Gasdynamics: Fundamentals And Applications

Prof. Srisha Rao M V
Aerospace Engineering
Indian Institute of Science - Bangalore

Lecture 45
1D Flows with Heat Addition Rayleigh Flows - I

So we are looking at flow through ducts and various different drivers which affect the compressible flow through ducts. One change that was we started with was the variation of area which is variable area ducts. Then we looked at friction, effect of friction separately, constant area duct with friction which was Fanno flow. And now we come to the case where we are taking a constant area duct and there is a heat transfer across the duct.

So, the duct is no longer adiabatic, in the previous cases there was no heat transfer. Now we take pure heat transfer which is this kind of flow is known as Rayleigh flow. We are taking them separately. So, that we can understand in essence what happens when the dominant effect is due to either one of them and then later on towards the end of this section I will give a quick introduction to how one can look at combinations of parameters in what is known as a generalized quasi 1D analytical procedure or equations but that would be just an introduction.

When we are considering these flows they are taken separately and the essence of these or the understandings obtained from these flows are very important and they do not change by great deal when we are looking at more complex flows.

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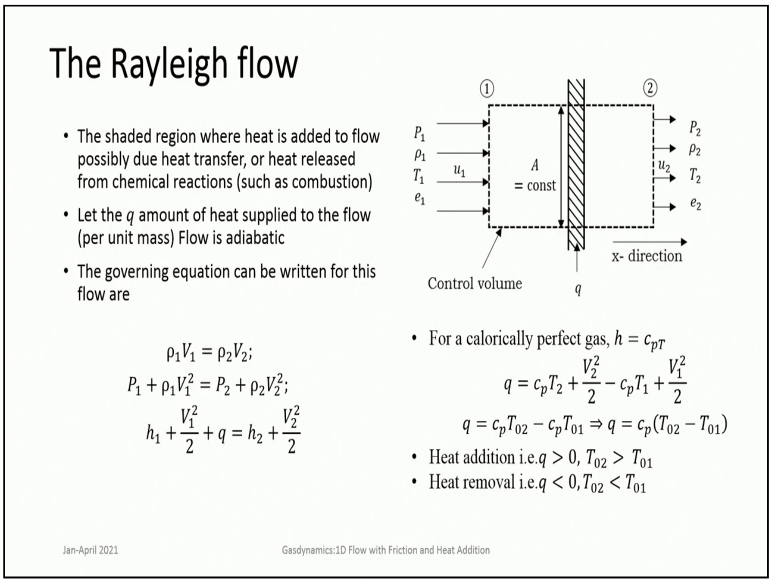

So, what are we looking at in this case this is the Rayleigh flow. And in Rayleigh flow we are looking at heat transfer or heat addition to the flow as the flow occurs in a constant area duct. Now the heat addition or it can be even heat removal. It is a non adiabatic process. So, that means heat can be transferred that can happen both heat addition or heat removal can happen.

It can be due to heat transfer or heat addition can also take place due to heat released from chemical reactions such as combustion. This can also be modelled in terms of a Rayleigh flow and let us take q as the amount of heat supplied per unit mass. So, the flow is not adiabatic. So this is wrong the floor is diabatic that is it allows heat transfer to happen. So, the governing equations they are the mass flow rate through the system remains constant.

So, $\rho_1 u_1 = \rho_2 u_2$; ρu = constant. Area is same.

Then,

$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$

So here we are considering frictionless flow. So, the frictional factors are not coming in picture that means momentum is conserved. What about the total enthalpy?

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

So, there is addition of heat q, q is actually $\dot{q}$ which is total heat added divided by $\dot{m}$.

So, there is a heat added term.

In calorically perfect gas,

h = Cp T.

So, q is essentially is ,

q = h_{02} –h_{01} = Cp(T_{02} –T_{01}). So, this one should understand. So, if there is heat addition. Then heat added that is q is greater than zero that is your stagnation temperature will rise,

$T_{02} > T_{01}$.

If heat is removed from the system then q is less than zero that is stagnation temperatures will drop, $T_{02} < T_{01}$. So, this is in essence what we are looking at.

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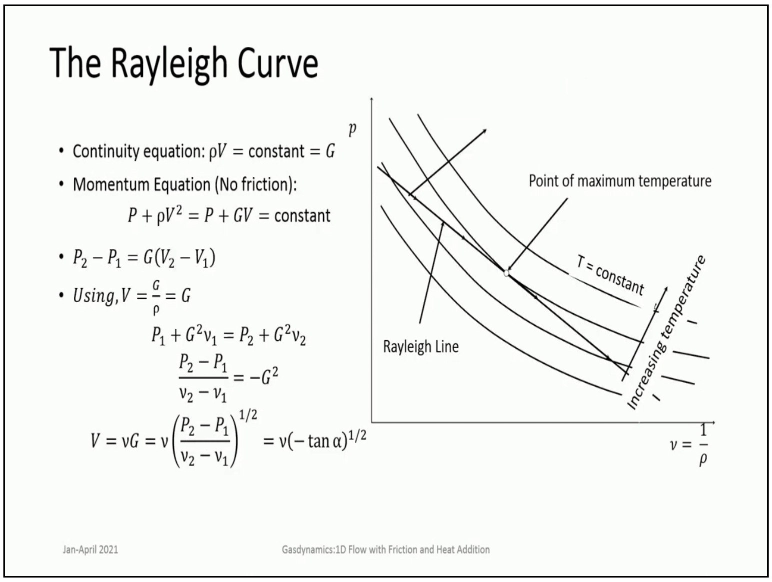

And similar to the Fanno flow, we will first start by looking at the thermodynamics of the process we will put some charts very easy to understand them with charts and that way we will understand almost everything about the flow with heat addition. So, here also

ρu = constant.

So, that is ρu = G ,again this parameter G comes here.

Momentum equation,

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

$$P + \rho u^2 = P + G u$$

$$P_1 + v_1 G^2 = P_2 + v_2 G^2$$

$$P_2 - P_1 = G(v_1 - v_2)$$

$$\frac{P_2 - P_1}{v_2 - v_1} = -G^2$$

So, the momentum equation can also be written in terms of G and specific volume.

Now this is the equation of a straight line. So, the Rayleigh lines or Rayleigh curves if written or if drawn on Pv diagrams, P and specific volume v is they are actually straight lines and from here you can understand this term, $\frac{P2\ - P1}{v_2 - v_1} = -G^2$

G^2 has to be a term which is positive that means $\frac{P2\ - P1}{v_2 - v_1}$ has negative slope. So, that is very important. So, they have always had negative slope.

Then you can relate G with the slope of the Pv, Rayleigh line in the Pv diagram if tan(α) represents the slope of the Pv diagram. Then G is nothing but,

G = $\sqrt{-\,tan(\alpha)}$

So, V then the velocity can also be represented now along the line,

V = $vG = -\,tan(\alpha)$

So, it is very nice way to represent this particular line is known as the Rayleigh line. So, in the Rayleigh line is described by looking at the continuity equation and the momentum equation.

And in Pv diagram they are straight lines and you can draw isotherms and isentropes the curves of constant entropy and curves of constant temperature along these same Pv plane and that will tell us about how temperature changes and pressure changes in these curves.

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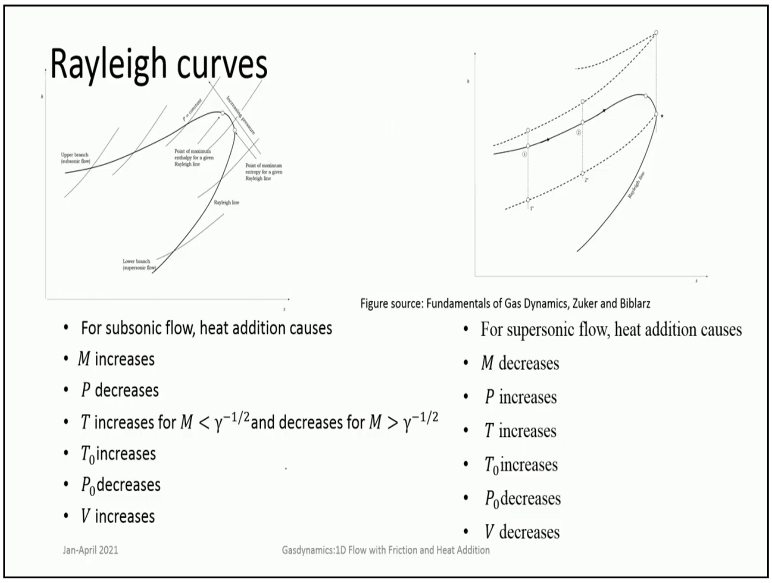

So, now they can also be plotted in hs diagram or Ts diagram similar to Fanno curves. These curves also have 2 branches there is an upper branch and there is a lower branch the way the curves look as of this kind. It is different from a Fanno curve of course but you can notice some important differences one is that if you look at the upper branch which corresponds to the subsonic flow, Mach number is less than one. So, if you take this upper branch; and you see how temperature varies in this branch. First the temperature increases up till a maximum of temperature.

See temperature maximum is achieved at this point and then temperature decreases until another point is reached this point corresponds to maximum entropy Smax, maximum entropy Smax. So, if you look at this curve the upper branch which is a subsonic branch initially there is an increase in temperature after reaching a point where T is equal to Tmax maximum temperature. Then its temperature decreases until it reaches Smax where maximum entropy is achieved.

Now if you look at lower branch which is here, here the temperature continuously increases. So, very important point in Rayleigh curves is there are 2 critical points one is maximum temperature the other one is maximum entropy. Now in order to understand this clearly you should also look at the stagnation temperature lines because it is or stagnation enthalpy lines.

The stagnation enthalpy line will now have 2 branches similar to the stagnation or rather similar to the Rayleigh curve. It will not be h not is not constant like that like it was for the case of Fanno flow . So, here if you add heat always the stagnation enthalpy increases or T_0 increases. So, going from 1 to 2 increasing in this direction there is an addition of heat. So, T_0 increases. So, the lower branch of this curve corresponds to subsonic flow.

Because this height here this difference should correspond to $\frac{u^2}{2}$ by the equation. So, this height should correspond to $\frac{u^2}{2}$. So, the lower one corresponds to Mach number less than 1 while the upper branch corresponds to Mach number greater than 1 and they reach a cusp rather they reach a point at which maximum heat can be added and that particular cusp corresponds to the point where entropy is maximum S is equal to Smax.

Then q is equal to qmax ,maximum heat that can be added. So, this point should be is very important. And also notice that here you have a line if you draw, an isentropic line for a point one you have T_0 corresponding to that and T* corresponding to the point where it can be isentropically brought to Mach number equal to 1. Even this T* is varying because now heat is getting added into the flow.

So, T* and T_0 are not constants. So, do not confuse between isentropic flow, Fanno flow and Rayleigh flow. Rayleigh flow is not an adiabatic flow . So, therefore T* and T_0 * or T_0 they are not constants. Now we know how pressures are depicted along the hs diagram. So, if you look at the curve itself you can make an understanding or you can get an understanding of what happens.

For subsonic flow when heat is added you see that this height increases that means velocity increases, Mach number increases, pressure decreases, temperature increases until a particular point, maximum the maximum enthalpy point and that particular point can be shown to be equal to $\frac{1}{\sqrt{\gamma}}$. After this point the temperature decreases why this is happening why should temperature increase at lid point and then decrease?

This is because actually what you see is that addition of heat affects total enthalpy which is T_0 or h_0, h_0 is a combination of , $h + \frac{u^2}{2}$

This is static enthalpy, h and $\frac{u_1^2}{2}$ is kinetic energy. So, for the initial region of this subsonic branch until the point of maximum enthalpy the increase the addition of heat goes largely to increase static enthalpy and less to increasing kinetic energy.

Of course V is increasing but it is to do with the relative increase of the 2 initially the relative increase is more towards increase of static enthalpy. But once you reach maximum enthalpy point further addition of heat actually increases kinetic energy enormously and some amount of it actually can also be taken from the static enthalpy. So, total enthalpy which is a combination $h + \frac{u^2}{2}$ that continue to increase but static enthalpy reduces because now more amount of energy is taken by the kinetic energy.

So, that is why this happens. So, you have to view this as a combination your h_0 is increasing all the time when you add heat but they are a combination of static enthalpy and kinetic energy. T_0 or stagnation enthalpy increases. P_0 actually decreases. P_0 decreases. Because you find here that even here not only just that entropy changes. So, you have to work out the equations here. So, that needs to be done.

In case of supersonic flows when you add heat the addition of heat causes Mach number decreases. So, velocity actually decreases pressure increases temperature increases T_0 ofcourse definitely increases, P_0 also decreases. So, this is evident from this Rayleigh curves.

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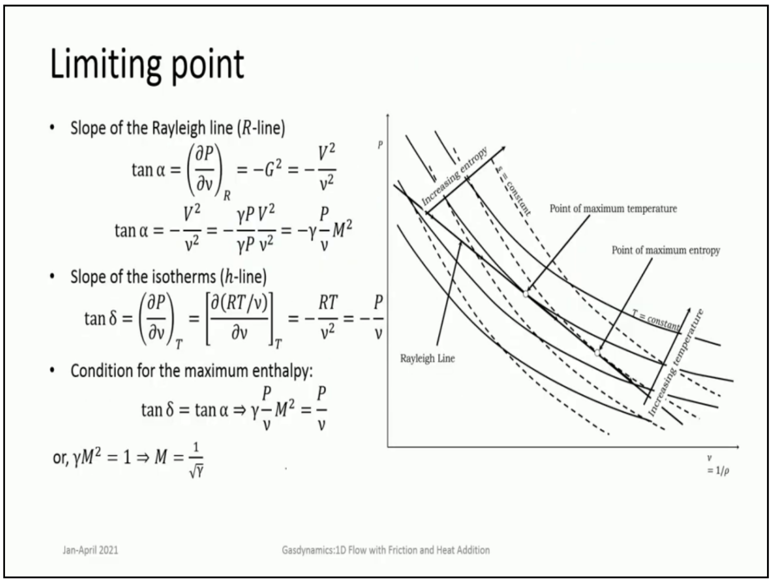

Now how do we get to these limiting points? So, this is what was meant. So, if you look at the Pv diagram these straight lines this particular straight line is the Rayleigh line. Now if you look at the curves at which h = constant. This is static points of sort of constant entropy S =constant and points at which temperature is constant that is enthalpy is constant.

There are 2 different curves and we know they are related to each other. So, the entropy curve has a higher slope than the enthalpy curve. So, this is T = constant and this is S = constant. This difference is there as a consequence. So, where is this point of maximum temperature? It is a particular point where this Rayleigh line becomes tangent to the temperature equal to constant curve.

Then whenever it becomes tangent to the entropy equal to constant curve S = constant. Then that corresponds to maximum entropy further beyond that you do not have any solution. So, you can see that these are 2 different points here. So how do we get that the slope? So, because we look at the tangency condition, $tan(\alpha)$ that is the slope of the Rayleigh line is $-G^2$ that is we have established that,

$$tan(\alpha) = \left(\frac{\partial P}{\partial v}\right)_R = -G^2 = -\frac{u^2}{v^2}$$

v that is specific volume.

This can be expressed in terms of Mach number if you do multiply and divide by γ P then this is a^2 . So, you get,

$$tan(\alpha) = -\frac{\gamma P u^2}{\gamma P v^2} = -\frac{\gamma P M^2}{v}$$

Now what about the slope of isotherms?

That is taken as $tan(\delta)$ which is $tan(\delta) = \left(\frac{\partial P}{\partial v}\right)_T$ at constant temperature.

And we know the ideal gas equation of state and from there we can find out when temperature is constant what is the slope of the isotherms or isotherms ,

$$tan(\delta) = \left(\frac{\partial P}{\partial v}\right)_T = \left(\frac{\partial(\frac{RT}{v})}{\partial v}\right)_T = -\frac{RT}{v^2} = -\frac{P}{v}$$

So, now you can at the point of maximum enthalpy that is a tangency condition between maximum or temperature constant curves and the Rayleigh line.

So, $tan(\alpha) = tan(\delta) = \frac{\gamma P M^2}{v} = \frac{P}{v}$

So, from here you get,

$$M = \frac{1}{\sqrt{\gamma}}$$

So, this establishes the relation for the point of maximum temperature.

Now what about the limiting point of maximum entropy we should look at the tangency condition for the Rayleigh line, $tan(\alpha)$ with the entropy line isentropes.

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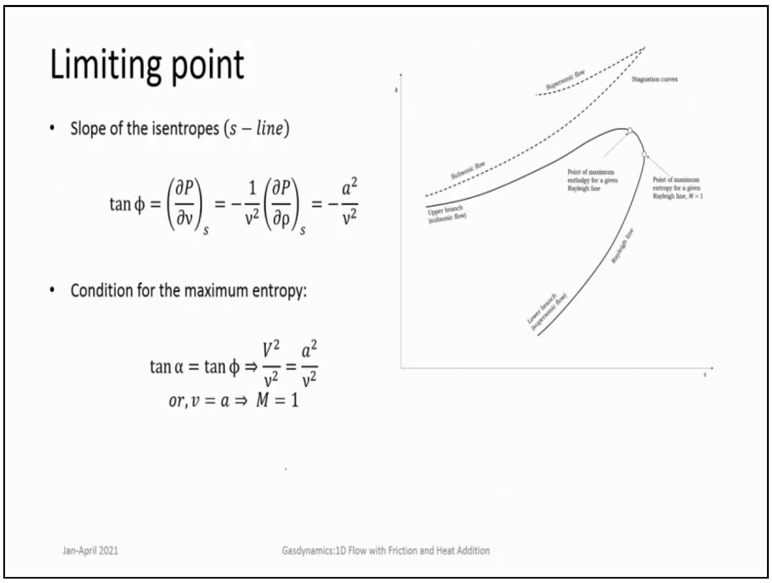

So, S = constant lines, so, if you look at that effect,

$$tan(\Phi) = \left(\frac{\partial P}{\partial v}\right)_s = -\frac{1}{v^2}\left(\frac{\partial P}{\partial \rho}\right)_s = -\frac{a^2}{v^2}$$

$\left(\frac{\partial P}{\partial \rho}\right)_s$ at constant entropy is speed of sound, a^2.

$$tan(\alpha) = tan(\Phi)$$

$$\frac{u^2}{v^2} = \frac{a^2}{v^2} \rightarrow u = a \rightarrow M=1$$

Again we find that at the point of maximum entropy Mach number equal to 1. So, and that also represents a switch over point now between subsonic flow and the supersonic flow. So in all the different cases variable area ducts or Fanno flow or Rayleigh flow Mach number equal to 1 was a critical point in all these cases it is a critical point but they are different descriptions. So, they are not the same point . So, in a Fanno curve it has to follow a Fanno curve to get to point Mach number equal to 1 and then it is a critical point in the final curve.

In a Rayleigh curve it has to follow a Rayleigh curve or a Rayleigh line and it has to reach Mach number equal to 1 which is a critical point. This point also corresponds to the maximum qmax, qmax that is maximum heat added kind of situation here. So, if you add heat in a subsonic flow it drives the flow towards Mach number equal to 1. Similarly if you add heat to a supersonic flow it also drives the flow towards Mach number equal to 1.

And the maximum heat that can be added corresponds the point where maximum entropy line lies and that also is equal to one. So, the point why you get Mach number equal to 1 qmax is also on the same line is slightly different from the point where you if you consider an adiabatic flow like the frictional Fanno flow with friction you cannot have the flow going smoothly from subsonic flow to supersonic flow in case of a Fanno flow.

This is a Fanno flow and you have Smax and Mach number equal to 1, subsonic branch is a supersonic branch and one cannot go from one branch to the other branch because this is an adiabatic flow and here entropy has to be always Δs has to be always greater than zero. So, this is the condition here but this is not an adiabatic flow but here only by means of heat addition one cannot go beyond Mach number equal to 1 because if you look at the stagnation curve that corresponds to maximum heat added.

So, you cannot add heat beyond q_{max} maximum heat there are no solutions beyond that . So, you cannot move beyond subsonic to supersonic just by means of heat addition or supersonic to subsonic just by means of heat addition the reason is somewhat it is a fine point it is somewhat different from the point why it is the same for the Fanno flow.

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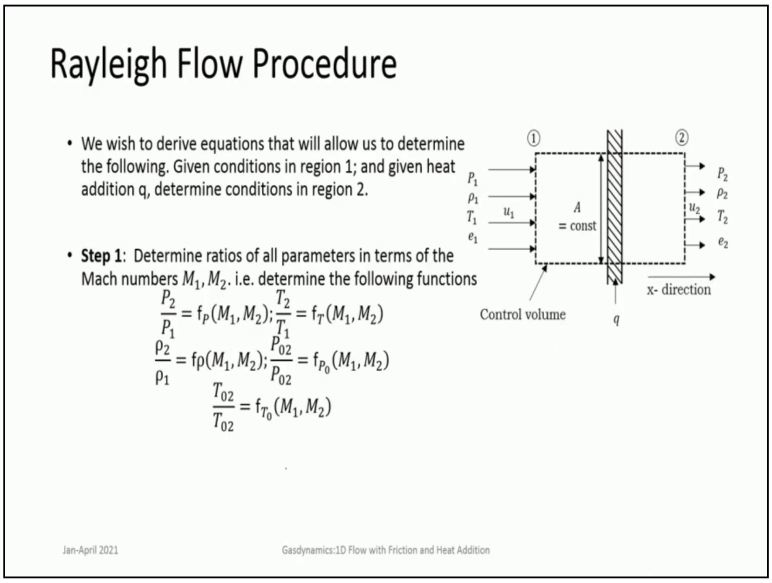

So, now when we look at Rayleigh flows similar to Fanno flow now this has been a qualitative understanding but more or less all the different conditions of Rayleigh flow can be understood by means of a Rayleigh curve similar to Fanno curve. If you understand this curve thoroughly then you understand almost all aspects of Rayleigh flow. So, please learn to draw these curves qualitatively and distinguish the various points.

So, now we what we want to do is solve situations where we come across Rayleigh flow. There the points would be that you would know how much amount of heat is added and you know points in region one. We would know want to know what happens to region 2. So, this is a typical problem . Then how would we go about this particular problem. So, the guiding principle here is that $Cp(T_{02} - T_{01}) = q$.

So, this is the guiding principle. Then we see all similar to the other cases we seek the ratios $\frac{P_2}{P_1}$, $\frac{T_2}{T_1}$, $\frac{\rho_2}{\rho_1}$ they should be expressed solely as functions of M_1, M_2 and γ. For a calorically perfect gas γ is a constant. Also $\frac{T0_2}{T_{01}}$. So, that is what we wish to do and $T0_2$ can be determined by using this particular equation. So, once you know $T0_2$, $\frac{T0_2}{T_{01}}$ can be determined if M_1 is known M_2 can be known.

And this can be done but we would look at the equation soon we will find that again solving them is quite tedious task. So, we always use a reference point for the Rayleigh curve. The reference point is going to be the maximum entropy point where Mach number will also become equal to 1. So, for a given Rayleigh curve this point is fixed. It is a fixed point for that particular Rayleigh curve.

So, that is a reference point you have it in terms of star values. So, $\frac{P}{P^*}$, $\frac{T}{T^*}$ and $\frac{\rho}{\rho^*}$. So, now again clearly distinguish between all the different processes the star value that you got in a variable area duct is due to an isentropic process. It is different from the star value you get when Mach number 1 is achieved in a Fanno flow, it is due to a process of having friction in a constant area duct.

Similarly you can drive the flow to Mach number equal to 1 in the case when there is heat being added to the flow and in that case you get the Rayleigh flow and you get a Mach number equal to 1 in the case of Rayleigh flow also. These three conditions are not the same. So, do not ever take them to be the same. When you are considering Rayleigh flow and Rayleigh curve the star condition is corresponding to Rayleigh curve.

When you are considering Fanno flow; it is corresponding to Fanno flow. So, this part you have to take care. So, the equations how to solve them we will discuss in the next class and thank you.