Gasdynamics: Fundamentals.. And Applications

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Lecture 44
1D Flow with Friction - Fanno Flow - Numericals

So we have discussed about Fanno flows. They are compressible 1D flow with friction. We looked at how they behave what are their equations and we solve the simple numerical in a subsonic case. Let us solve a couple of numericals now. You can also look at what happens in supersonic case and if there are shocks and so on.

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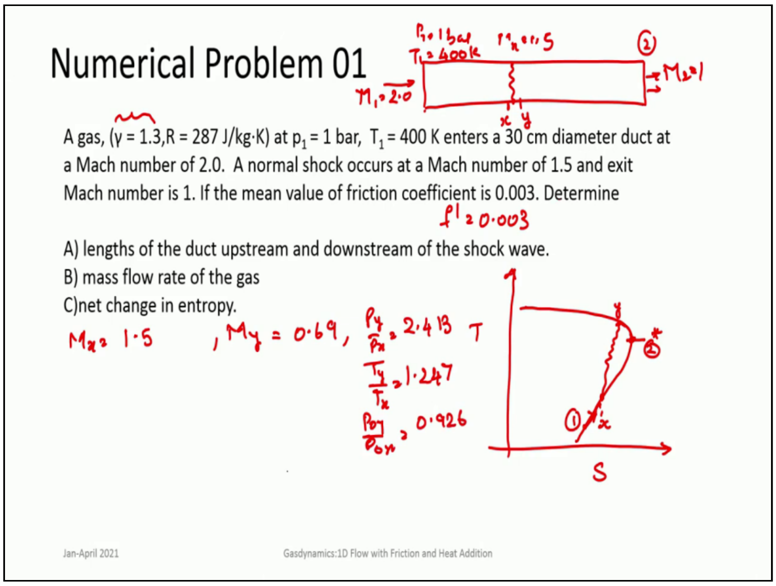

So, let us look at this problem a gas γ is given γ =1.3, R = 287 J/kgK . So, please make note of γ it is slightly different from what we usually use. So, if you are using online calculators then there is an option to change γ . So, at P_1 = 1 bar ,T_1 = 400 Kelvin. So, it is given. So P_1 is given T_1 is given. Enters a 30 centimetre diameter duct, duct diameter is given at a Mach number of 2, M_1 is given. So, let us draw the duct.

We know, P_1 = 1 bar ,T_1 = 400 Kelvin and M_1 =2.0. A normal shock occurs at a Mach number of 1.5. So, it is a Fanno flow with so in supersonic conditions Mach number decreases. So, at somewhere in the duct some location we do not know the location in terms of length but we know the Mach number. Mach number is 1.5 at which the shock occurs. So, let us call this particular place I call it as x and after the shock as y and what is known is at the exit at location 2 the Mach number is 1.

So, here exit Mach number M_2 = 1. Very effective if you are able to draw a TS diagram of this particular case. So, what we are looking at is essentially something of this kind this is the star point. So, initially you started with Mach number of 2. So, this is point 1 and follows the Fanno curve reaches a point x where there is a shock. So, the shock transfers the Fanno curve. So, the line that is being followed from x to y which is located on the subsonic branch of the Fanno curve and it reaches point 2 which is exactly equal to the star condition.

So, it is useful to draw this. If you understand this then, it is once this is understood, it is reasonably easy to calculate all the other things. Friction coefficient the mean value of friction coefficient is 0.003. So, f' = 0.003, termed in the lengths of upstream duct and downstream duct or the shock wave. So, what is the location of the shock wave? We know the Mach numbers at which they are located. So, this is at 1.5.

So, Mx = 1.5 . We know this value. So, and we also need to find the change in entropy, mass flow rate of the gas and so on. So, let us look at this problem. Now so, we know that normal shock if it is located at So, M x = 1.5 then we know My it is a normal shock but please make sure that γ you change it is 1.3, My = 0.69 ,pressure ratio across the shock is $\frac{Py}{Px}$ = 2.413 and $\frac{Ty}{Tx}$ = 1.247 and $\frac{P0y}{P0x}$ = 0.926 and we know M_1 = 2. So, how do we proceed from here?

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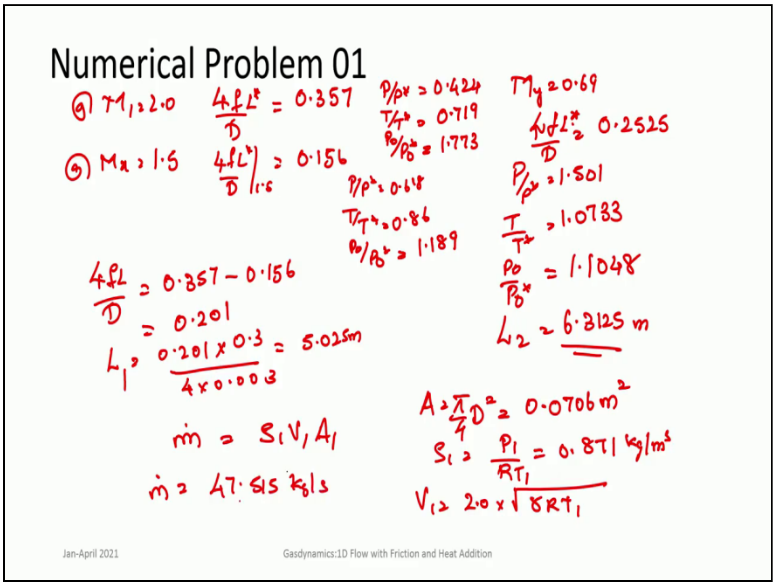

So, what we should find out is what is L_1? So, at Mach number equal to M_1 = 2.0 the value of $\frac{4fL^*}{D}$ = 0.357 This is known .You can also make note of, $\frac{P}{P^*}$ = 0.424 , $\frac{T}{T^*}$ =0.719

and $\frac{P_0}{P_0^*}$ = 1.773. So, it is better to make note of them.

And at M x at Mx = 1.5.

So, $\frac{4fL^*}{D}$ = 0.156 , $\frac{P}{P^*}$ = 0.618 , $\frac{T}{T^*}$ =0.86 and $\frac{P_0}{P_0^*}$ =1.189.

So, what is the length now before the shock it is easy to find that out.

So, $\frac{4fL}{D} = \left|\frac{4fL^*}{D}\right|_1 - \left|\frac{4fL^*}{D}\right|_x = 0.357 - 0.156 = 0.201.$

$L_1 = \frac{0.201 \, x \, 0.3}{4 \, x \, 0.003} = 5.2025$ m

Similarly now across the shock you know the conditions.

Now across the shock the condition is that your Mach number is My = 0.69 ,

$\frac{4fL^*}{D}$ = 0.2525, , $\frac{P}{P^*}$ = 0.1.501 , $\frac{T}{T^*}$ =0.1.0733 and $\frac{P_0}{P_0^*}$ =1.1048

So, similarly so, downstream it is directly L*. So, we know L*. So, because it is going to Mach number 1 at the exit, So, L_2 = 6.3125 meters. So, that solves the problem.

The second case is what is mass flow rate. For this you need to know the mass flow rate $\dot{m} = \rho_1 A_1 u_1$ is known because diameter is given 30 centimetres or 0.3 meters.

$A_1 = \frac{\pi}{4} D^2 = 0.076 \text{ m}^2$

Density ρ_1 , P_1 and T_1 are given. So, you can find out ρ_1

$\rho_1 = \frac{P_1}{RT_1} = 0.871$ kg /m^3

What about V_1? V_1 is a Mach number is given 2.0 and $u_1 = a_1 M_1 = M_1 \sqrt{\gamma R T_1}$, T_1=400 K.

So, substitute all of them here put them and $\dot{m}$ turns out , $\dot{m}$ = 47.515 kg/ sec.

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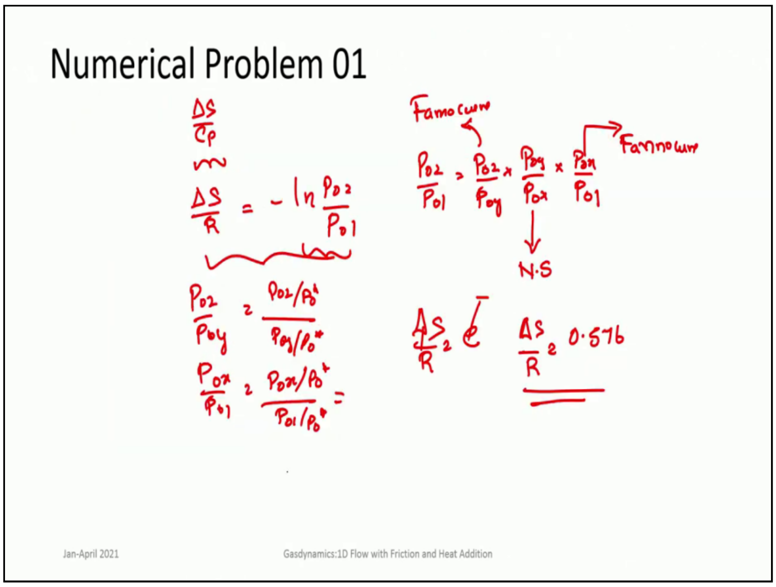

Now how to get net change in entropy $\frac{\Delta S}{Cp}$ so, for this we can use or $\frac{\Delta S}{R}$ you can use. It should be related to the change in entropy.

$$\frac{\Delta S}{R} = -\ln \frac{P_{02}}{P_{01}}$$

So, if you know the total change in $\frac{P_{02}}{P_{01}}$ does change or ratio of the stagnation pressures then we can find out what is the entropy change.

So, here what are the stagnation changes if you like write $\frac{P02}{P01}$, this can be expressed as

$$\frac{P_{02}}{P_{01}} = \frac{P_{02}}{P_{0y}} \times \frac{P_{0y}}{P_{0x}} \times \frac{P_{0x}}{P_{01}}$$

Where $\frac{P_{02}}{P_{0y}}$ comes from Fanno flow Fanno curve or those equations $\frac{P_{0y}}{P_{0x}}$ is normal shock the stagnation pressure ratio across normal shock and $\frac{P_{0x}}{P_{01}}$ is again by Fanno curve.

Though you can get these by different you know Mach numbers at each particular point. So,

$\frac{P_{02}}{P_{0y}}$ can be expressed as $\frac{P_{02}}{P_{0y}} = \frac{\frac{P_{02}}{P_0^*}}{\frac{P_{0y}}{P_0^*}}$

and similarly $\frac{P_{0x}}{P_{01}} = \frac{\frac{P_{0x}}{P_0^*}}{\frac{P_{01}}{P_0^*}} =$

You can express these all these in terms and finally you can arrive at $\frac{P_{02}}{P_{01}} = e^{-\frac{\Delta S}{R}}$

So, $\frac{\Delta S}{R} = -\ln \frac{P_{02}}{P_{01}} = 0.576$

So, this looks at a problem in which we have a supersonic flow coming in there is a normal shock in the middle of the duct at some location and finally at the exit Mach number equal to 1. In these situations always it is useful to draw the TS diagram and then it is very easy once you know the TS diagram you know what you are looking for.

And rest of it is through charts or you can use the online calculators to get the other numbers. The essential concept one has to understand the location of shock and how to represent it on a Fanno curve.

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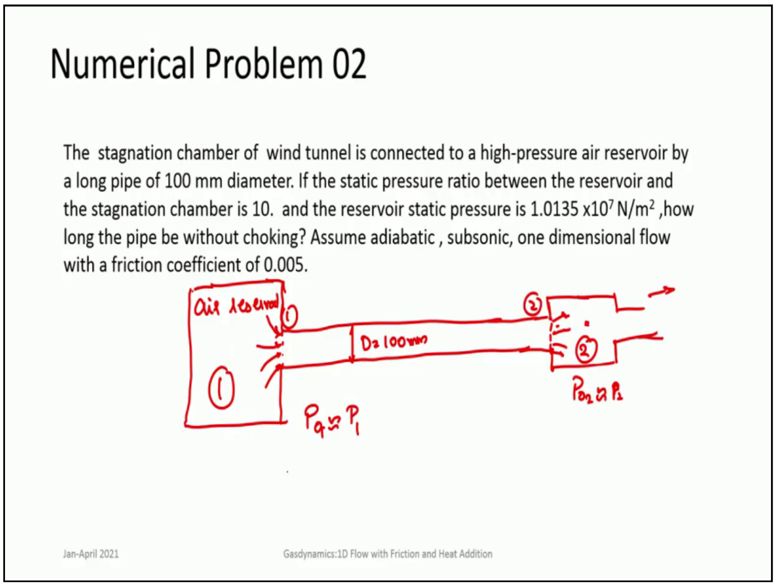

Now if you look at the next problem the stagnation chamber of wind tunnel is connected to a high pressure air reservoir by a long pipe of 100 mm diameter. If the static pressure ratio between the reservoir and the stagnation chamber is 10 and the reservoir static pressure is 1.0135 x 10^7 N/m^2, how long the pipe is without choking. So, what should be the length of pipe so that there is no choking.

Assume adiabatic subsonic one dimensional flow with a friction coefficient of 0.005. So, let us represent it schematically this is a sort of applied kind of a problem where you are considering wind tunnels . And we had looked at how wind tunnels look like in the previous classes. They usually have air reservoirs which are located quite far away from the test section. So, relevant problem and they are connected by pipes.

So, you have a long pipe and this is air reservoirs where a large amount of air is stored air reservoir and this diameter is given D = 100 mm and here you have a smaller stagnation chamber is a smaller stagnant chamber and then further it goes down. You have the wind tunnel there. So, what is given is the ratio of pressures across this. Now if you consider reservoirs what you would say as static pressure.

Inside the reservoir the velocities are going to be extremely small. So, you can almost take that P_0 is approximately equal to P. So, if this is 1 and this is 2 then what you are essentially given here is P_{01} is approximately equal to P_1 and P_{02} is very close to P_2 inside the reservoir or inside the stagnation chamber somewhere at this location you can say. But please mind that at the entry to the pipe and the exit to the pipe you can have different static pressures.

Because flow will obviously be coming in gushing in from all directions. Similarly flow will be exiting over here. So, they are not talking about these entry points here which is 1 and 2 for the pipe. What we look for is 1 and 2 of the pipe but what you are given of course is about what is there inside the reservoir. So, can we now look at this problem?

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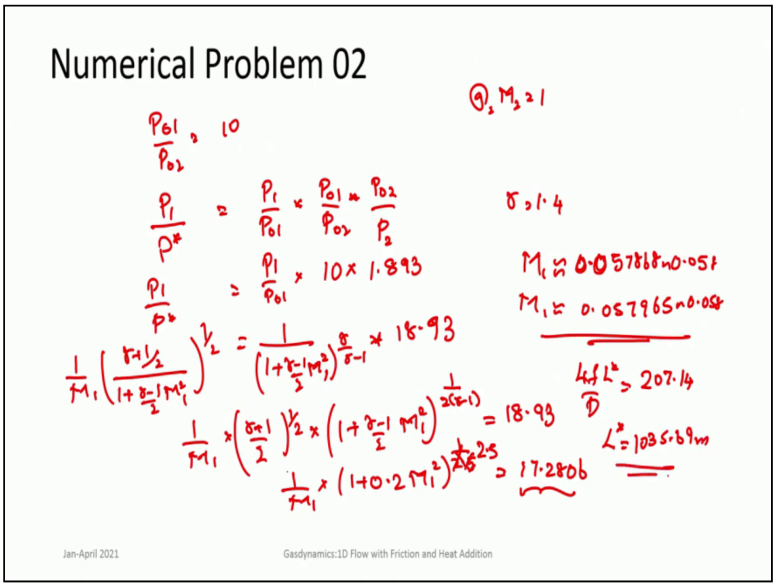

So, what we are given is basically $\frac{P_{01}}{P_{02}} = 10$ is given. Now we are considering that what should be the length that it does not choke. So, what is the maximum limit, maximum limit Lmax is the point when it chokes. So, M = 1, suppose we take the maximum limit that is Lmax, M = 1, any length which is less than Lmax will satisfy that it is not choked. So, we take the limiting condition Lmax is. So, this is equal to Lmax.

So, Lmax. So, what do we know about that we know that it is $\frac{P_{02}}{P^*} = \frac{P_{02}}{P_2}$ 1.893 or P_2 is actually P* in this case, this is equal to this and $\frac{P_{01}}{P_{02}}$ is given. And also we know that at 2 Mach number M_2 =to 1 therefore $\frac{P_{02}}{P_2}$ is also known at, at that particular point.

$$\frac{P_1}{P^*} = \frac{P_1}{P_{01}} \text{ x } \frac{P_{01}}{P_{02}} \text{ x } \frac{P_{02}}{P_2} = \frac{P_1}{P_{01}} \text{ x } 10 \text{ x } 1.893$$

From Fanno flow,

$$\frac{P_1}{P^*} = \frac{1}{M_1}\left[\frac{(\gamma+1)}{2+(\gamma-1)M_1^2}\right]^{\frac{1}{2}}$$

What is $\frac{P_1}{P_{01}}$? $\frac{P_1}{P_{01}} = \left[\frac{1}{\left(1+\frac{\gamma-1}{2}M_1^2\right)}\right]^{\frac{\gamma}{\gamma-1}}$

So, $\frac{1}{M_1}\left[\frac{(\gamma+1)}{2+(\gamma-1)M_1^2}\right]^{\frac{1}{2}} = \left[\frac{1}{\left(1+\frac{\gamma-1}{2}M_1^2\right)}\right]^{\frac{\gamma}{\gamma-1}}$ x 10 x 1.893

$$\frac{1}{M_1}\left[\frac{(\gamma+1)}{2}\right]^{\frac{1}{2}} \text{ x } \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{1}{2(\gamma-1)}} = 18.93$$

$$\frac{1}{M_1} \text{ x } \left(1+0.2M_1^2\right)^{1.25} = 17.2806$$

Now we know that Mach number is extremely small. So, this of course you have to solve but it needs an iterative solution but one fact we know Mach number is extremely small at the entry.

So M_1^2 is going to be very small. So, if we take that approximation then this parameter in bracket is going to be 1. So, Mach number M_1 will approximately be equal to 0.057868.

$M_1 \approx 0.057868$.

So, we get a ballpark number. This is a number where the Mach number of point at point 1 and we can then look at iterating this slightly. So, so as to satisfy even more closer, this particular condition. So, you can do a little bit of iteration and exact number is very close to 0.057965. So, you see if you take the third decimal place they are almost the same 0.058 because this is 0.058.

So, this is known. So, once you know this you know that L* , that is L*you can get by finding out what is $\frac{4fL^*}{D}$ for this Mach number. This is turns out to be, $\frac{4fL^*}{D}$ = 207.14 .
Therefore $L^* = 1035.69$ meters.
So, any length which is smaller than this L^* will not have a choked flow. So, this is a nice applied problem where pressure ratio was given across the reservoirs and then the length was the maximum length at which choking would occur is taken.

So, there are 2 problems. So, we have covered a problem in subsonic flow a problem involving supersonic flow and shocks and an applied problem. And this covers Fanno flow which is one of the driving changes of compressible flow in ducts. So, we have discussed 2 drivers one is variable area the other one is flow with friction. Now one more driver is flow where there is heat transfer.

When you consider heat transfer you do not consider varying area or friction that kind of a flow is known as Rayleigh flow which we will see in the next class, thank you.