**Gasdynamics: Fundamentals and Applications**
**Prof. Srisha Rao M V**
**Aerospace Engineering**
**Indian Institute of Science – Bangalore**

**Lecture 41**
**1D Flow with Friction - Fanno Flow-I**

So, in this module we will move from essentially invisible descriptions. Until now we were looking at quasi 1D assumption. We looked at shock waves and expansion waves and also the variable area ducts. So, under quasi 1D assumption the variation of a compressible flow can be through various driving potentials or drivers. One driver of course is the change of duct area and a compressible flow happens through the varying area duct. And we spent quite a good amount of time discussing various aspects of this flow including nozzles diffusers and so, on. So, for a large part it is true that we do not have to consider viscous effects because they are quite small but that is not always correct. So, viscous effects have to be considered in real flows. So, under quasi 1D assumption under, the 1D flow the driver for that is friction, fluid friction, at the walls and we are considering 1D flow with friction.

So, here we do not consider any change in area of the duct. Examples of these kind of flows can be flow through pipes. Compressible flow through pipes. Gaseous flows through pipes. And this kind of the development of the flow or analysis of the flow was done by Fanno and it is also named as Fanno flow. So, 1D flows with friction or Fanno flow.

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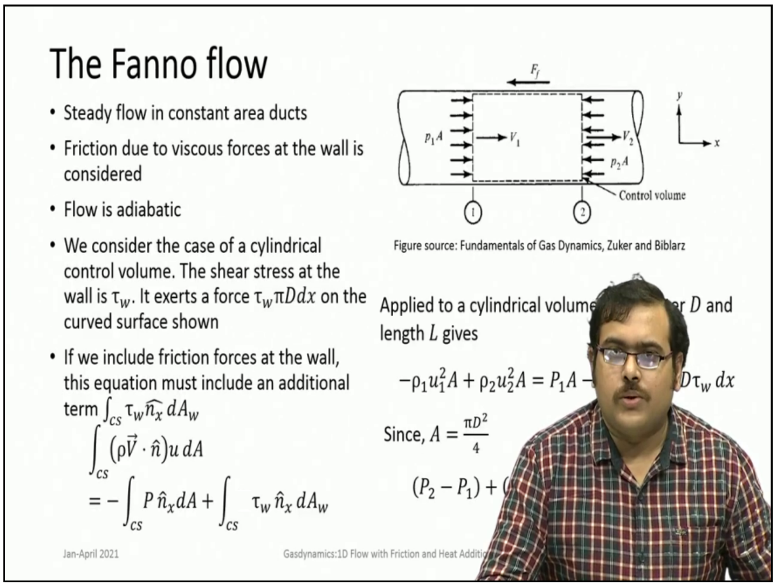

So, what we are looking at is we are looking at a constant area duct um. So, first we would look at the thermodynamics of this process and then we will move on to flow, how we would do the flow equations and so, on. And these are well described in very good books, text books of gas dynamics. So, several thermodynamic charts and such kind of figures will be taken directly from such text books. So, here we look at steady flow in constant area ducts.

So A is constant and we are considering flow friction due to viscous forces at the walls and there is no heat transfer. So, the flow is adiabatic. So, 1D flow, uniform flow properties that is quasiminus1D assumption and flow is adiabatic it is steady. So, friction always occurs in the opposite direction of the flow or it opposes the flow and this is occurring along the wall. So, if you consider pipe with circular cross section the force of friction would occur on the lateral surface area.

So, for a small element if we can consider this as an element dx, small element dx or it can be a typical length L then the lateral surface area is πD dx and if shear stress on the wall is $\tau_w$ then the force is $\tau_w$ πD dx. So, this force acts in the direction opposite to the flow. So, once again the analysis of this flow has to be considering all the three conservation equations.

Conservation of mass, momentum, and energy. Here conservation of mass is straight forward, $\dot{m} = constant$. So, it is constant area duct. So, ρV is constant is a constant because it is a constant area duct and $\dot{m} = \rho V = constant$. So, we look at now the momentum equation. So, in the momentum equation you have forces, you have the flux of velocity is equal to flux of momentum is equal to the forces.

That is here is the pressure forces and the frictional forces. So, here is friction and this is pressure. And we are doing 1D flow so we are considering only the x component of these velocities. Now this can be integrated for a cylindrical volume . So, here the incoming flux is $\rho_1 u_1^2 A$ = constant .Outgoing is $\rho_2 u_2^2 A$ and the pressure forces are a $P_1$ A on the left hand side and $P_2$ A on the right hand side.

$$-\rho_1 u_1^2 A + \rho_2 u_2^2 A = P_1 A - P_2 A - \int_0^L \pi D \tau_w \, dx$$

Well friction force which acts against the flow is $\pi D \tau_w$ dx and it is an integral 0 to L because in general $\tau_w$ can vary along the duct. So, this is the factor now, additional factor, that comes in the equation. Since we are taking a constant area duct, D is constant. So, $\pi D$ can be taken out A is also constant. So, dividing it by area you get, since, A = $\frac{\pi D^2}{4}$

$$(P_2 - P_1) + (\rho_2 u_2^2 - \rho_1 u_1^2) = \frac{-4}{D} \int_0^L \tau_w \, dx$$

So, the major change that happens is in the momentum equation. What about the energy conservation equation? There is no heat transfer occurring over here. So, if you take,

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

or it essentially says total enthalpy or total temperature remains a constant.

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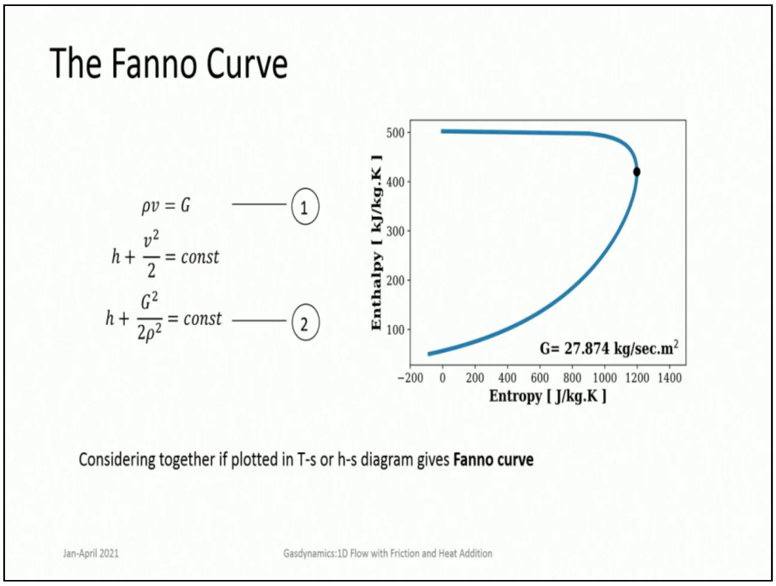


So, if you take the Fanno flow and first what we shall do is look at the thermodynamic description of Fanno flow as a process and see what we can learn from this description. In fact almost all aspects of Fanno flow will be covered just by means of one single curve which is the Fanno curve. How do we get to this curve? It is you consider 2 equations which is the mass flux or $\rho V$ =constant and it is given a parameter G.

$G = \rho V$

Now this parameter G will come again and again in the descriptions now and the other one is total enthalpy or $h + \frac{u^2}{2} = constant$. So, total enthalpy is a constant. Now this can be expressed. So, here given $\rho V$ = G

we can express, V = $\frac{G}{\rho}$ and this can be substituted in the energy equation and we get ,

$$h + \frac{G^2}{2\rho^2} = constant$$

So, now if you take this particular equation then it consists of only thermodynamic variables along with of course ideal gas equation of state and so on.

So, now if we can plot all possible states beginning from an initial point which satisfies these conditions then such a curve is a Fanno curve. So, Fanno curve is a curve for adiabatic flow through constant area ducts and it is expressed here in terms of T s that is temperature versus entropy T s diagram. Now it get, this is also enthalpy if you are considering a calorically perfect gas it is equivalent to enthalpy.

So, enthalpy entropy diagram. So, now if you look at this curve that is represented by these equations you see it has a particular shape. Now take note that first if you if I start from the upper side of this curve, it actually you can see that the curve slowly drops down in temperature until it reaches a point. So, here you can see that the entropy is increasing, temperature is decreasing until a point over here which is the maximum entropy point. So, it is maximum entropy $Smax$. After this you see that if you consider this particular curve, here I again starts from the left side if I go and I see here that the temperature increases entropy, increases up till the same point which is $Smax$. Now there is a specific reason why we went only in one direction the reason is that in fact if you plot. So, this is all static variables. In case of Fanno curve the entropy or the stagnation enthalpy remains constant. So, it let us take it is a straight line this is $h_0$ = constant.

So, if you look at this essentially this particular distance describes $\frac{u^2}{2C_p}$ or if you take enthalpy it is $\frac{u^2}{2}$ here. So, what you see is this part of the curve which I have described here as is the upper branch, that curve corresponds to small velocities or it is for a subsonic flow. While if I take the same corresponding point if I take it here, right here, then $\frac{u^2}{2C_p}$ is very large. So, the bottom branch is a supersonic branch.

So, this branch is M > 1, this branch is M < 1 and we will soon find out that this point at which the entropy is maximum is actually a critical point which is M = 1 and this is an adiabatic flow and so one would, one should realize that it is not possible. So, from here if you take any particular point one and flow happens within a duct where there is friction essentially you are going to go along this curve.

And as you go along this curve what you find really is that Mach number is increasing because you get a point which is M = 1 here. But from this curve it should be readily understandable that you cannot pass beyond this maximum entropy point. So, a subsonic flow can when it happens in a duct with friction then it is Mach number actually increases and it can increase up to Mach number one.

But if you look at a supersonic flow, similarly if I take another point here which is supersonic here beginning starting point to supersonic as the flow happens in the duct it is Mach number decreases but it can decrease maximum to a point ,1 and it is not possible to go beyond that the reason is that it is a maximum entropy point and if you try to go beyond this particular point entropy decreases which is not possible in an adiabatic duct, entropy has to always increase.

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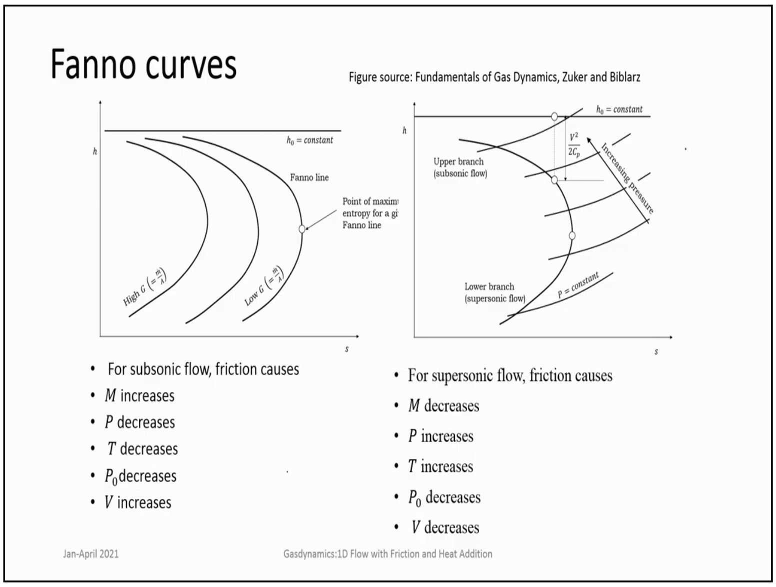


But this particular diagram tells you a lot more than this actually tells you the whole story. So, we will go into further details. So, these are qualitative curves and you should actually practice to draw these qualitative curves of Fanno curves and in the later modules you will learn another one which is Rayleigh curves and looking at them on looking at problems on described on Fanno curves or Rayleigh curves T s diagrams gives you a better feel for how to solve them?

What are the interrelationships between different variables? Because in these problems there is not a single variable that change as one parameter changes all other parameters undergo change. So, these diagrams are effective ways to understand the interrelationship between all these variables. So, if you draw an hs diagram then we know how pressure lines look like on a hs diagram.

Schematically it is drawn over here. You can see, these are basically pressure lines. So, as you go from left to right pressure actually decreases. So, also here on the right hand curve you have different plots where the only parameter that you saw in the Fanno curve was G which is the mass flux basically G. So, if you vary G how does the Fanno line shift? As essentially it shifts in this manner; that as you go from left to right, G decreases you get lower and lower G.

So, and at all these points you have this maximum entropy point which corresponds to Mach number equal to 1. So, if you look at this curve immediately what happens to pressure and density will become obvious because the pressure decreases as you go from left to right but be mindful they are not parallel lines. So, they are curved. So, actually if you look at the way the exact way in which the Fanno line occurs then in the subsonic branch actually you have a decrease in pressure as you go along the Fanno curve, P decreases in the subsonic branch.

But you can notice the supersonic branch is rather steep and if you draw the pressure lines along this particular curve supersonic branch then you will find that pressure will increase. And this corresponding the height over here this is essentially $\frac{u^2}{2}$. So, from that you are able to understand that in subsonic flow velocity actually increases in a Fanno flow as the flow goes through a long duct with friction.

While in supersonic flow if the initial velocity entering the duct is supersonic the velocity decreases. Now this is an adiabatic flow. So, for adiabatic flow $h_0 =$ constant or $T_0 =$ constant. Also, the other fact that becomes constant is the value $T^*$ that is the temperature at which it reaches Mach number equal to 1. So, even $T^*$ actually is a constant. So, this particular point corresponds to $T^*$ but please do not confuse it with isentropic flows. So, this is a flow in which there is friction.

So, friction always introduces entropy that is why you see that entropy always increases and it reaches a maximum. If a flow occurs through a duct with friction and Fanno flow is considered then the critical point $T^*$ is located here at the maximum entropy point and $T^*$ would correspond to this $T^*$ that is obtained in an adiabatic flow because it is an adiabatic flow. So in that sense only the $T_0$, $T^*$ relationships will be the same but pressures are completely different.

So, $P^*$ is not the same as $T^*$ in an isentropic flow. So, please bear that in mind and do not confuse between these 2 Fanno flow is an entirely different flow where there is change in entropy and entropy increases. So, that is an important point. So, now we can completely distinguish between the upper branch which is a subsonic branch and the lower branch which is the supersonic branch. In the subsonic branch, friction causes Mach number to increase and pressure temperature decreases . So, density also decreases and velocity increases and there is an increase in entropy.

So, whenever there is increase in entropy and it is an adiabatic flow, the stagnation pressure will decrease. This is directly from our first fundamental principles where we looked at the thermodynamics of process and how stagnation pressure is related to entropy. So, in an adiabatic flow if there is increase in entropy stagnation pressure will decrease. Stagnation pressure decreases across a shock it also decreases in the case of Fanno curves Fanno flows.

Now what happens in supersonic flow friction it causes Mach number to reduce velocity also decreases Mach number decreases and pressure, temperature will increase consequently you have increase in density as well and $P_0$ that is $P_0$ will decrease because even in this case the entropy continues to increase along the supersonic branch. So, understanding if you have understood Fanno curves ,these curves very well then you will be in a position to understand Fanno flow very well because just by means of this particular curve one can completely describe the Fanno flow.

So, whenever flow is happening, so, as point moves from so, initial point one here to another point 2 here represents a flow through a duct of length corresponding length is L. So, you can say this is the duct of length L. Now how we will get to these lengths and what are the equations that will help us solve them for flow through pipes and so on. We will soon come to it.

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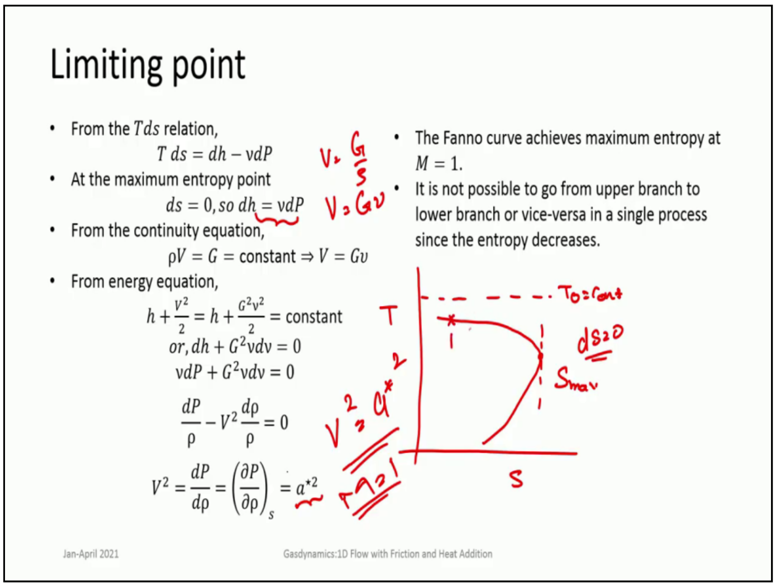


Here we are discussing thermodynamic aspects of Fanno flows and just by means of a thermodynamic curve hs or T s diagram. We can know all that is to know about this particular flow. Now we have to look at that particular point. So, we are looking at this particular point where there is a maximum of entropy $S_{max}$ and $T_0$ is a constant. So, constant what is the condition at which $S = S_{max}$, entropy is maximum what is this particular condition?

So, for this we look at the relation, Tds = dh –vdP

here v specific volume. So, at maximum entropy point if you look at the neighbourhood of a maximum entropy point essentially it is a maximum. So, ds should be equal to 0, ds =0. This is a condition for maxima. So if you put ds = 0 you get dh = v dp and from continuity equation or this is the definition of the mass flux, $\rho V$ = G and G is a constant.

So, you can express v that is the velocity is V = $\frac{G}{\rho}$ , v is specific volume. So, it is Gv specific volume. Then the other term which remains constant is the total enthalpy,

$h + \frac{u^2}{2} = h_0$ $constant$. This $u^2$ is now written solely in terms of G and specific volume G square $u^2$ = constant. And differentiating this equation we get,

dh + $G^2$ V dV = 0

So, now, dh =v dP , that is from the maximum entropy point at the maximum entropy point dh = v dp.

So now from here we can easily proceed $G^2$ ,this can $G^2$ can be written as $G^2 = \frac{u^2}{\rho^2}$ . So, it can be written as $\rho^2$ , $u^2$ , $G^2$ . but you also have a v in a specific volume factor over here and considering all of them together what you arrive at,

$$\frac{dP}{\rho} - V^2 \frac{d\rho}{\rho} = 0$$

Or $V^2 = \frac{dP}{d\rho} = \left(\frac{dP}{d\rho}\right)_s = a^{*2}$

That is happening at this particular point where ds = 0 or entropy is constant.

So, $\left(\frac{dP}{d\rho}\right)_s = a^{*2}$ is constant is nothing but the speed of sound. So, $V^2 = a^{*2}$ or M=. So, this shows that at the critical point Mach number is equal to 1 or at maximum entropy Mach number is 1. And precisely from the nature of this curve it should be understood that one cannot move from the subsonic branch. So, you if you start initially from this point you are flowing the gas is flowing through duct and through a length L.

So, L can be varied it can be having different lengths. So, as you go through different lengths you are getting to different points. So, you have say poin 2 you can vary this. So, until you reach this maximum entropy point which is equal to M = 1. Beyond this it cannot change because entropy will decrease. Similarly if you start from a supersonic branch you can go until maximum entropy point but you cannot go from supersonic to subsonic.

So subsonic to supersonic or supersonic to subsonic flow via a Fanno curve only via Fanno curve is not possible. It is limited by entropy.

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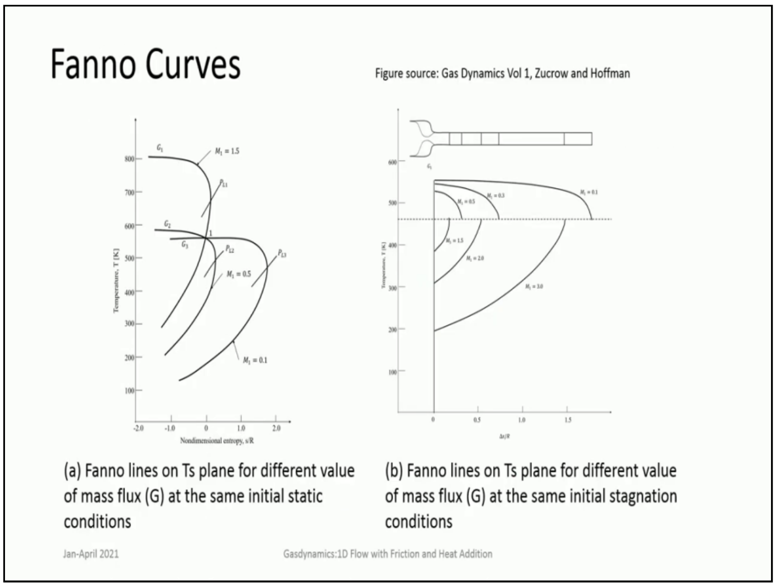


So, here there are different kinds of Fanno curves that are plotted. This one it is taken from this particular textbook and these are curves beginning from the same point 1 which is having the same static pressure and temperature same static conditions but having different Mach numbers. So, different initial Mach numbers, this for example this particular curve belongs in the subsonic branch in G2 as well as in G3.

For G3 it is 0.1, G 2 it is 0.5 but another curve G1 ,it is having a supersonic velocity. So, it belongs in the supersonic branch. While the curves on the right show different Fanno curves which start with the same stagnation conditions. So, initial conditions, the stagnation conditions are the same but they have different Mach numbers. So, consequently you have different Fanno curves over here.

So, this is how it looks like and a general understanding qualitative understanding of how to quickly draw a Fanno curve is very useful in understanding this particular topic and in solving problems also.

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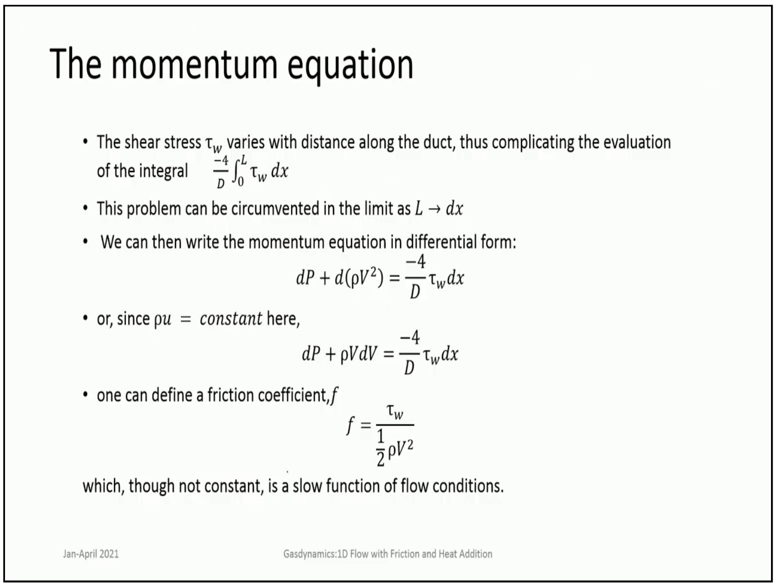


So, now we come to the point that we want to now look at getting numbers, quantitatively understanding Fanno flow. When trying to do this we did an initial integration but we came up with the factor this particular factor $\frac{-4}{D}\int_0^L \tau_w\, dx$ because the shear stress in general varies along x direction. So proceeding this way would be difficult. So, rather under 1D assumption itself with uniform flows, let us look at small lengths, dx.

So, from integration integral kind of an approach we move towards looking at small control volumes with the length dx we have to just look at the differential form. So, on the left hand side you have,

$$dP + d(V^2\rho) = \frac{-4}{D}\tau_w\, dx$$

Now $V^2\rho$ can be differentiated also considering the fact that $V\rho$ is a constant.
So,

$$dp + \rho V\, dV = \frac{-4}{D}\tau_w\, dx$$

Now we need some way to express the shear stress on the wall. For this we take the definition of coefficient of friction. This something we can easily borrow, $C_f$ or f is equal to it, is also written as Cf or f ,coefficient of friction,

$$f = \frac{\tau_w}{\frac{1}{2}\rho V^2}$$

Usually f is also not a constant but it is a slow function of flow condition. So, it does not vary very rapidly. It is somewhat slow in it’s variation.

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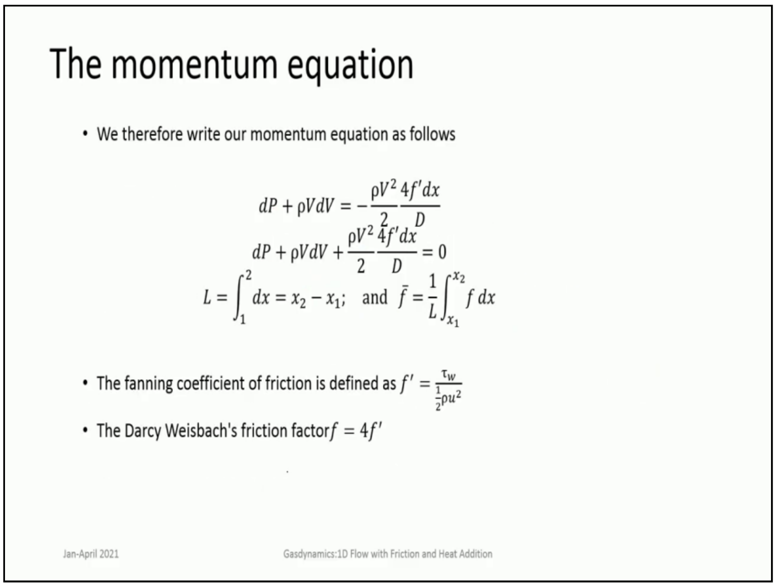


So, if you put that, we write then this in terms of $f$

$$dp + \rho V\ dV = \frac{-\rho V^2}{2}\frac{4f'}{D}dx = 0$$

The term $f'$ is written because soon we will discuss there are 2 descriptions of this coefficient of friction and friction factor they are not the same. Coefficient of friction we represent by $f'$ while friction factor is represented as f. So, we will soon come to it. So,

$$dp + \rho V\ dV = \frac{-1}{2}\ \rho V^2 \frac{4f'}{D}dx$$

So, now this can be taken to the left hand side and taken together. So, you get this particular equation. Now since coefficient of friction varies very slowly you can consider an average coefficient of friction defined in this way,

$$\bar{f} = \frac{1}{L}\int_{x_1}^{x_2} f\,dx$$

So, and consider that the average friction remains constant. So, this is what has been; what I was just now mentioning is that sometimes many literatures describe friction in terms of fanning coefficient of friction which is the definition of coefficient of friction itself, $f'$.

But in many places the friction factor particularly in pipe flows and such kind of descriptions friction factor is given which is f= $4f'$ ,4 times Cf. So, look at the wordings carefully when you look at literature relevant to Fanno flows whether they are talking in terms of coefficient of friction or friction factor. So, correspondingly this factor multiplication should be considered.

So, but in general what we understand from here is that we are looking at average coefficient of friction which remains constant.

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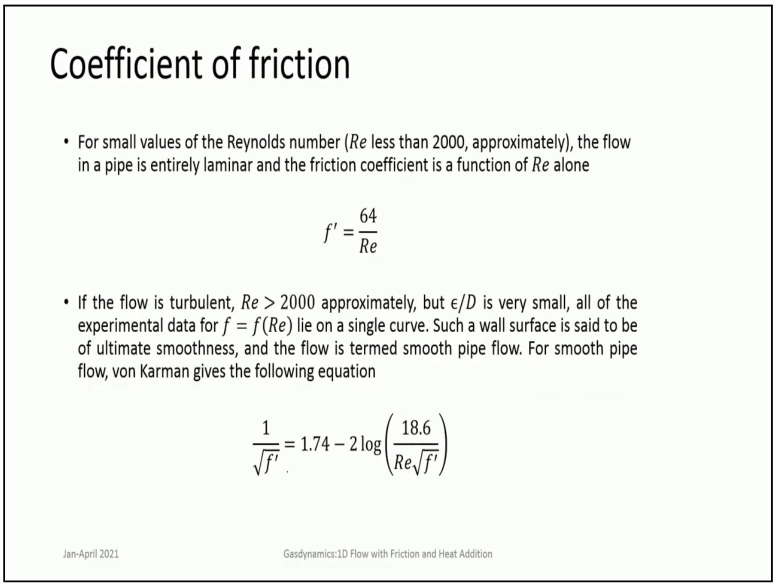


So, now what about this, so, here we said we will represent it in terms of coefficient of friction but we need to know coefficient of friction. For this we can look at several correlations several relations that are available that is for laminar flows and turbulent flows they are expressed in such relations. For laminar flow of course this $f' = \frac{64}{R_e}$ and for turbulent flows also there are various descriptions for a smooth wall this is a particular equation.

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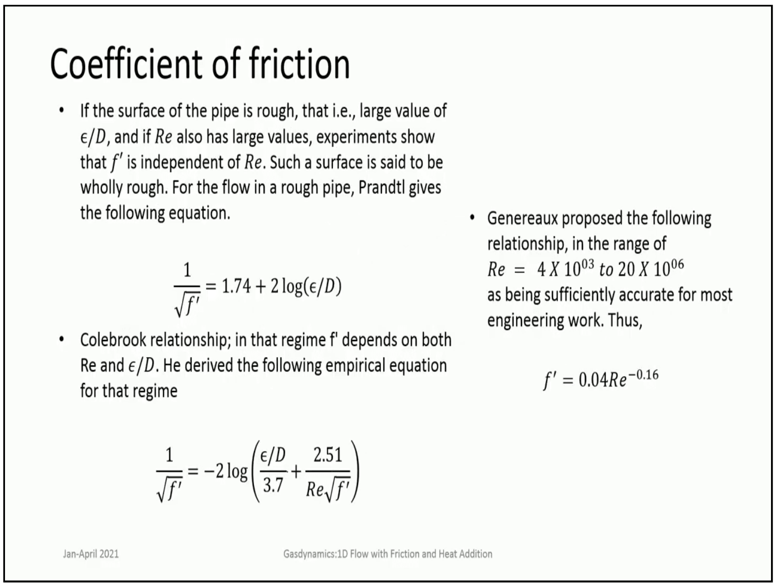


While for walls which have certain roughness's we have different equations. A suitable equation can be chosen.

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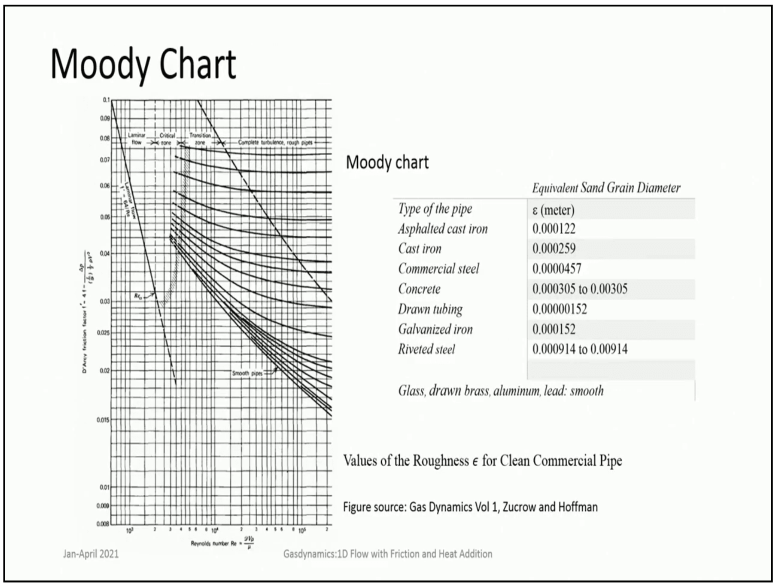


Also, a common chart which sort of depicts all the variations of friction factor or friction coefficient is the Moody chart where you have the laminar flow and the turbulent flows for different surface roughness's because turbulent flows are affected by surface roughness. So, you have all these are available either; if one is doing some of the problems and is using an approach where it can be evaluated analytically.

Then we can use such analytical expressions or rather relations to get the coefficient of friction or if one is doing it using calculators and charts then Moody's chart is available. But when we are looking at these problems of friction flow through pipes then we will generally know f, f would be given and then we would solve the equation. So, that is how most of the textbook problems are given.

For applications you can refer to these charts. So, now from here we understood in a qualitative sense what is happening in Fanno flows. And there are differences in how the flow behaves in subsonic flow and supersonic flow with the point Mach number equal to 1 being the critical point , where entropy is maximum. Now in the next class what we will look at is look at getting equations in terms of Mach number. So, that term such as pressures, temperatures change of Mach number can be evaluated in a flow through the constant area duct with friction.