Gasdynamics: Fundamentals and Applications
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Lecture 40
Varying area flow – Numericals- IV

So, in this particular class we will look at problems on variable area ducts that involve several concepts together. They include not only variable area ducts, include shock waves maybe oblique shocks but also an application level problems, two of them, where first one is in a two dimensional supersonic nozzle is designed to give a uniform flow at Mach 3 with air as flowing fluid.

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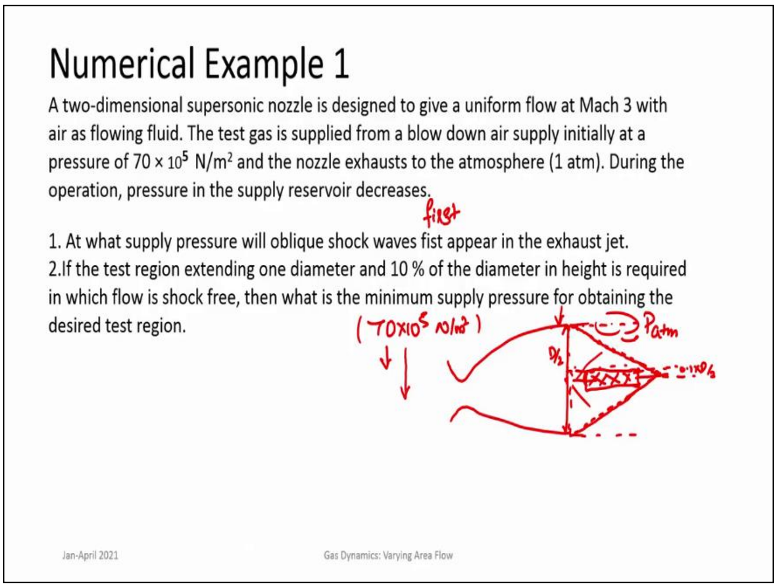

Test gas is supplied from a blow down air supply initially at a pressure of $70 * 10^5 \frac{N}{m^2}$ the nozzle exhaust to atmosphere. During the operation, pressure in supply reservoir decreases. So, you see that here supply pressure decreases. At what supply pressure will oblique shock waves first appear in the exhausted? If the test region extending one diameter and 10 % of diameter in height is required in which flow is shock free, then, what is the minimum supply pressure for obtaining the desired test region?

So, this is an applied problem which is normally used in internal applications. The good thing with supersonic flows is that because there is a kind of information propagation in supersonic flows is limited. So, the flow if it is uniform at the exit, the flow is uniform at the exit. Then, if any changes happen downstream they will not be affecting the flow until certain waves actually pass through the flow.

So, for example, for until a wave for if there are no shock waves then the corresponding wave is a Mach wave which is a weak shock. So, the Mach wave for this particular for this particular Mach number may proceed for in along these directions. Let us take them to be symmetrical. So, in this particular triangle, you can see that they form a kind of triangle the ma the Mach number and flow properties will remain uniform.

So, and the flow will not get affected by any changes. Any changes that happen over here in this region will not affect this region. This is a nice thing about supersonic flows and testing in supersonic flow. So, you can have a model inside here or a certain dimensions which are of this kind, of this nature and it will be in uniform flow. So, this is what this problem is looking at but what it says is now this wave is an oblique shock is given clearly that it is an oblique shock.

And it says that the oblique shocks should, the limitation of the object shock should be such that this is always within the uniform flow, where the diameter that is given is about $10\ \%$ percent the diameter. So, this is the diameter of the exit. So, this is if I say this is $D/2$, so, here you have $10\ \%$ of $D/\ 2.\ 0.1D/2$ is the height which is symmetric over here. So, the oblique shock should at least touch this at this particular point.

So, this is what is given so it is a kind of application in internal operation and a model in a tunnel. So, how do we find this, these kinds of condition? Before that, so, the way this internal is operated is that initially $70 * 10^5 \frac{N}{m^2}$ of pressure is given and it exhausts through the nozzle and on to the model. But then, the supply pressure is continuously decreasing, so, what we are interested in these kinds of testing, is Mach number.

So, as long as the Mach number remains constant, we expect certain flow features like shock waves around the body to remain the same. But obviously if stagnation pressure decreases exit pressure will also decrease. So, this has to be borne in mind. But pressure ratios will not change because pressure ratios will depend only on Mach number. If the nozzle is operating in design condition and Mach number is maintained, then, ratios of pressure will be maintained. Absolute values are changing because upstream pressure is changing.

So, that is something that needs to be understood. So, first question is what supply pressure will oblique shock waves first appear in the exhaust jet, appear in the exhaust jet. So, that is at the point ah, if the pressure drops below just below the correct operating pressure of the nozzle. Then, oblique shocks will appear a slightly over expanded operation. So, the correct operating pressure will decide the required stagnation pressure for this point.

And what is known is that this nozzle is just exhausting to atmosphere so, at the exhaust jet so, it is just going out to atmosphere so here it is P atmosphere. And we are finding out this particular region in which the flow is going to be uniform, more or less uniform.

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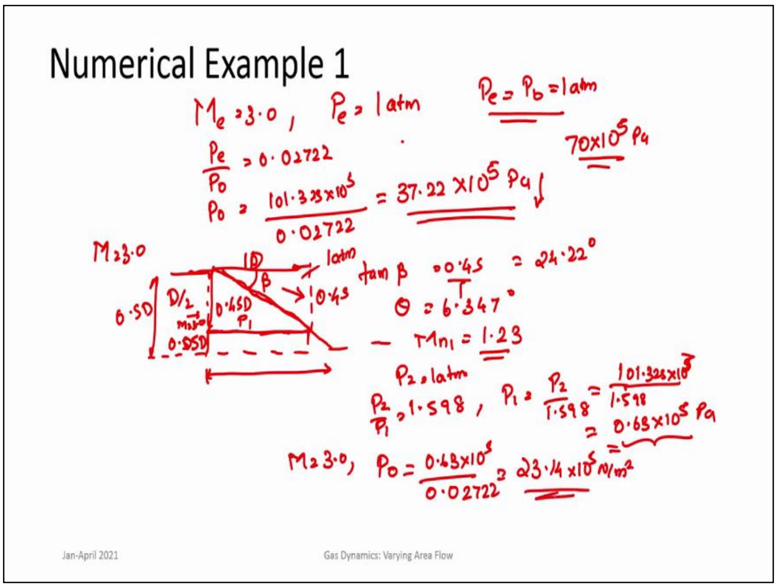

So, we know that what is the Mach number is 3. So, Mach number of nozzle is three. So, $M_e = 3$ at point 0 for this and pressure at the back pressure is pressure at exit is 1 atm . So, exactly at the condition when the pressure at exit becomes equal to back pressure, equal to 1 atmosphere, that is the correct operating optimum operating point of the nozzle.

If the pressure stagnation pressure reduces, beyond this particular point, then oblique shocks will appear. So, the correct operating pressure is stagnation pressure is given by $\frac{P_e}{P_0}$. For ah, So $\frac{P_e}{P_0}$ for Mach 3 is 0.02722. So, P_0 is $\frac{101.325*10^3}{0.02722}$ which is $37.22 * 10^5\ Pa$. So, if the pressure drops below $37.22 * 10^5\ Pa$, then, oblique shocks will occur.

But already we know that this internal start operating with $770 * 10^5\ Pa$. So, this is nearly half of that. So, there is a long time to go before we get to this point $37.22 * 10^5\ Pa$. So, if it falls below this value then oblique shocks will occur. Next question here is that this is the exit of the nozzle. So, let us take it symmetric and this is $\frac{D}{2}$. So, this is $\frac{D}{2}$ and 10 % of $0.5\ D$ is $0.05D$.

So, you are having a model here the dimensions of the model are that length is 1 diameter. So, 1 D and the height that this model is 0.05 D and this is point 5, (D, 0.5D). So, this height here is 0.45D. Now, if an oblique shock is formed. Then, if it just touches graces the model, the end of the model, the edge of the model that is the limiting condition.

Then, the entire model will still be in uniform flow of $M = 3.0$. Only after the oblique shock passes, the flow turns towards and there is it becomes a different flow. So, if you are testing for Mach 3, then, there is this much amount of space available to do the testing. So, question is, what is the condition at which this particular condition is achieved. So, now we know this is an oblique shock and the geometry of the oblique shock is given.

So, this is 0.45 and this angle is beta for the oblique shock. So, $\tan\beta = \frac{0.45}{1} = 0.45$. So, beta is $24.22°$. Now, corresponding θ can be found out because Mach number is upstream Mach number is 3, $M = 3$, so θ is $6.347°$ ok. So, now we know $M - \beta - \theta$ so we can find M_{n1} which is 1.23. So, once you know M_{n1} the pressure. Here is $1\ atm$ now after the oblique shock.

So, we need to find pressure in this region. This is P_1. So, P_1 we have to find, P_2 is P_2 is $1\ atm$. So, $\frac{P_2}{P_1}$ is 1.598, therefore $\frac{P_1}{P_2}$ by $\frac{1}{1.598}$ which is $P_1 = \frac{1.01325*10^5}{1.598}$. So, which is equal to $0.63 * 10^5\ Pa$. So that is the pressure in this uniform region. Now, Mach number is three, so Mach number is still three $Mach\ number = 3.0$ so we can find P_0 corresponding to this particular pressure.

This because $\frac{P_e}{P_0}$ is known. So, this will be $\frac{0.63*10^5}{0.02722}$ which is $23.14 * 10^5\ Pa$. So, this had two concepts here involved one is that unlike previous cases where we had stagnation pressure as constant and changes were done to back pressure. Here back pressure is maintained constant at one atmosphere and changes is done to stagnation pressure.

And we were interested in when does the nozzle just go to over expanded regime. But in the over expanded regime, we are still interested in a particular point up to which the oblique shock will just graze the surface of a model, which is placed in front of the nozzle, as it just expands into the atmosphere. So, all the regions from the exit of nozzle till that particular point is a uniform flow region which can be used for testing.

And the corresponding pressure at which this particular condition is achieved is what we were looking at. So, this is involved multiple such concepts and geometry derived from the geometry of model and the condition that oblique shock will hit at a particular location.

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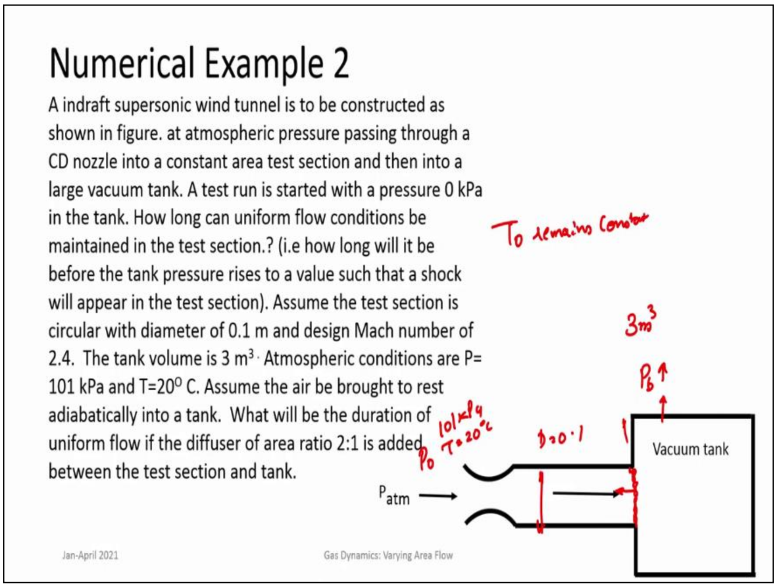

So, next we look at another example. So, here there is an indraft supersonic wind tunnel it needs to be constructed as shown in figure. So, indraft means you are taking the air from ambient atmosphere and then pulling it into a vacuum tank passing through a convergent diversion nozzle into vacuum tank. The test run is started with a pressure of 0 kPa in the tank that means vacuum levels are very high.

So, pressure is very, very small compared to the atmosphere hence it is taken as 0 kPa. How long can uniform flow conditions be maintained in test section? So, the implication here is so the moment so already a very large pressure differential is produced across the between the test section and through a nozzle. So, the moment let us say there was a valve here the moment the valve is opened immediately, the nozzle chokes, because the pressure ratio is very high um.

And it will choke and give the desired conditions but what happens the pressure in the vacuum tank will now start increasing. Earlier it was very, very low vacuum and it starts increasing. So, back pressure is now increasing but P_0 is held constant here. So, initially there is a supersonic flow and then pressure in the vacuum tank will keep increasing. At a certain pressure shock will start appearing, at this point, at the point, at the end of the test section and then start moving.

So once this shock appears here, essentially, beyond that point flow in the test section is going to get disturbed, as this shock passes over. So, the test time or for the duration of test for which the uniform flow can be maintained here is the time from, for the pressures in the vacuum tank to rise from very low values to the particular back pressure at which this condition is me.

So assume test section is circular with diameter(D) of 0.1 m and design $Mach\ number\ is$ 2.4. So, this is 2.4 and this section diameter is given D is 0.1m and vacuum tank has volume 3 m^3, so volume of vacuum tank is 3 m^3. And atmospheric conditions is given 101 kPa and T is 20 °C. Assume that air be bought to rest adiabatically into the tank.

So, here air is brought to rest adiabatically that means there is no change in stagnation temperature T_0 remains constant. What will be the duration of uniform flow?

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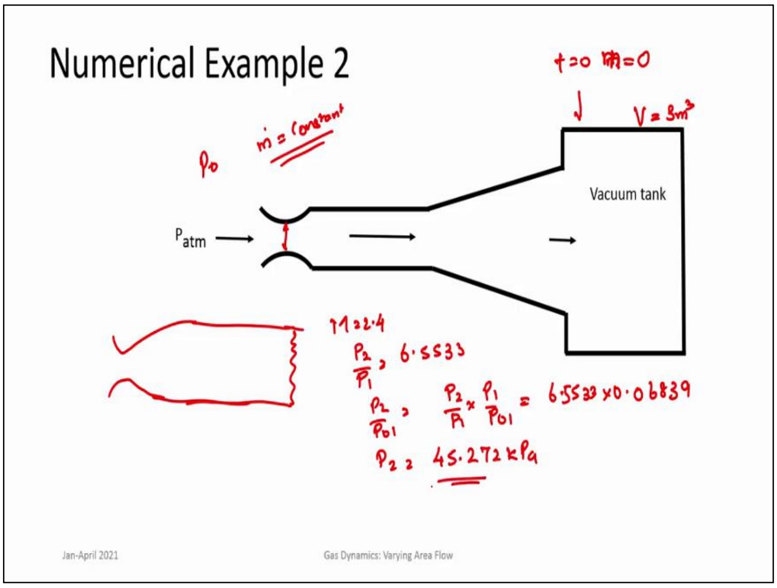

If that is so, there is an additional question that if instead of a straight duct that is shown over here, a diffuser divergent diffuser of area ratio $\frac{2}{1}$ is added between test section and tank what is the change that it makes to the test time? So, here the question is that you already have. So, let us take the case one, where you have the duct and you have the nozzle and long duct which is the test section.

At the exit of the duct section there should be a normal shock so, initially at starting very, very low pressures. So, mass inside the tank can be taken to be 0. So, $\dot{m}$ inside the, at start $t = 0, \dot{m} = 0$ or mass inside the tank is 0. Then, as air is pushed into it m will increase. The moment it is started, there is a large pressure difference P_0 is constant. So and flow is choked so that means $\dot{m}$ that is mass flow rate through the test section is a constant.

So, these are some things that have to register once you look at this problem. $\dot{m}$ is a constant it will continue to be a constant as long as the normal shock does not come and completely disturb the flow. So, that is the idea here so, how much mass needs to be added to the tank so that the back pressure rises to a certain level? The volume of the tank is constant υ is 3 m^3. So, this is something that is useful. We can look at this thing.

So, what is that particular condition at which the back pressure the there is a shock at the test section, this is what we need to know. So, M is 2.4, from normal shock relations we know what is $\frac{P_2}{P_1} = 6.5533$. What we require is $\frac{P_2}{P_{01}}$, $\frac{P_2}{P_{01}} = \left(\frac{P_2}{P_1}\right)\left(\frac{P_1}{P_{01}}\right)$. For Mach 2.4, $\frac{P_1}{P_{01}}$ is known 0.06839 so this is $(6.5533)(0.06839)$. So, from here we get P_2 is $45.272\ kPa$.

So that means the vacuum tank pressure should increase back pressure that is getting applied on this section should increase to $45.272\ kPa$. Then, this tunnel will start having non uniform flows.

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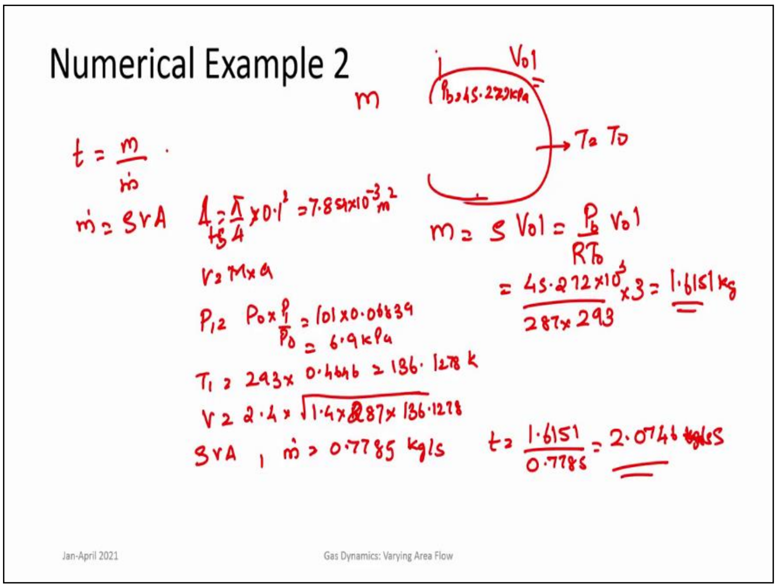

So, how do we look at that problem? We find out what is the mass of mass that gets accumulated inside the vacuum tank because volume is known and the temperature we know that it goes to rest adiabatically. So, temperature here will be T_0 volume is known and the pressure is now known. Back pressure is known $P_b = 45.722kPa$. So, the total mass that would give this particular value is $\rho * \upsilon$, density$\rho = \frac{P_b}{RT_0}$ is $m = \frac{P_b}{RT_0}\upsilon$.

This is T_0 is $293K$, $m = \left(\frac{45.27*10^3}{287*293}\right)3$. So, this mass turns out to be 1.6151 kg. Now, how to get time? How do we get time when this is achieved if we can divide this mass by the mass flow rate that is flowing through the wind tunnel will get time? So, what is mass flow rate to the internal? This is ρV A.

So, that is because from the start, the nozzle is choked. So, $\dot{m} = \rho VA$ is constant. so here area of the wind tunnel is given diameter is $\frac{\pi}{4}0.1^2$ is the area of test section area of test section. That is $7.854 * 10^{-3}m^2$ and velocity is $M * a$. This is speed of sound and here we need to find pressure and temperature.

This is 2.4, so, you have $P_1 = P_0\left(\frac{P_1}{P_0}\right)$, P_0 which is atmospheric pressure. So, $101 * 0$ 0.06839. This is 6.9 kPa and $T_1 = 293 * 0.4646$. That is 136.1278 K. So, density can be found and then velocity is $2.4(\sqrt{1.4 * 287 * 136.1278})$ so you can get this velocity. So, multiplying all

$\rho V\ A$, you get mass flow rate as $0.7785\ kg/s$. So, time is total mass that needs to be achieved $\frac{1.6151}{0.7785}$ which is $2.0746\ s$.

So, there are certain simplifying assumptions assumed here. But this is a very good estimate $2.0\ s$ very good estimate about $2\ s$ of test time. Now, why this comparison is done is what is the use of having a diffuser?

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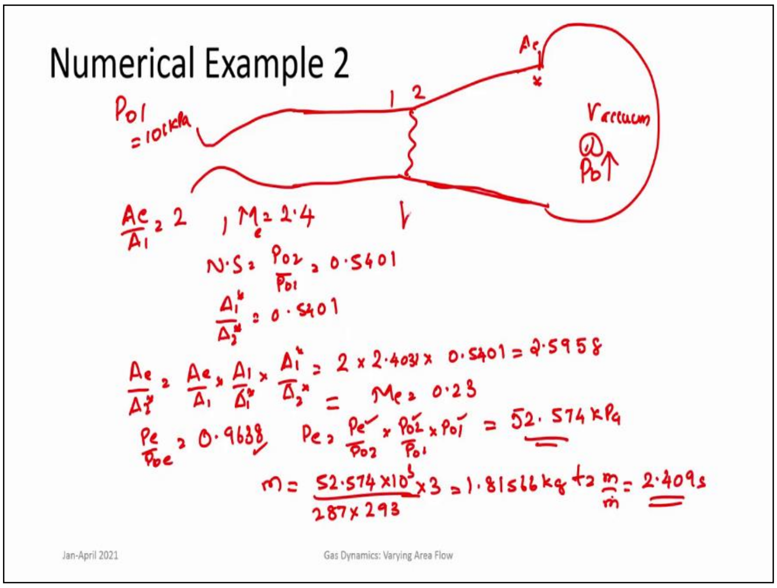

If you have a diffuser of this kind there just a diverging diffuser is placed there, how will it affect this test time? So, there is a diffuser at this portion. You have normal shock now at the exit of the test section just the same case as before. But now there is a diffuser attached ok. The diffuser allows further pressure recovery. That means the pressure at which this particular condition is now achieved for having a normal shock at the test section is slightly different from the earlier case.

So, if we do the calculation for that, the area ratio of the diffuser is given. $\frac{A_e}{A_1} = 2$ and $M = 1.4$. So, now you can use the normal shock relations in from normal shock you know P_{02}. So, this is 2 & this is 1, $\frac{P02}{P_{01}} = 0.5401$. So, $\frac{A_1^*}{A_2^*} = 0.5401,\ A_2^* = A_e^*$. Now as a consequence, now the exit now is here A_e.

So we need $\frac{A_e}{A_2^*} = \left(\frac{A_e}{A_1}\right)\left(\frac{A_1}{A_1^*}\right)\left(\frac{A_1^*}{A_2^*}\right)$. This is $(2)(2.4031)(0.5401) = 2.5958$. So, we get what is the Mach number at this exit. We know this exit Mach number. $M_e = 0.23$. So $\frac{P_e}{P_{0e}}$ at this particular point $\frac{P_e}{P_{0e}} = 0.9638$ is known.

So, now what is P_e we should find out. This P_{01} is known it is $101\ kPa$. We need $\frac{P_e}{P_{01}}$. So, will $P_e = \left(\frac{P_e}{P_{0e}}\right)\left(\frac{P_{0e}=P_{02}}{P_{01}}\right)P_{01}$. All these values are known $\frac{P_{02}}{P_{01}}, \frac{P_e}{P_{02}}$ is known here and P_{01} is known. If this is done it is $52.574\ kPa$. So, the pressure at which this is achieved is much higher than the previous pressure which was for $45.272\ kPa$.

As a consequence the pressure in the tank that needs here that back pressure is higher and you can find the mass that in the tank at that particular time m_{tank} is now $\left(\frac{52.574*10^3}{287*293}\right)3$. This is $1.81566\ kg$. So, now time is $\frac{m_{tank}}{\dot{m}}$, $\dot{m}$ does not change because P_{01} is constant.

And here you get time is $2.409\ s$ so because of the addition of a diffuser which can recover pressure it is possible to increase the test time from the earlier $2\ s$ to now close to $2.4\ s$. So, diffusers help recover pressure and they help reduce energy for operating such facilities. So, with this, we have covered extensively on varying area ducts, nozzles, diffusers their applications in various engineering devices, inlets, intakes and supersonic wind tunnels.

So, this essentially, covers the majority of variable area varying area duct problems. In actuating, actual engineering devices, there is additional efficiency that is considered for these devices. Nozzles operate at very high efficiencies because they have a favourable pressure gradient but diffusers because they have adverse pressure gradients and there are issues with boundary layers in adverse pressure gradient problems.

They have lower efficiencies compared to nozzles. But the ideal principles have been discussed thoroughly here various operating conditions and ways to calculate those operating conditions for both nozzles and diffusers and the facts that in diffuser the most important problem is starting of diffusers. If you do not design for starting then diffuser will never start. That is the main issue in diffusers.

So, with this, from next class we will go on to the forcing now we move from varying areas to having a friction inside ducts. So, area is maintained constant but friction is introduced into the duct and when then we see what happens to compressible flow in such ducts. That is a particular kind of problem known as Fanno flow. And further we will move to another problem where you can have only heat addition which is termed as Rayleigh flow.

And we will just have a small class on how to see if there are combined effects of all these area, area change and friction and the heat addition. So, that covers majority of the class of flows and gives good mathematical models to do simple designs of engineering devices. But to know complete details of flow, we need to work with differential equations, which will be covered further on. With that thank you.