Gasdynamics:
Fundamentals and Applications

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Lecture-04
Thermodynamics II

Welcome, now in this lecture we will continue from where we left off discussing thermodynamics in the context of gas dynamics. So, we had looked at two very important concepts, the concept of continuum and the local thermodynamic equilibrium. And also looked at how thermodynamics comes into picture, when looking at gas dynamics.

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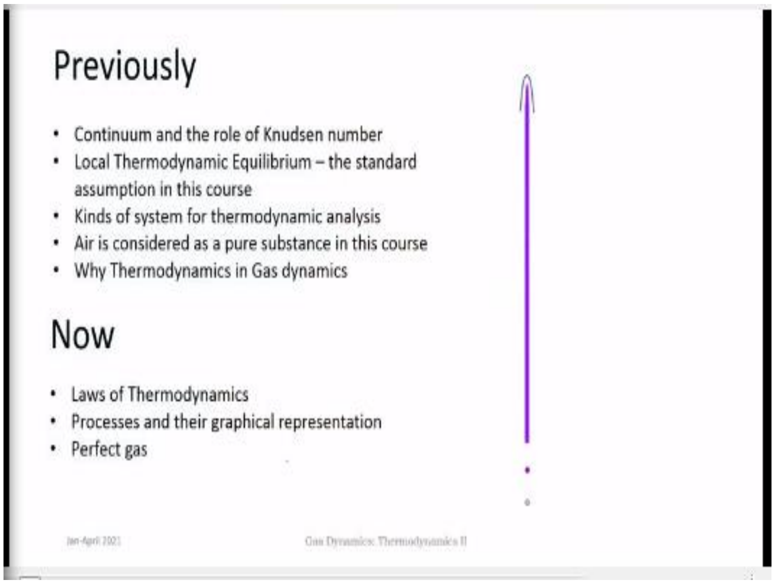

Previously

- Continuum and the role of Knudsen number
- Local Thermodynamic Equilibrium – the standard assumption in this course
- Kinds of system for thermodynamic analysis
- Air is considered as a pure substance in this course
- Why Thermodynamics in Gas dynamics

Now

- Laws of Thermodynamics
- Processes and their graphical representation
- Perfect gas

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Now we will just go through the laws of thermodynamics and processes and their graphical representation this is very useful. Because we look at different kinds of gas dynamic flows and they are you can consider them to be represented on a thermodynamic chart like PV or TS diagrams. And that will immediately help you understand several things about the gas dynamic flows.

So, the processes and their representations and as I said in the previous class, we are going to consider a continuum flow and of a perfect gas. So, what are these definitions of a perfect gas? .

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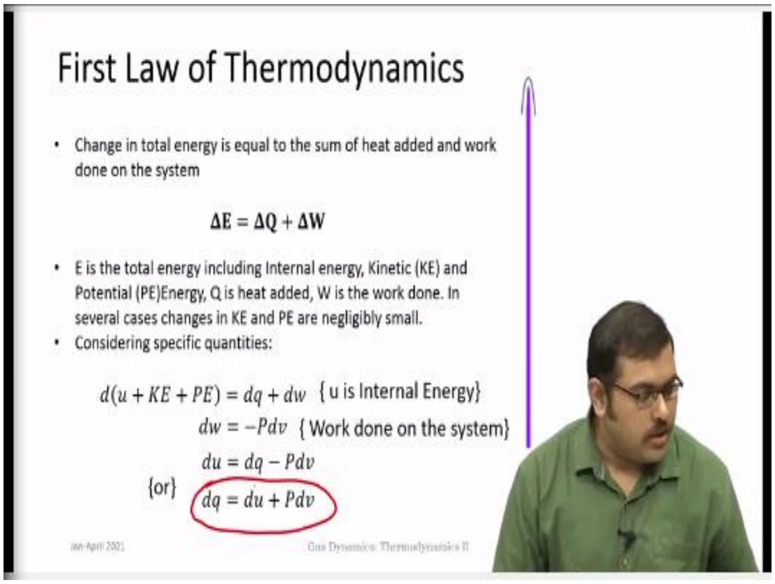

First Law of Thermodynamics

- Change in total energy is equal to the sum of heat added and work done on the system

$$\Delta E = \Delta Q + \Delta W$$

- E is the total energy including Internal energy, Kinetic (KE) and Potential (PE)Energy, Q is heat added, W is the work done. In several cases changes in KE and PE are negligibly small.
- Considering specific quantities:

$$d(u + KE + PE) = dq + dw \quad \{\text{u is Internal Energy}\}$$

$$dw = -Pdv \quad \{\text{Work done on the system}\}$$

$$du = dq - Pdv$$

$$\{\text{or}\} \quad \boxed{dq = du + Pdv}$$

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So, the laws of thermodynamics essentially relates to exchanges of energy, heat and work. And the first law of thermodynamics is a statement that the energy is basically conserved we can only transfer it from one form to the other. So, the total change of energy is the sum of heat added to the system and work done on the system. So, the energy that we are considering is it includes internal energy, kinetic energy, potential energy.

And in many cases, these changes to the potential energy and kinetic energy can be quite small. Then when you look at the change in energy, it is essentially you are looking at change in internal energy. And work done on the system is the pdv block which is it has the negative sign corresponding to the sign convention this is the work done on the system.

So, the main equation that comes about from this for a system is in $dq = du + pdv$ where v is the specific volumes. All these quantities are in specific terms that is per kg. So, this is the important equation that comes about from first law of thermodynamics.

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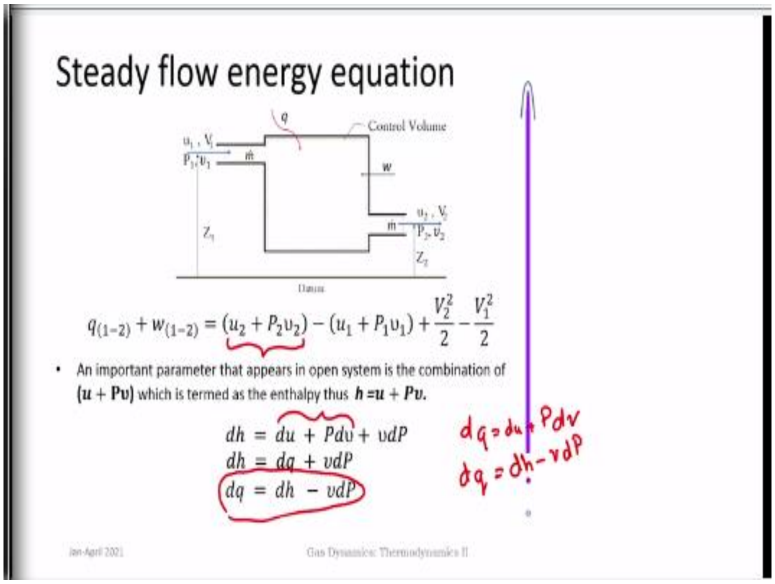

Steady flow energy equation

$$q_{(1-2)} + w_{(1-2)} = \underbrace{(u_2 + P_2 v_2)} - (u_1 + P_1 v_1) + \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

- An important parameter that appears in open system is the combination of **(u + Pv)** which is termed as the enthalpy thus ***h* =*u* + *Pv*.**

$$dh = \overbrace{du + Pdv} + vdP$$
$$dh = dq + vdP$$
$$\boxed{dq = dh - vdP}$$

$dq = du + Pdv$

$dq = dh - vdP$

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But when we consider open systems, when there is exchange of mass happening then the corresponding equation to be considered is the flow energy equation when you have flows that is coming into a control volume and going out of a control volume then when you look at the balance of the energies and the work done and the heat exchange. Then there is the quantity called the flow of work that always comes into picture and this gets clubbed with the internal energy, they form a group $u + pv$.

And this term is known as the enthalpy and it appears when considering flow kind of situations. So, enthalpy is $u + pv$ and if you differentiate this equation and apply the previous equation, that was dq equals $du + pdv$. This equation is also included here, so you get the $du + pdv$ term here. So, you can write the enthalpy $dh = dq + vdp$ or you get the next equation which is $dq = dh - vdp$.

So, both these equations $dq = du + pdv$ and $dq = dh - vdp$, both these equations are coming from the considerations of the first law of thermodynamics. But the first law of thermodynamics always relates to only exchanges of energy during a process as it goes from state A to state B. And it never tells anything about the direction whether it goes it can be whether the process can happen or not.

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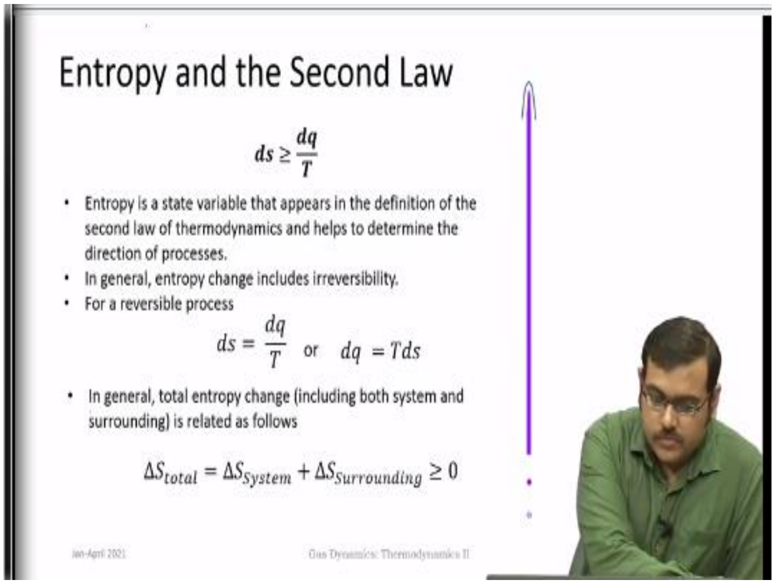

Entropy and the Second Law

$$ds \geq \frac{dq}{T}$$

- Entropy is a state variable that appears in the definition of the second law of thermodynamics and helps to determine the direction of processes.
- In general, entropy change includes irreversibility.
- For a reversible process

$$ds = \frac{dq}{T} \quad \text{or} \quad dq = Tds$$

- In general, total entropy change (including both system and surrounding) is related as follows

$$\Delta S_{total} = \Delta S_{System} + \Delta S_{Surrounding} \geq 0$$

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And this is clarified by the second law of thermodynamics, and in the second law of thermodynamics an important state variable that gets defined is the entropy of the system, the entropy is a state variable. And the definition of entropy helps us to determine the direction of the process the entropy of the system or the change in entropy (ds) is always greater than or equal to $\frac{dq}{T}$, where dq is the amount of exchange of heat energy.

So, you should notice that this is equal to and greater than it is not exactly equal to. This because during processes there can be irreversibilities that are introduced due to various things like friction and so the entropy change includes such irreversibilities. And in general it will be greater than $\frac{dq}{T}$, but if one considers a reversible process then an $ds = \frac{dq}{T}$, or you can write $dq = T\, ds$.

Now the important point is that entropy is a state variable, that means it can be defined for two separate points are A and B and it does not depend on the process, that is followed to go from A and B. So, this way of defining $dq = T\, ds$ for a reversible system will give us certain directions on how to estimate entropy. Now in general we should remember that the total entropy change actually includes both the system and surrounding and the second law relates to the total entropy change.

So, Δs the total here is even including they change due to mass also,
so $\Delta s_{total} = \Delta s_{system} + \Delta s_{surrounding}$. And this change in total entropy has to be always greater than or equal to 0 ($\Delta s_{total} \geq 0$). But when there are interactions of heat between system and surrounding one has to be careful in looking at this second law. And do not just conclude by just looking at the system and saying that it is greater than or equal to 0.

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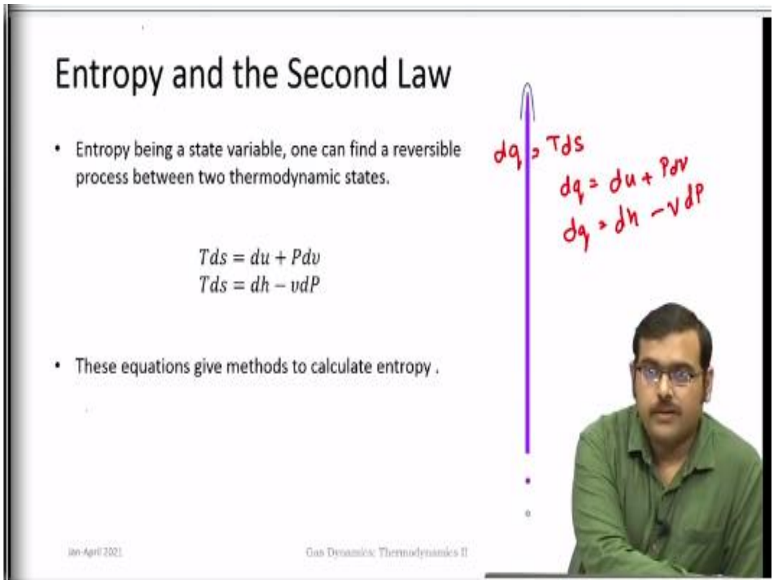

Entropy and the Second Law

- Entropy being a state variable, one can find a reversible process between two thermodynamic states.

$$Tds = du + Pdv$$
$$Tds = dh - vdP$$

- These equations give methods to calculate entropy .

$dq \geq Tds$

$dq = du + Pdv$

$dq = dh - vdP$

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So, now since entropy is a state variable we can always look at a reversible process between two thermodynamic states A and B. And you can write $dq = T\,ds$. And from the first law considerations we had the equations $dq = du + pdv$ and $dq = dh - vdp$. Now here dq can be replaced with Tds and these two equations provide us means to calculate entropy or estimate the entropy between two states. So, how to clearly go about doing it? We will come within a few slides.

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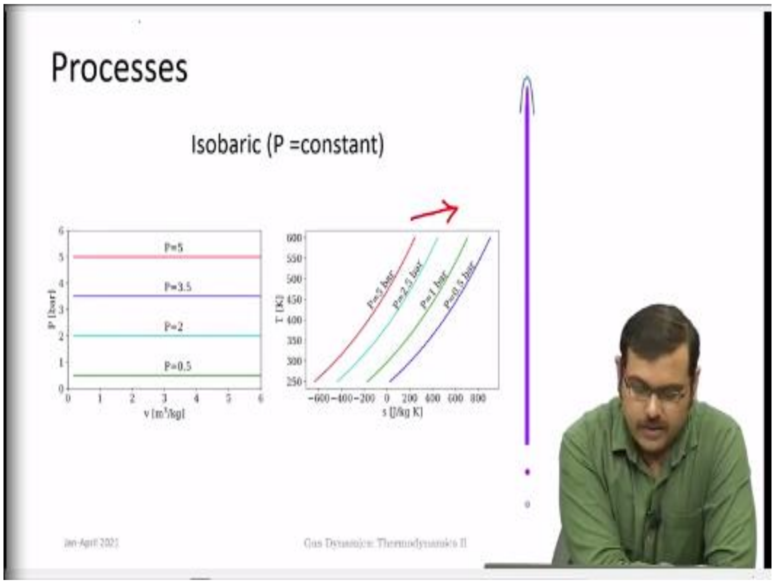

Processes

Isobaric (P =constant)

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So, now let us look at different kinds of processes and how they are represented on thermodynamic charts or thermodynamic diagrams. It is very useful to do this because immediately it is visually available to look at various aspects of the process. And the common types of representations are the pressure specific volume pv diagrams or the temperature and entropy diagrams Ts diagrams.

And one should also know that the area under the curve of a pv diagram is the work done while in the case of Ts diagram, it is the heat that is added or removed from the system. So, the representation can also be directly related to the laws of thermodynamics that we had just discussed. So, these processes some typical examples are of the isobaric process which is the pressure is remaining constant.

In a pv diagram, the pressure constant is just horizontal lines and it is immediately observable. While you have curves of the pressure lines in a Ts diagram and one should understand that as you go from the left to right that is you are increasing an entropy the pressure actually decreases. So, that you have to keep in mind when we are going to discuss using qualitative charts, then this kind of understanding will be useful.

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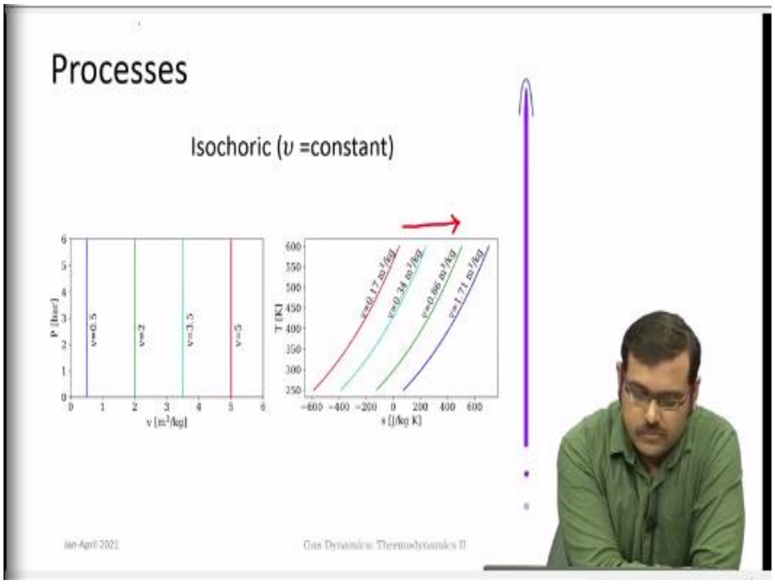

Processes

Isochoric (υ =constant)

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Now isochoric processes are processes where the specific volume or density is held constant. And in a pv diagram they are vertical lines and in Ts diagram they are curves again. And the curves are in such a fashion that as you go from left to right or increasing in entropy, the specific volume will increase. The specific volume increasing means density is decreasing.

So, combining the two you see that you previously you had pressure that is decreasing, while here you have a specific volume that is increasing. You should also bear in mind the slopes of these curves, in general the slope of the isochoric line in a Ts diagram will be higher than the isobaric line. So, that also you should be bearing in mind, so that you are able to understand these representations.

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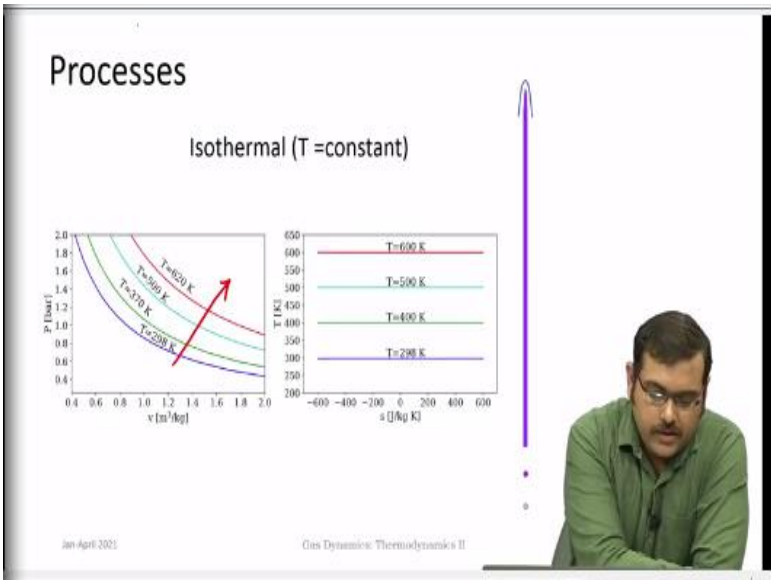

Processes

Isothermal (T =constant)

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Now let us come to the isothermal process, where in an isothermal process the temperature is held constant. So, in a pv diagram these are represented all these charts that are drawn here are for air as a perfect gas. So, these come about to be rectangular hyperbolas and their direction is such that in along this direction they are increasing. So, you have 298 K and goes up to 620 K, and in a Ts diagram the constant temperature lines or isothermal lines are just horizontal lines .

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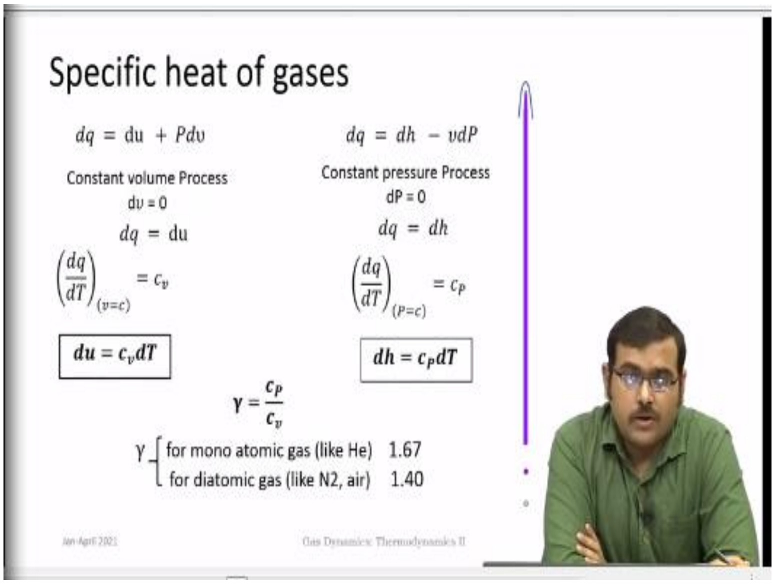

Specific heat of gases

$dq = du + Pdv$

Constant volume Process

$dv = 0$

$dq = du$

$\left(\frac{dq}{dT}\right)_{(v=c)} = c_v$

$\boxed{du = c_v dT}$

$dq = dh - vdP$

Constant pressure Process

$dP = 0$

$dq = dh$

$\left(\frac{dq}{dT}\right)_{(P=c)} = c_P$

$\boxed{dh = c_P dT}$

$\gamma = \frac{c_P}{c_v}$

γ for mono atomic gas (like He) 1.67

γ for diatomic gas (like N2, air) 1.40

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Now when you consider these different processes, then you are interested in trying to understand how much heat is added or how much heat is removed, what is the work done and such kind of processes or interactions. Then the important definitions that come about is the specific heat of gases. Now the specific heat by definition is the amount of heat that it needs to be added for a unit mass of the substance to increase it is temperature by 1 Kelvin.

But for gases the addition of heat involves changes in pressures and other state variables also pressures and the density or the specific volume. So, then the addition of heat should be done using a certain you should look at the process under which this addition of heat happens. So, if you take the first law of thermodynamics and look at the different definitions of it and you should consider the internal energy equation then you have $dq = du + pdv$.

Now if you consider a constant volume process heat addition under a constant volume process, then in that case there is no change in this specific volume or this part is 0. So, then you get $dq = du$ that is the change in the heat added will directly go and result in a change of internal energy. Then the specific heat defined indirect constant volume process is known as the specific heat at constant volume constant volume specific heat c_v .

So, the internal energy then can be written directly as $c_v dT$, so $du = c_v dT$. Similarly if you consider a the equation for enthalpy $dq = dh - vdp$. Then for a constant pressure process or an isobaric process, dp is 0, so you get that the heat that is added to the system should directly affect only the enthalpy. So, you get the specific heat under constant pressure as c_p and the enthalpy can be written as $dh = c_p dT$.

So, now these provide you means to calculate the internal energy and enthalpy of the gas under consideration. Another important parameter that comes from here is the ratio of a specific heats γ, which is $\frac{c_p}{c_v}$ that is a ratio of the specific heat at constant pressure to this specific heat at constant volume. And these values for gamma is dependent on the type of gas, so for mono atomic gases it has a value of 1.67 like inert gases like helium, argon so on. And diatomic gases like nitrogen, oxygen and so on including air which is mainly constituted by nitrogen and oxygen, the value of γ is 1.4.

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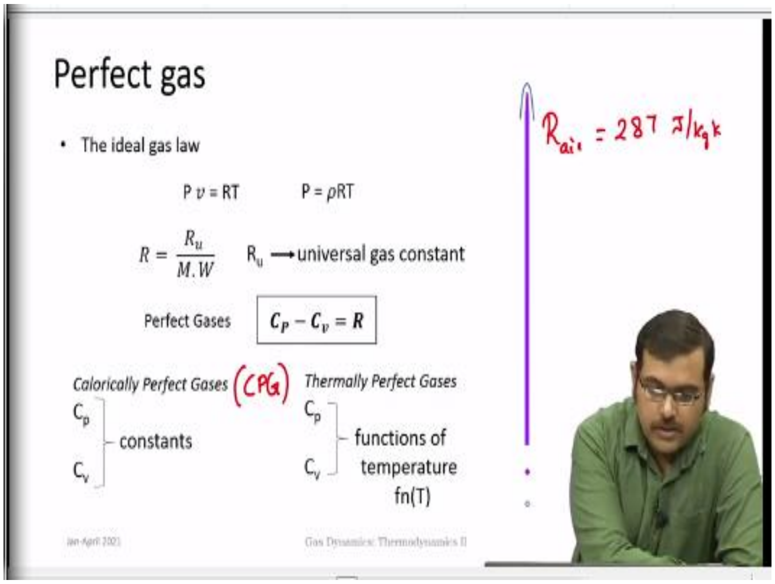

Perfect gas

- The ideal gas law

$P v = RT \qquad P = \rho RT$

$R = \frac{R_u}{M.W}$ R_u → universal gas constant

Perfect Gases $\boxed{C_p - C_v = R}$

Calorically Perfect Gases (CPG) *Thermally Perfect Gases*

C_p, C_v – constants

C_p, C_v – functions of temperature fn(T)

$R_{air} = 287$ J/kg K

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So, now we are considering only perfect gases in our descriptions here in this course. So, they follow the ideal gas law which is $pv\ =\ RT$ where R is the specific gas constant, this specific gas constant is related to the universal gas constant which is R_u which is a constant to by dividing it by the molecular weight of the gas . So, for air the typical value for this is around 287, so R for air is taken to be 287 $\frac{J}{kg\ K}$.

So, then the other important equation that is comes about is that for perfect gases difference between the specific heat at constant pressure and constant volume is R. So, you see that $c_p - c_v = R$, now in general the c_p and c_v need not be constant they can change and in perfect gases, the changes happen in such a way that a $c_p - c_v = R$.

So, I mean those conditions which satisfy these relations they belong to the perfect gas. Specifically in that c_p and c_v can be constants are taken as constant and that is then called as a calorically perfect gas. Calorically perfect gas sometimes you can also use the abbreviation CPG. Under this condition c_p and c_v are constants this is often used while describing the processes in gas dynamics, .

The other case is that of a thermally perfect gas, in thermally perfect gases c_p and c_v they are functions of temperature. So, as temperature changes c_p and c_v change but they change in such a fashion that they always obey $c_p - c_v = R$.

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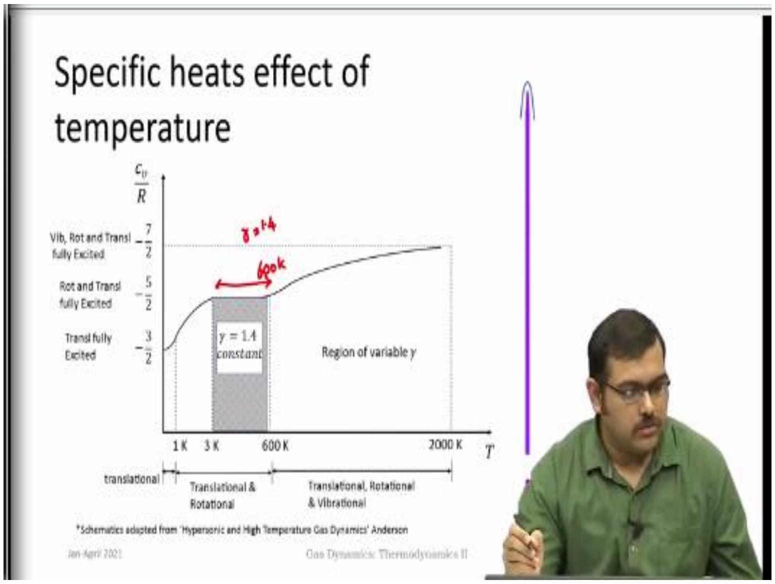

Specific heats effect of temperature

*Schematics adapted from 'Hypersonic and High Temperature Gas Dynamics' Anderson

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So, that is possible because of the different modes of energy that we had discussed. So, this is a chart which looks at c_v as a function of temperature and it is seen that only for moderate values of temperatures is about 600 Kelvin . So, close to around 600 Kelvin, you can consider that c_v is a constant or γ is exactly equal to 1.4. But after that what happens is that during this regime, the main modes that are in of energy that are in play are translation and rotational energy.

Because the energy needed to excite vibrational energy is quite high but once temperatures start increasing beyond these numbers, then vibrational modes also get excited. And because of that your CP will start changing with temperature. But even those c_p and c_v change with temperature they still behave as perfect gases. And only in conditions that there are say chemical reactions happening at extremely high temperatures beyond this region, you may find that they do not obey the perfect gas relations.

So, but in our discussions in this course, we will only consider perfect gases. So, and in majority of cases for discussions, we will consider the calorically perfect gas that is c_p and c_v are constants.

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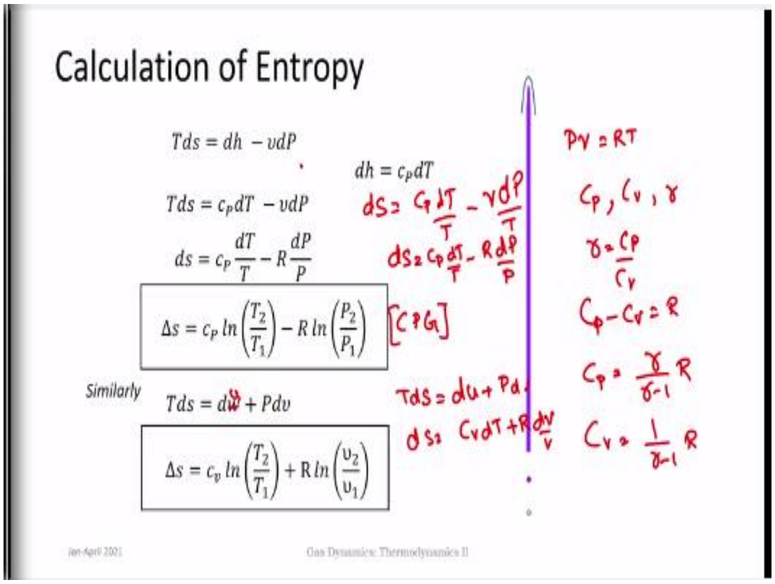

Calculation of Entropy

$Tds = dh - vdP$

$dh = c_p dT$

$Tds = c_p dT - vdP$

$ds = c_p \frac{dT}{T} - R\frac{dP}{P}$

$$\boxed{\Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)}$$

Similarly $Tds = du + Pdv$

$$\boxed{\Delta s = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right)}$$

$ds = C_p \frac{dT}{T} - \frac{vdP}{T}$

$ds = C_p \frac{dT}{T} - R\frac{dP}{P}$

[CPG]

$Tds = du + Pd$

$ds = C_v dT + R\frac{dv}{v}$

$Pv = RT$

C_p, C_v, γ

$\gamma = \frac{C_p}{C_v}$

$C_p - C_v = R$

$C_p = \frac{\gamma}{\gamma - 1} R$

$C_v = \frac{1}{\gamma - 1} R$

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And so, now that we have understood definitions of c_p and c_v and also looked at the second law and how and the equations to calculate entropy. We can actually come up with few formulae to calculate the entropy as the gas undergoes certain processes. And that is when you consider the equation for enthalpy $Tds = dh - vdp$ then you can do this changes these are algebraic changes you can write$ds = \frac{c_p dT}{T}$.

And so if you divide this by T it will be $\left[\frac{c_p dT}{T} - \frac{vdp}{T}\right]$. Now you can use the perfect gas law $pv = RT$ to write $\frac{v}{T}$ in terms of $\frac{R}{p}$. So, you get $ds = \left[\frac{c_p dT}{T} - \frac{Rdp}{p}\right]$. Now it is a matter of integration of this equation. So, you are looking at change in entropy, so Δs, this change in specific entropy is equal to this integration gives rise to $[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)]$.

Here of course you have taken that c_p is not a function of temperature, so this is valid for calorically perfect gas CPG . So, now similarly if you consider the equation $Tds = du + pdv$. Then you can again do the same algebraic manipulations to get $ds = \left[\frac{c_v dT}{T} + \frac{Rdv}{v}\right]$. And you can write the entropy in terms of change in entropy in terms of temperature and specific volume. So, these equations provide us means with by which we can calculate the entropy in a calorically perfect gas.

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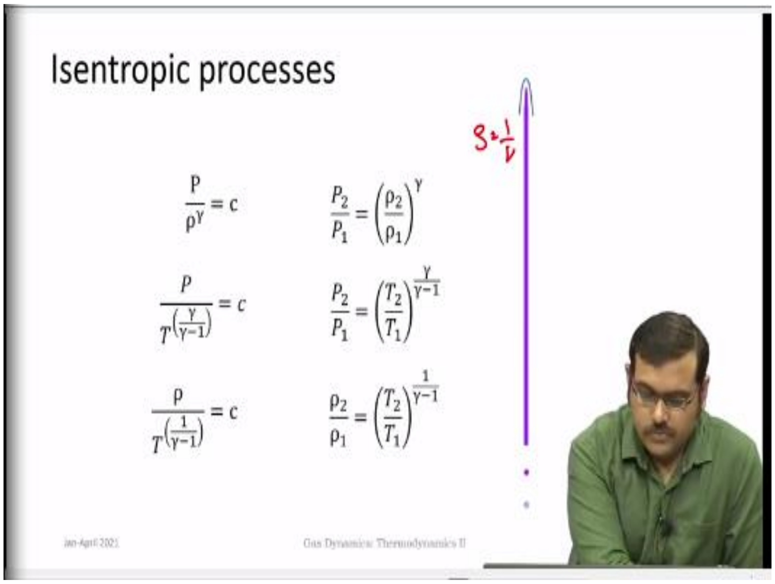

Isentropic processes

$$\frac{P}{\rho^{\gamma}} = c \qquad \frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

$$\frac{P}{T^{\left(\frac{\gamma}{\gamma-1}\right)}} = c \qquad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho}{T^{\left(\frac{1}{\gamma-1}\right)}} = c \qquad \frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

$\rho = \frac{1}{v}$

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So, an important kind of process is the isentropic process these processes are when the change in entropy is 0. And you can really see from here directly that if you put $\Delta s = 0$ and then try to write these and solve for the temperature ratios and pressure ratios you can get these numbers. If you combine the definition of c_p, c_v with γ, where $\gamma = \frac{c_p}{c_v}$ with the equation $c_p - c_v = R$.

You can write c_p and c_v in terms of R or c_p is $\frac{\gamma}{\gamma-1}$ and c_v is $\frac{1}{\gamma-1}$; multiplied by R, you can put them in these equations. And from them you can derive these conditions for isentropic process where change in entropy is 0, you get $\frac{p}{\rho^{\gamma}}$ then ρ is nothing but $\frac{1}{v}$. We use density because density will is the one which is important when considering gas dynamic flows.

So, $\frac{p}{\rho^{\gamma}}$ is constant or $\frac{p}{T^{\left(\frac{\gamma}{\gamma-1}\right)}}$ is constant or $\frac{\rho}{T^{\left(\frac{1}{\gamma-1}\right)}}$ is constant. So, that gives us the relations between the pressure ratios, temperature ratios and density ratios when a system undergoes an isentropic process. This will become really important when we discuss certain modules of this gas dynamic course.

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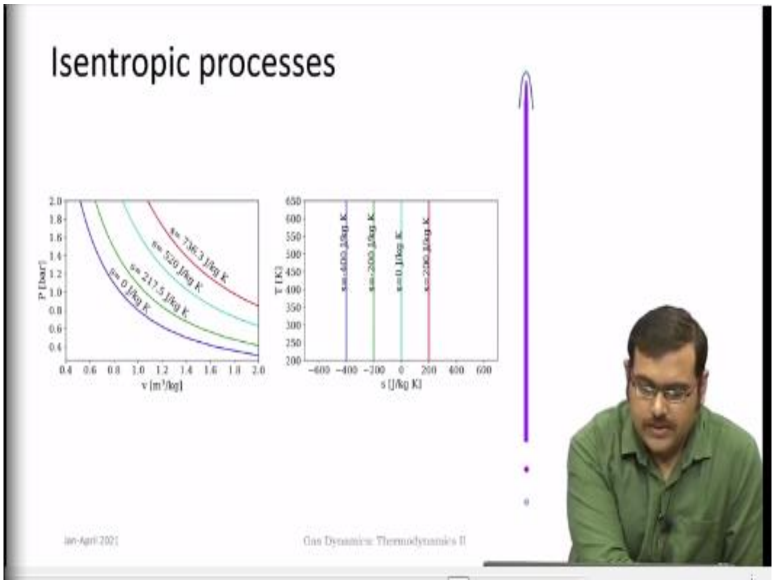

Isentropic processes

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So, these you can represent the isentropic processes on the pv and the Ts diagram just as we had done the other processes , so here this is for perfect gas for air. So, again they follow a similar curve as is for the isothermal process but it is not a rectangular hyperbola. They have a slope which is different, they have a higher slope than that for constant temperature processes and in a Ts diagram, they have vertical lines.

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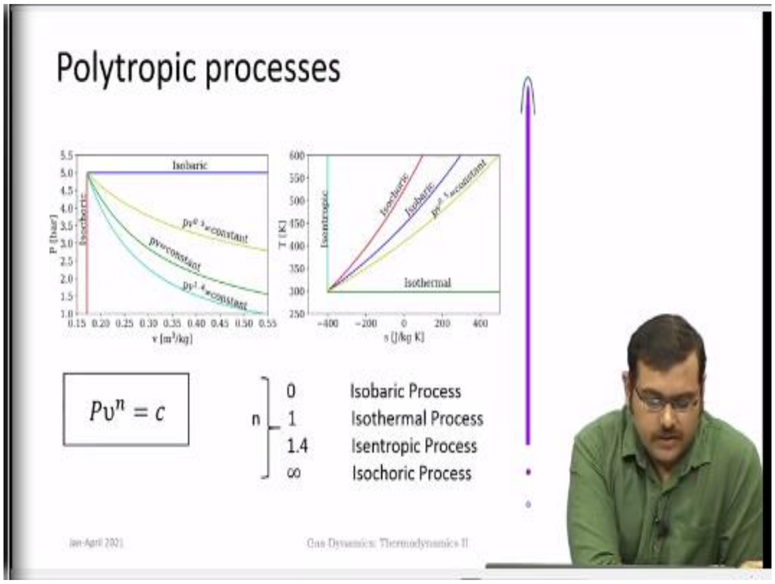

Polytropic processes

$$Pv^n = c$$

n
0 Isobaric Process
1 Isothermal Process
1.4 Isentropic Process
∞ Isochoric Process

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So, you can also consider a generic kind of description where it is like a pv^nequal to constant, a polytropic process. And the power n can be varied between different values starting from 0 to infinity if it is 0 then a p = constant when n is 0 because you will get 1. So, p is constant when n = 1 then you get 'pv' equal to constant which is an isothermal process, when n = γ the value of ratio of specific heats then this is a isentropic process, when it goes to infinity it is a isochoric process.

So, by varying in a typical polytropic process, you can consider all these different processes. And you can have an arbitrary number also like 0.5 and this represents their representation in the pv and Ts diagrams.

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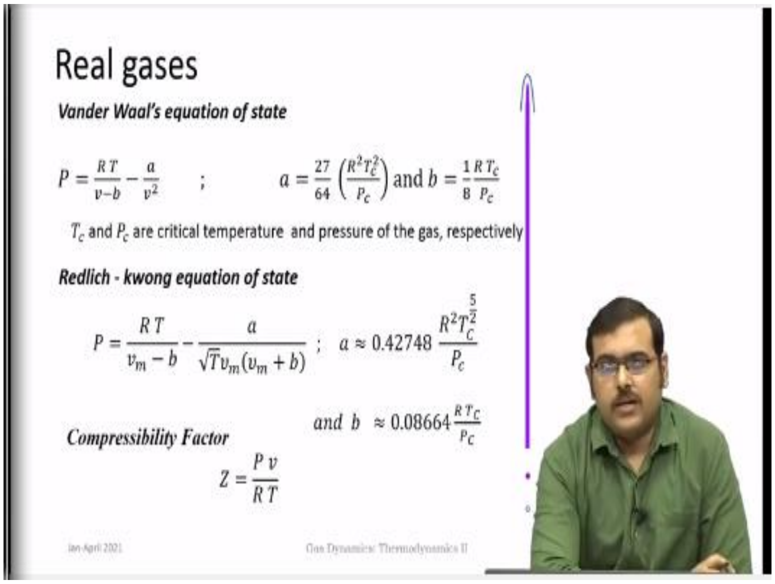

Real gases

Vander Waal's equation of state

$$P = \frac{RT}{v-b} - \frac{a}{v^2} \quad ; \quad a = \frac{27}{64}\left(\frac{R^2T_c^2}{P_c}\right) \text{ and } b = \frac{1}{8}\frac{RT_c}{P_c}$$

T_c and P_c are critical temperature and pressure of the gas, respectively

Redlich - kwong equation of state

$$P = \frac{RT}{v_m - b} - \frac{a}{\sqrt{T}v_m(v_m + b)} \quad ; \quad a \approx 0.42748\,\frac{R^2T_C^{\frac{5}{2}}}{P_c}$$

$$\text{and } b \approx 0.08664\,\frac{RT_C}{P_C}$$

Compressibility Factor

$$Z = \frac{Pv}{RT}$$

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So, in so as I said that in almost all the cases that we consider in this course, it will be a perfect gas and we will use ideal equations of state for the gases. These ideal equations of state air valid when you consider that the intermolecular forces of attraction are negligibly small or almost equal to 0. But in many cases in real applications it may not hold good, then you need other equations of state.

So, these are real gas equations of states, I have listed a few of them Vander Waal's equation, Redlich kwong equation of state. Sometimes it is described using a compressibility factor. Now they can also be used along with the equations that are the fluid flow equations, energy equation and the equation of state. So, the equation of state can become a different one from the perfect or the ideal gas equation.

But we do not consider such flows where these are important in this particular course. But these equations have been put up here for understanding that there may be cases where the ideal gas law may not hold. And in order to solve the compressible flow equations, you may need to use a different equation of state.

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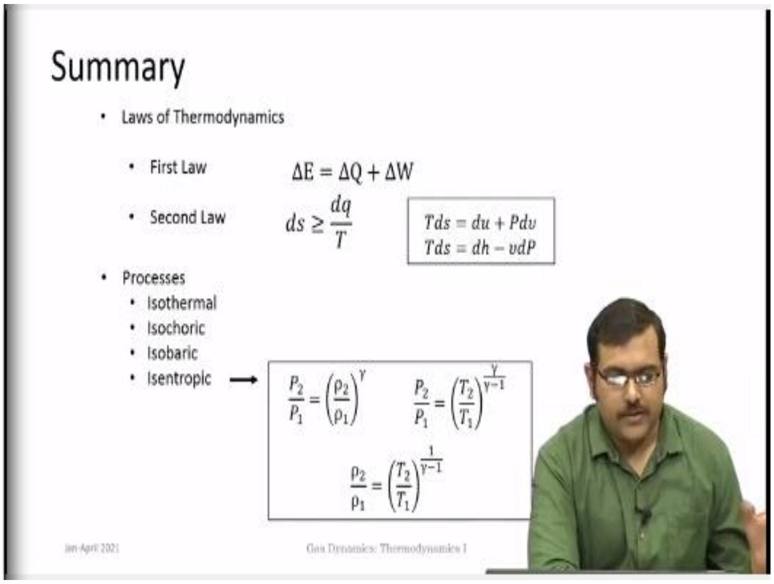

Summary

- Laws of Thermodynamics
  - First Law $\Delta E = \Delta Q + \Delta W$
  - Second Law $ds \geq \frac{dq}{T}$

$$\boxed{\begin{aligned} Tds &= du + Pdv \\ Tds &= dh - vdP \end{aligned}}$$

- Processes
  - Isothermal
  - Isochoric
  - Isobaric
  - Isentropic →

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \qquad \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

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Gas Dynamics: Thermodynamics I

So, if you look at this entire summary, so we have looked quickly looked at thermodynamics, it is a really vast field in itself. And in gas dynamics, we take it that we understand thermodynamics well. So, in case you need to go and read it up again please do it and only the essential part that to relating to perfect gases has been described as of now in these two lectures that we had .

So, the basic first law, second law and the equations which become very helpful for us in defining several things and different processes and their representations. The representations of processes in graphical manner is important because you will see that in gas dynamics we have several cases where the flow behaves like a isentropic process. And then we can describe it on a Ts diagram and that will help us to understand many things of the flow and that is often used in description of gas dynamics processes.

So, please understand the process and their graphical representations also. So, with that in the next class, we will move towards the basic fluid flow equations, the integral flows and things like that. Before we go to fluid flow, the next class we will look at a few numerical examples of familiarizing ourselves with the equations of thermodynamics. So, we will do a couple of numericals and then move over to fluid flow equations.