Gasdynamics: Fundamentals.. And Applications
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Lecture 43
1D Flow with Friction - Fanno Flow- III

We have been looking at 1D flow with friction that is Fanno flow. We looked at the Fanno curve which thermodynamically shows how these flows behave when they are starting from subsonic flow and supersonic flow. Then we looked at equations of Fanno flow. And now we will look at how these look at these equations. And then come to the concept of what is known as frictional choking.

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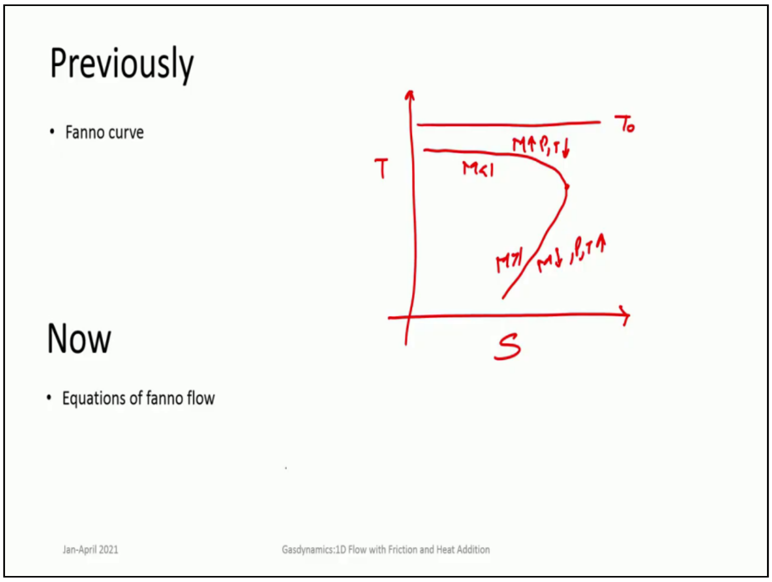

So, bear in mind and there is it can be repeated many times because this is very an essential concept that you look at whenever you look at Fanno flows remember this curve. Qualitatively it looks like this T_0 is a constant. And this is the way Ts looks like. Maximum entropy point. Mach number is less than 1, Mach number is greater than 1. Here Mach number increases, pressure, temperature decreases. Here Mach number decreases, pressure and temperature increases.

So, this is a supersonic branch, this is subsonic branch. This concept is known. Then let us go ahead now and go towards different concepts of this friction.

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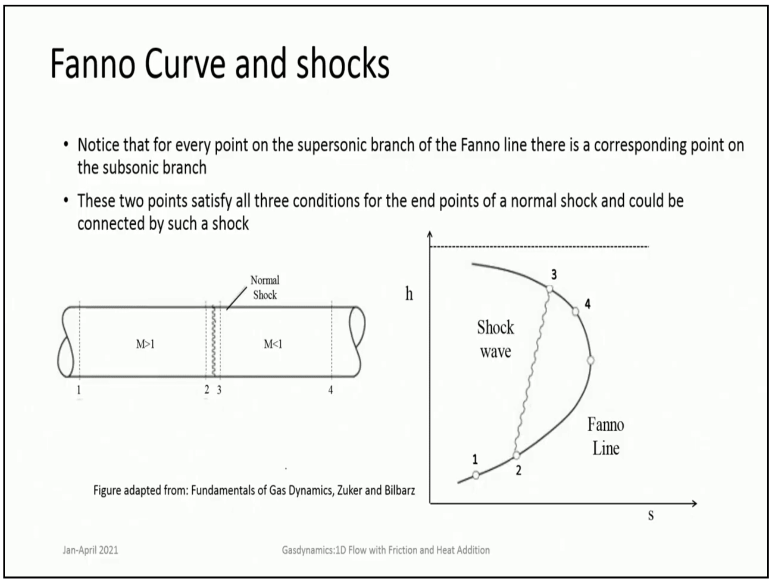

The first thing that we have to understand here is look at now in this it is a 1D flow . And we know that in supersonic flows one can always expect shock waves. And in 1D flow we look at normal shocks. Now the question we can ask here is can a normal shock be present in a 1D duct with friction ?. So, when there is a frictional flow happening can there, be a normal shock. If there is a normal shock how will it get represented.

And that can be immediately understood because one thing is shocks are discontinuity. So, they do not have any specific length or their discontinuous. So, since that is the case. And the other cases they saw they also satisfy the equation,

$\rho_1 u_1 = \rho_2 \; u_2$; ρu = constant.

And also, they also satisfy the equation,

$$h + \frac{u^2}{2} = constant$$

that is total enthalpy; it is an adiabatic flow.

Now how did we draw the Fanno curve? We drew the final curve by just considering these 2 equations and then we consider G as a parameter. And rho can be varied correspondingly whatever fits in that formed the Fanno curve. And therefore if you look at a normal shock and look at the equations that depict normal shock they are the same. So, that means normal shock, the states of normal shock can be located on a Fanno curve.

So, that is a very straightforward inference that you can make out of this. And this point you can also understand by the fact that there are 2 branches in a Fanno curve. If you take this branch this is less than 1. And there is a corresponding supersonic branch for M greater than 1. So, one can always find 2 points right, one, say 2 and 3 here that corresponds to a normal shock states across a normal shock. So, if you have a duct and there is a the inlet is supersonic.

And there is a possibility of a shock occurring within the duct. Then that shock can be located on the Fanno curve. So, if you look at this particular the problem or this problem that is given over here the entry Mach number is greater than 1 that is a supersonic, $M > 1$. Then first it passes through a duct of length L1. And a reaches a Mach number M_2 at which there is a shock. And across the shock you get conditions M_3.

So, M_2 conditions 2, conditions at 3 there is no change in length because shock is a discontinuity it is at that same location. And then you have a length L2 .Now this L2 at this particular point is having subsonic flow. So, this particular description of this problem can be represented on a Fanno curve like this. So, starting from 1 it went along this Fanno line to the point 2 , the flow is still supersonic.

But at this point a shock occurred and therefore you have this wiggly line of course you cannot put any line or anything across a shock it just jumps from point 2 to point 3. So, it is located on the same Fanno curve because G is the same. So, same can occur. Look at the point 2, 3 which is across the shock. Then further there is another flow or the Fanno flow which happens for length L2.

So, initially Mach number decreases in the supersonic flow there is a shock, there is a sudden jump Mach number becomes subsonic. Then after that Mach number increases pressure slightly decreases. So, is possible to locate a shock in the context of supersonic flow in constant area duct with friction or in the context of Fanno flow.

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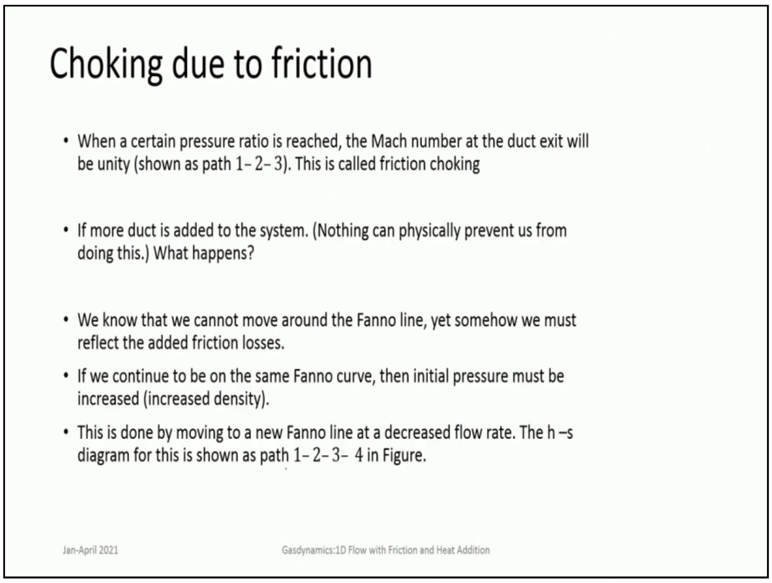

And what happens actually if there is a shock that actually will see in a few moments. So, associated with this is the very important point of choking due to friction. So, in this context what is meant by choking? So, here if you had taken a look at the Fanno curve depicted here you know that there is a maximum entropy point which is corresponding to Mach number equal to 1. If I take any particular, I draw once again T-s.

And this is the Fanno curve and I take any arbitrary starting point say this is point number one. So, this is flow inside a duct. And let me take that at the end of the duct. So, L corresponds to L* which is the maximum length or length of L* where at the end of the duct Mach number equal to 1. Mach number is 1 at the end of the duct. Now this is L* what it corresponds to is a maximum entropy point.

Now if you look at this curve and ask the question at this point if I increase the length by a small length delta L, what happens? Of course this cannot be done because there is no solution on the right hand side. And also the other fact is that an initially subsonic flow can only go up to a maximum of Mach number equal to 1. And you cannot go beyond that. So, there is a maximum length of the duct that one can give with a given friction for a particular in inlet condition or for a particular inlet condition which is L*.

If one gives a length which is greater than L*, you do not have a solution in the context of this Fanno curve. Something must change otherwise that flow cannot happen. So, this fact; that there is a maximum length for a given Fanno curve. Now Fanno curve is parameterized by G. G is mass flux which is $\frac{\dot{m}}{A}$. So, for given mass flow rate there is A. And given static conditions at the inlet there is one particular maximum length condition.

So you cannot change this particular fact. So, how can this be changed? Then it changes, if you have this particular condition that you are a L*. And you want to change the length of the duct you can do it by one case is by changing the initial conditions you can move. The initial condition back which is equivalent to saying that there is an increase in pressure at one. So, if I say this is one prime.

So, then now if I move this back then my total length now increases. G is remaining the same in this case that means mass flow rate is remaining same for the duct. But now I have to give higher pressure. So, that means increased pumping. So, if that is not possible if you cannot increase pressure. Then what you should what can be done is change in mass flow rate.

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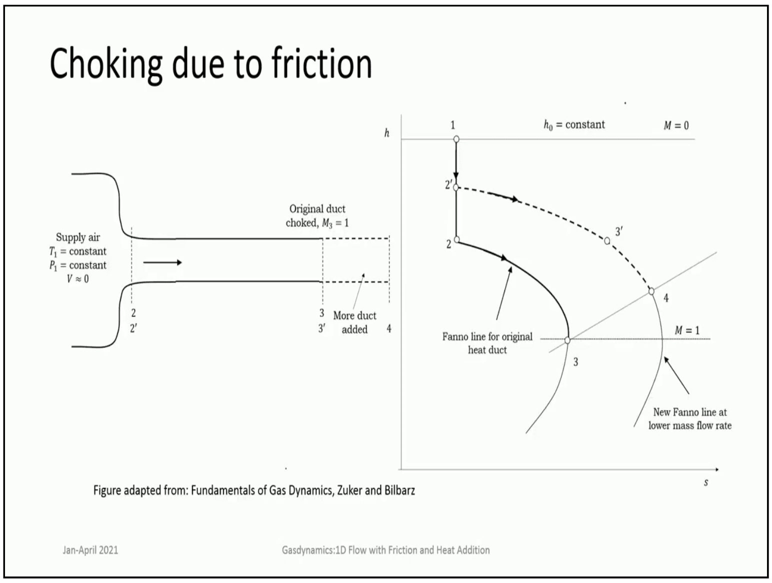

Because you can move to a different G curve which is a different Fanno line which has lower mass flow rate. So, this is schematically represented here. So, you have a stagnation given stagnation conditions. So; where we have a certain P_1 , T_1 which is remaining constant. So, initially this is the initial point started with 2 and goes to 3 here along the particular Fanno curve along this curve.

And if it goes to Mach number equal to 1 at 3. Then it is the maximum length. Now the question is if we want to increase this length or suddenly you want to add more ducts. Then what will happen? Then this is not supported by these conditions one was of course to move it to another condition located somewhere here. But that is also not possible because here stagnation conditions are getting fixed over here.

So, that the P_0, T_0 are fixed. So, that is also not going to change. Then the only change that can be done here is to vary the duct length. So, though vary the G that is mass flow rate. So, if mass flow rate is varied. Then G can change, G decreases. So, since G decreases now you can you can see that the L *corresponding to this G' which is the new Fanno line at G which is smaller prime G' is less than G if G is for this. And G' is for this.

Then $L^{*,'}$ prime is greater than L*. So, you can add more ducts. So, if you add more ducts with all other conditions remaining constant mass flow will decrease. So, the idea here is in subsonic flow initial subsonic flow if you have approached the choking point at the end of the duct that is the maximum possible length that can be sustained by that particular condition. If you want to change anything to that particular like add more duct length.

Then it affects the upstream either you have to change the pressure increase the pressure thereby move along the same Fanno curve. So, that you get additional length of the duct. Or the other cases move to a different Fanno line but you would see that mass flow rate decreases in the other Fanno line which is G' decreases. So, that is this is frictional choking.

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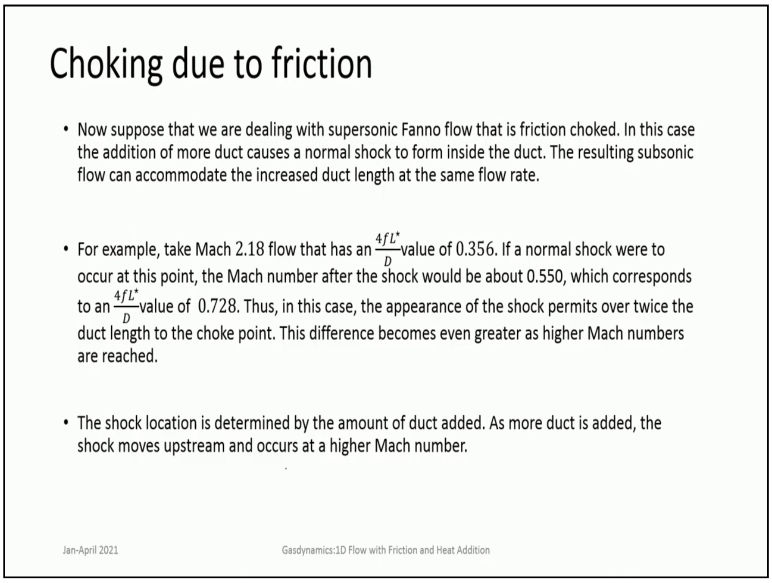

And now you see what happens if the initial flow inside is supersonic. And you get a choked flow at the end of the duct then how can you look at this particular problem. In supersonic flows you have another medium or another solution which can help in this manner or in this matter what is that? That is the shock, because you have this sort of condition. So, if you are at already at L* and then you try to increase the duct length. Then you can have shocks formed in the duct and transferring the supersonic flow to a subsonic flow.

And then again you have an additional L* associated with the subsonic flow. So, for example it is as is given here if you take a Mach , M = 2.18 flow, it has a value of $\frac{4fL^*}{D} = 0.356$. But if a normal shock were to occur at this particular point. And then the value of $\frac{4fL^*}{D}$ after the normal shock would be 0.728. So, there is an additional length that is now possible to be accounted for but you would of course get a normal shock inside the duct.

So, in supersonic flows if you have a choke condition that is Mach number equal to one at the exit length is maximum L*. And you try to increase the length a little bit. Then it can form shocks within the duct. And amount of duct added will lead to how where the shock is located. And it can go further upstream.

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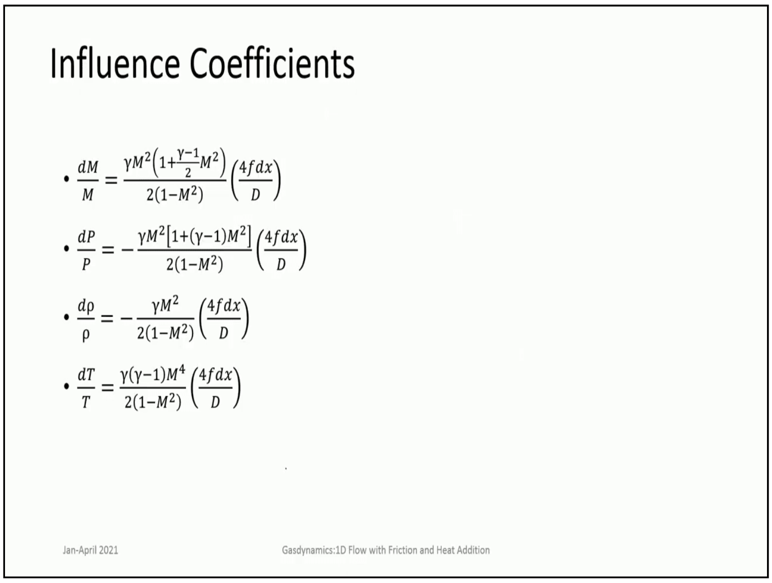

So, that is about the concept of frictional choking and now. We look at these influence coefficients. How these influence coefficients are derived is a matter which we discussed in the case of varying area ducts. You can use, you can write down the equations for variations of Mach number, pressure, density temperature. Express them as an equation as a matrix equation. And then use Kramer's rule to solve them with $\frac{4fdx}{D}$.

So, here this term $\frac{4fdx}{D}$ is the governing parameter or the driver. And that drives all these influence influences all these parameters.

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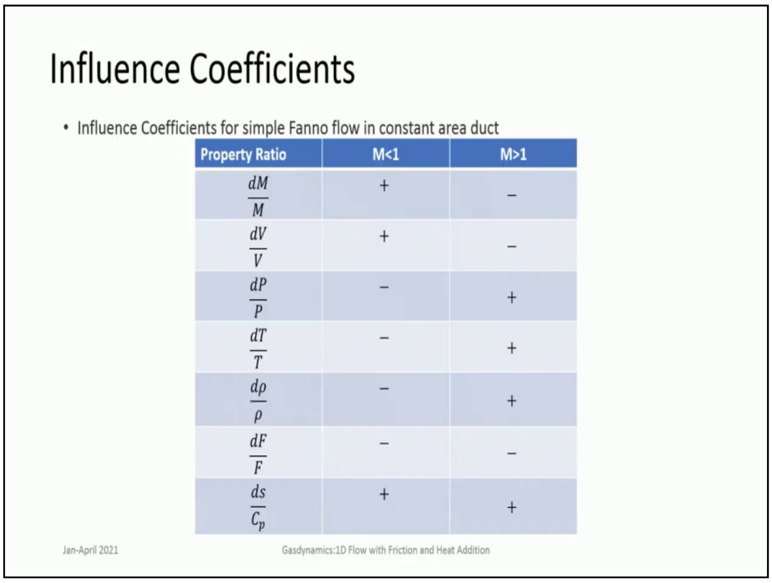

Property Ratio	M<1	M>1
$\frac{dM}{M}$	+	–
$\frac{dV}{V}$	+	–
$\frac{dP}{P}$	–	+
$\frac{dT}{T}$	–	+
$\frac{d\rho}{\rho}$	–	+
$\frac{dF}{F}$	–	–
$\frac{ds}{C_p}$	+	+

And how does it affect this something we discussed in detail. That when Mach number is less than 1, initially subsonic flow you get a increase in Mach number, increase in velocity, pressure, temperature, density decreases. And entropy will increase. F is the impulse function that decreases. Now if you consider the case of a supersonic flow, starting with a supersonic flow. Then Mach number and velocity decrease, pressure, temperature, density will increase and entropy always increases.

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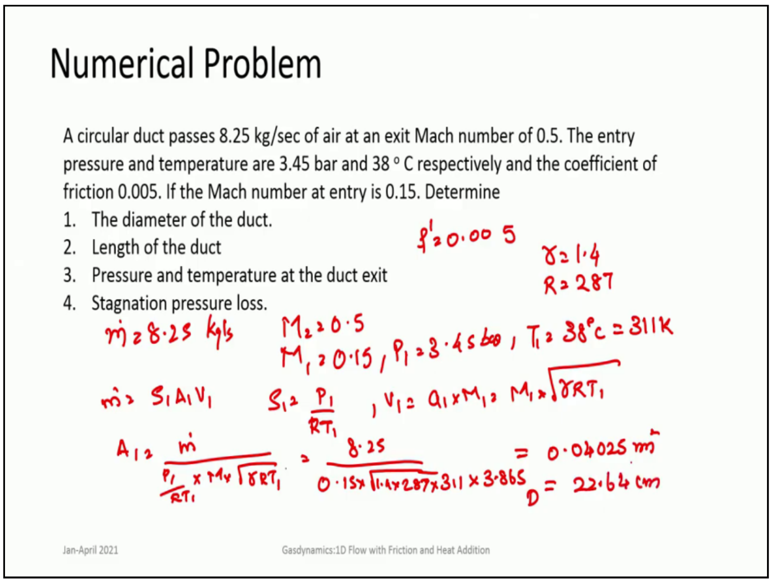

So, quickly let us look at a very simple problem before going on to more complex problems in the subsequent sections. So, a circular duct passes 8.25 kg/sec of air. So, $\dot{m}$ is given here $\dot{m} = 8.25\ kg/sec$,at an exit Mach number , $M_2 = 0.5$ .Entry pressure and temperature are three point four five bar and thirty eight degree centigrade respectively. And the coefficient of friction, $f' = 0.005$.

So, $f' = 0.005$, M_1 is given, Mach number at entry is 0.15. P_1 =3.45 bar.

And $T_1 = 38$ °C .

$\dot{m} = \rho_1 u_1$

So, temperature $T_1 = 311$.

So, now we need to find diameter. So, we need to find A.

$$\rho_1 = \frac{P_1}{RT_1}$$

And u_1 is given by , $u_1 = a_1 M_1 = M_1\sqrt{\gamma RT_1}$.

So, you can express this. So, you get $A_1 = \dfrac{\dot{m}}{\frac{P_1}{RT_1} M_1\sqrt{\gamma RT_1}}$. So, substitute here this is for air.

So, air $\gamma = 1.4$, R = 287 substitute these numbers here.

$A_1 = \dfrac{8.25}{3.865 x 0.15 \sqrt{1.4x287x311}} = 0.04025\ m^2$

So, diameter you can find out from here,

D = 22.64 cm

So, this is the diameter of duct.

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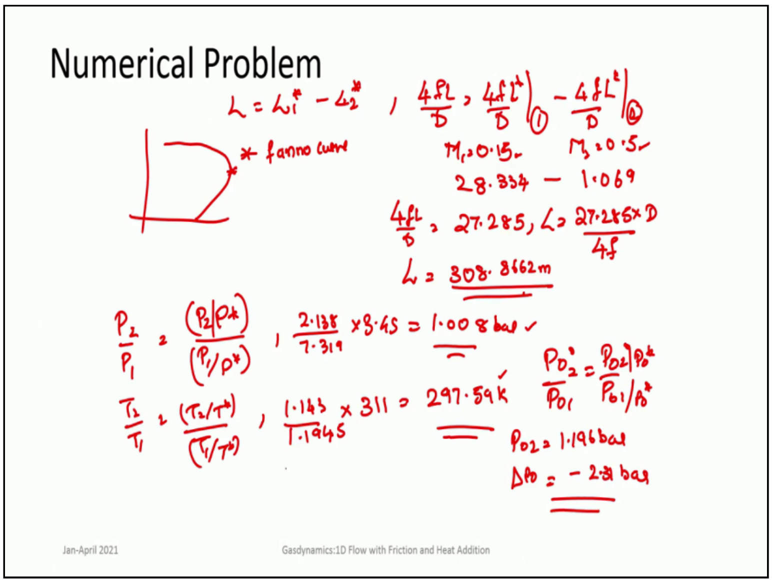

Now what is the length of the duct? Length of the duct, We have the particular equation which is L is equal to L1 * - L2 * or you can write this as,

$\frac{4fL}{D} = \left|\frac{4fL^*}{D}\right|_1 - \left|\frac{4fL^*}{D}\right|_2 =$. Now M_1 we know it is 0.15 and M_2 we know it is 0.5,

therefore we just have to find this $\frac{4fL^*}{D}$ you can look at tables or you can take a calculator you can look at the online calculator you can plug in this values γ and M.

$\frac{4fL}{D} = 28.334 - 1.069 = 27.285$

So, from here L can be calculated,

i.e. $L = \frac{27.285 \times 0.2264}{4 \times 0.005} = 308.8662$ m

Now what are the pressures and temperatures how do? We get to that. So, we need $\frac{P_2}{P_1}$ this is nothing but $\frac{P_2}{P_1} = \frac{\frac{P_2}{P^*}}{\frac{P_1}{P^*}}$. And these values can be directly found out.

$P_2 = \frac{2.138}{7.319} \times 3.45 = 1.008$ bar

Similarly $\frac{T_2}{T_1}$ can be expressed as,

$\frac{T_2}{T_1} = \frac{\frac{T_2}{T^*}}{\frac{P_1}{P^*}}$

$$T_2 = \frac{1.143}{1.1945} \times 311 = 297.59\ K$$

And please understand that these star values are for the Fanno curve that is this star value is corresponding to this is the star value in a Fanno curve. And don't confuse this with star values in an isentropic flow. So, they are different but this is completely tabulated. So, you can get these values. So, from here if you substitute the values you can get .

So, this should be keeping with the fan of flow that. We had learnt that initially subsonic flow Mach number is 0.15. Across the duct Mach number will increase, pressure decreases , temperature decreases. So, we are good it is in keeping with that particular equation. Now what is about stagnation pressure loss, that stagnation pressure loss is , $\frac{P_{02}}{P_{01}} = \frac{\frac{P_{02}}{P^*_0}}{\frac{P_{01}}{P^*_0}}$

And from this you can get the same way that you did for $\frac{P_2}{P_1}$. Now this is in Fanno flow. have You refer to Fanno flow conditions get $P_{02} = 1.196\ bar$. And therefore you can find out what is the ∇P_0 = -2.31 bar, P_0 decreases. So, decrease of pressure stagnation pressure which. So, this can be found. So, this is a very simple problem of a subsonic flow now. We look at other problems in the next class, thank you.