}}$ and we know what is M_2. So, from there we should be able to get what is $\frac{T_{01}}{T^*}$ and then therefore get M_1 , T_{02} known. it is this temperature that is static temperature is known.

So, T_2 is 1273 Kelvin and T_{02} is not known but Mach number is known.
So, we directly go and look at the isentropic relations and find out what is $\frac{T_{02}}{T_2}$
and $\frac{T_2}{T_{02}}$ = 0.8916 for M2 equal to 0.9, that means T_{02} from here you can find.
It is T_2 = 1427.77 Kelvin. So, T_{02} is you can be easily found from here. Now we know T_{02} and we know $\frac{T_{02}}{T_{01}}$. So, T_{01} can be found,

$$T_{01} = \frac{T_{02}}{\frac{T_{02}}{T_{01}}} = \frac{1427.77}{3.74} = 381.756 \text{ K}$$

So, we have found what is the; inlet stagnation temperature. Now how do we get what is the inlet Mach number.

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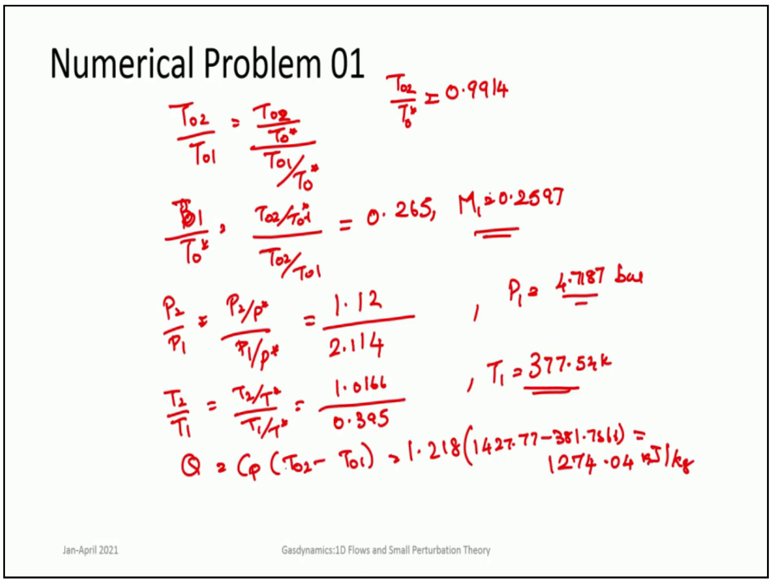

So, $\frac{T_{02}}{T_{01}}$ is,

$$\frac{T_{02}}{T_{01}} = \frac{\frac{T_{02}}{T_0^*}}{\frac{T_{01}}{T_0^*}}$$

So, they now lie on the Rayleigh curve this T_0^* is the same. So, $\frac{T_{02}}{T_0^*}$ can be found for Mach number is 0.9, i.e. $\frac{T_{02}}{T_0^*} = 0.9914$. So, we get $\frac{T_{01}}{T_0^*}$ is $\frac{T_{01}}{T_0^*} = \frac{\frac{T_{02}}{T_0^*}}{\frac{T_{02}}{T_{01}}}$. So, this is correct. So, we will get this value now $\frac{T_{01}}{T_0^*}$, this is it turns out to be $\frac{T_{01}}{T_0^*} = 0.265$.

Now we can go back to the calculators or the charts and find out what is the corresponding Mach number and Mach number $M_1 = 0.2597$. So, we now have the value for the Mach number 0.2597. So, now how do we get the pressures and the temperatures at the entry for that we need $\frac{P_2}{P_1}$. This is, $\frac{P_2}{P_1} = \frac{\frac{P_2}{P^*}}{\frac{P_1}{P^*}}$ for the Rayleigh flow and $\frac{P_2}{P^*}$ is known for this particular case of $M_2 = 0.9$. So, $\frac{P_2}{P^*} = 1.12$, $\frac{P_1}{P^*} = 2.114$.

and therefore from here you can get,

$$\frac{P_2}{P_1} = \frac{1.12}{2.114} = 0.5298$$

$$P_1 = 4.7187\ bar$$

So, you see that P_1 is higher than P_2 which is to be expected in a subsonic flow . So, it is 0.9 it is a subsonic flow and therefore P_1 is greater than P_2.

What about $\frac{T_2}{T_1}$, similar lines , $\frac{T_2}{T_1} = \frac{\frac{T_2}{T^*}}{\frac{T_1}{T^*}}$ and $\frac{T_2}{T^*}$ we have the value it is $\frac{T_2}{T^*} = 1.0166$ while for $\frac{T_1}{T^*}$ for this case which is $\frac{T_1}{T^*} = 0.395$.

$\frac{T_2}{T_1} = 2.5736$

$T_1 = 377.54$ Kelvin

And therefore you can get T_1, $T_1 = 377.54$ Kelvin

So, that is now a smaller temperature that is in keeping with heat addition. What is the amount of heat added q?

$q = Cp\,(T_{02} - T_{01}) = 1.218 \times (1427.77 - 381.7566) = 1274.04$ kJ/kg, this is the amount of heat added .We know Mach number is 0.9. So, it must be very close to the Mach number 1 which is the maximum heat being added but slightly less than that.

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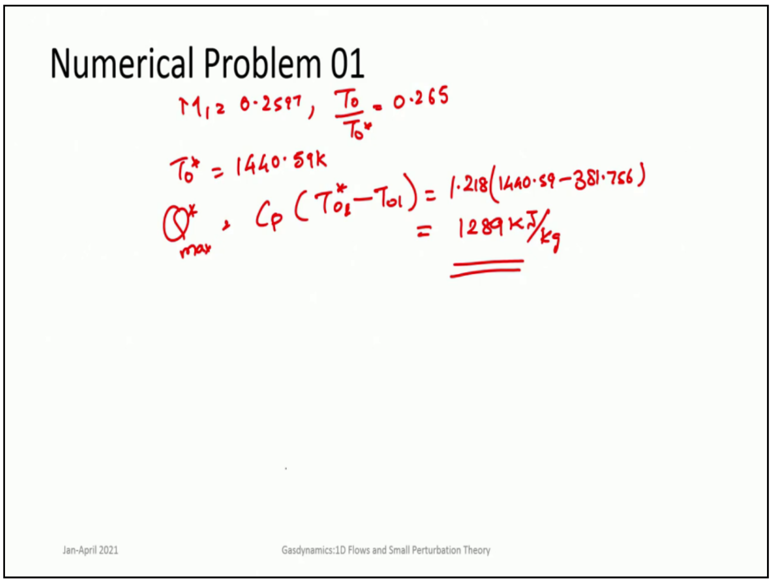

$q_{max} = Cp\,(T_0^* - T_{01}) = 1.218 \times (1440.59 - 381.7566) = 1289$ kJ/kg

So, what is the maximum heat that can be added we know the Mach number M1, M_1 is known it is $M_1 = 0.2597$. So, corresponding to this we can get $\frac{T_{01}}{T_0^*}$ it can be found and that is $\frac{T_{01}}{T_0^*} = 0.265$ and from here from this point we can calculate what is T_0^* which is becauseT0 is known. So, $T_0^* = 1440.59$ Kelvin . Therefore maximum heat q* that is maximum heat is

q_{max}= Cp (T_0^* - T_{01}) = 1.218 x (1440.59 - 381.7566) = 1289 kJ/kg

So, and this is 1289 kilo joules per kg. So, it is very close to since we have Mach number at M_2 is 0.9 we should expect that this value will be very close to the q that was added in the previous case and they are quite close 1274 and 1289. So, this is a direct problem in Rayleigh flow.

So it is always useful to keep this Rayleigh curve in mind when you are solving such problems and look at the charts look at corresponding values of $\frac{T_0}{T_0^*}$, $\frac{P_0}{P_0^*}$ idea is the sonic condition remains a reference point.

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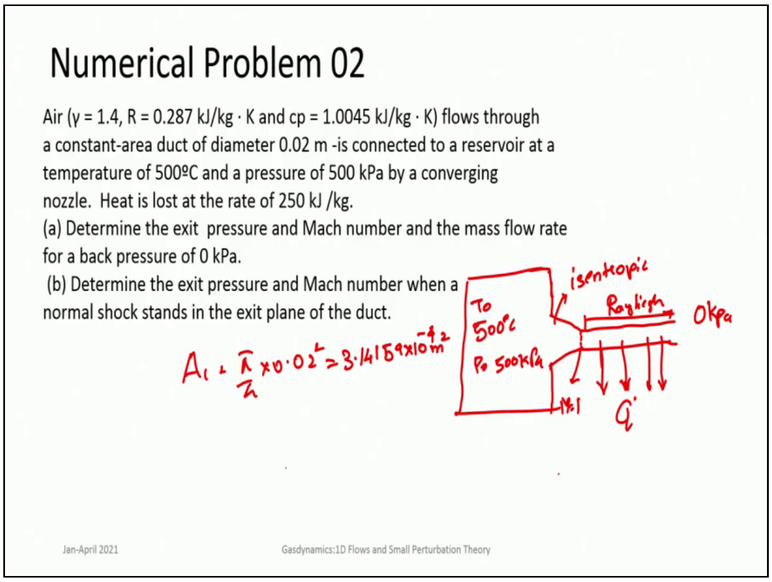

So, now we go to the second numerical. In this air properties are given γ =1.4,

R = 0.287 kJ/kg, Cp is given, and the Cp =1.005 kJ/kgK is also given flows through a constant area duct of diameter 0.02 m and that is connected to a reservoir at a temperature of 500 degree centigrade and a pressure of 500 kilo Pascal by a converging nozzle now heat is lost at the rate of 250 kilo kJ/kg.

Determine the exit pressure and Mach number and the mass flow rate for a back pressure of 0, kilopascal what is meant by 0, kilopascal here is very low pressures at the exit of this determine the exit pressure and Mach number when a normal shock stands in the exit plane of the duct. So, what is given here is that there is an air reservoir a much larger reservoir and that is getting connected to a duct by means of a convergent nozzle and then you have a duct where there is heat is getting removed.

So, heat is removed. So, you can see that in Rayleigh flows both heat can be added or heat can be removed. It is not there is nothing constraining heat removal process. So, we are having two processes here and there is a convergent nozzle this is an isentropic flow. So, it is isentropic here in the nozzle and thereafter you have a Rayleigh flow here. So, a Rayleigh flow here and what is given is reservoir pressures and temperatures 500 degree centigrade. So, this is T_0 = 500 °C and P_0 = 500 kilopascal.

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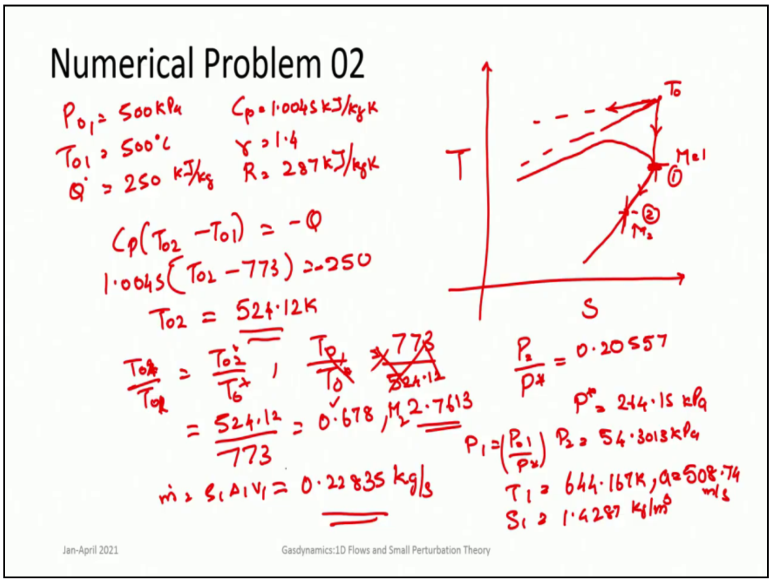

So, always useful to draw the Rayleigh curve and if in this case you are representing this Rayleigh curve. So, what is given is that at the exit the pressure is very low and considered as 0, kilopascal what does this imply? The flow should expand rapidly if you have a conversion nozzle and you have a large pressure differential across it then the obvious case is that you will get Mach number 1 at the minimum area point.

So, at this point Mach number should be equal to 1 and after that you have a Rayleigh flow and there heat is removed. Let us just see this point on a Rayleigh curve Ts diagram. So, this is the case that is being discussed. Draw the Rayleigh curve. This is the maximum stagnation temperature point and what you find is that at the entry to the Rayleigh curve Mach number is M = 1.

So, the first case it should have gone from a stagnation temperature say T_0 over here from that point it has gone there has been an isentropic process to the point where Mach number equal to 1 thereafter heat is removed. So, it follows ,this is a subsonic branch but now if you remove heat from this point it will follow the supersonic branch because Mach number will increase now.

So, it will go to further supersonic values if you remove heat here will go to supersonic values. So, it follows the supersonic branch over here to another. So, T_0 will be lesser. It will be a lower T_0 because heat is being removed. So, what else is given here. So, P_0,T_0 is given P_{01} = 500, kPa, T_{01} = 500 ºC and heat removed is given $\dot{q}$ = 250 kJ/kg.

Cp is given; Cp = 1.0045 kJ/kg K and γ = 1.4, R =287 J/kg K. So, what should be the Mach number at point 2, M_2 ,how do you find this out? We know that heat is being removed.

So, -q = Cp (T_{02} - T_{01}) = 1.0045 x (T_{02} - 773) = -250

T_{02} = 524.12 K

From here we get T_{02} = 524.12 K

So, this corresponds to directly if you do

$$\frac{T_{02}}{T_{01}} = \frac{T_{02}}{T_0^*} = \frac{524.12}{773} = 0.678$$

So, this is directly the case where you have. So, always we look at heat addition problem. M_2 is lower. So, this should be correct. This is equal to $\frac{T_{02}}{T_0^*}$ = 0.678 and the Mach number corresponding to this is M_2 = 2.7613 .We also have to look at the supersonic branch. So, if you look at the supersonic branch you will get the solution to 2.7613. So, M_2 is known.

So, now if M_2 is known then it is easy to find the rest of the values directly. This is the star value that we are looking at. So, if we know $\frac{P_2}{P^*}$ then we will know the pressure for this the $\frac{P_2}{P^*}$ =0.20557. So, and we need to find the static pressure at this particular point at point 1 and this is at point 2. We know it is Mach number one and the stagnation pressure is given from there we can find using isentropic relations P1.

P1 is nothing but $\frac{P_0}{P^*}$ = 1.8939 from there we can find out using isentropic relations what is P* or what is this pressure P * = 264.15 kPa . So, once P* is known then P_2 can be found and P_2 = 0.20557 x 264.15 = 54.3013, kPa. So it is consistent with the Rayleigh flow curve pressure will reduce. So, it has reduced now what about mass flow rate? Mass flow rate can be found m dot.

$\dot{m} = \rho_1 u_1 A_1$ and the area of the duct is given, 0.02 meter diameter is given. So, area is

$A_1 = \frac{\pi}{4}D^2 = 3.14159 \times 10^{-4}$ m^2. Then P_1 is known, T_1 is also known, T_1 can be found out therefore T_1 is from $\frac{T_{01}}{T_0^*}$ isentropi process.

T_1 = 644 Kelvin. And therefore you can find the density ρ_1.

ρ_1 = 1.4287 km/m^3 and velocity corresponds to Mach number equal to 1 which is acoustic velocity at 644 Kelvin which is acoustic speed is 508.74 m/s. Therefore mass flow rate is $\dot{m}$ = 0.228345 kg/sec.

So, this was a different kind of a problem where there was heat removal. So, it is negative heat that is taken out minus q dot that is 250J is taken out.

So, therefore T_{02} reduces that has to be understood here and we should be careful when we look at this problem because the direction of heat transfer is in the opposite direction. But it is starting from the reference value. So, T_{01} is T_0^*. So, $\frac{T_{02}}{T_{01}}$ if you take it becomes $\frac{T_{02}}{T_0^*}$ and therefore $\frac{T_{02}}{T_0^*}$ = 0.678 and therefore you get M_2. And from M_2 and M_1 we can get all other values.

So, with these two problems we come to the end of discussions on Rayleigh flow. So, we have now looked at all kinds of different cases of flows in ducts. It can have variations in area it can have friction or it can have heat addition with constant area. Now in real applications there is no real distinction between all these 3 they may occur together. So, is it possible to analyze such flows. Here when we looked at only single drivers.
Like if you consider only constant area there is no friction, no heat addition then you are able to do isentropic flows and get close form solutions. Then you consider the frictional flow where you considered a constant area duct and there only friction and there you are able to again get close form solutions in Fanno flow. Similarly in Rayleigh flow but what if there is a combination. ?

So, we will introduce the topic which is a generalized quasi-1D analysis where you can have combinations of driving potentials. How do we analyze this problem what are some interesting kind of results that might come about in the next class. So, thank you.