Gasdynamics: Fundamentals and Applications
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Lecture 39
Varying area flow Numericals- III

So, we continue our applications of concepts of variable area ducts shock waves expansion waves in problems.

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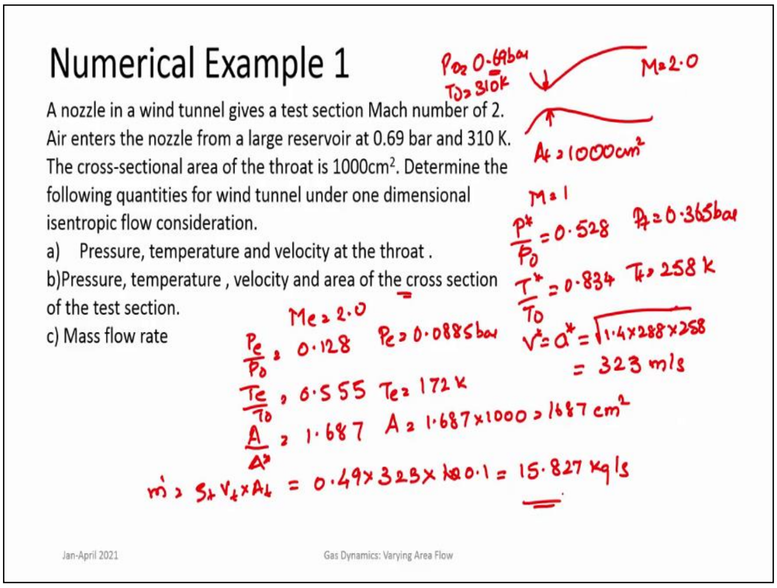

So, here we take simple problems not too many variety of concepts are used. So, a nozzle in a wind tunnel gives a test section Mach number of 2, air enters the nozzle from a large reservoir at $0.69\ bar\ and\ 310\ K$. Cross sectional area of the throat is $1000\ cm^2$. Determine following quantities for internal and one dimensional isentropic flow con consideration that is quasi-1D flow , pressure, temperature and velocity at throat. Pressure, temperature, velocity and area of cross section at the test section and mass flow rate.

So, this is direct application of varying area duct principles. So, what is known is throat area is given. So, this is a nozzle problem. So, you have a nozzle and exit Mach number is given. So, Mach number is 2.0 and that is also known that at $0.69\ bar\ and\ 310\ K$ are P_0 and T_0 and area at throat is known.

So, A_t is 1000 cm^2. So, let us see how to go about this problem pressure, temperature and velocity at throat for this tunnel at operating conditions for a supersonic nozzle throat Mach number is one. So, a Mach number is one. So, $\frac{P^*}{P_0}$ and $\frac{T^*}{T_0}$ are the values to consider and this is 0.528 and this is 0.834. So, these values you can get from isentropic tables or calculators.

So, then pressure at throat is just multiplication by P_0 which is 0.69. So, this is 0.365 bar and T_{throat} is 258K. So, we get 258 K here. And from this, we can also calculate what is the velocity at the throat? Velocity at throat is equal to a^* , that is speed of sound as Mach number is 1, $= \left(\sqrt{\gamma RT}\right) = \sqrt{1.4 * 288 * 258} = 323\ m/s$.

This is the value. Now, we go to what should be the pressure, temperature, velocity at the exit. So, it is an isentropic flow completely an isentropic flow. Mach number is 2 at the exit. So, M_e is 2.0. So, $\frac{P_e}{P_0} = 0.128$ and $\frac{T_e}{T_0} = 0.555$ and since we need to find the area of cross section, we need to know $\frac{A}{A^*}$. This, for Mach 2 it is 1.687. So, pressure at the exit, P_e is 0.0885 bar.

Temperature at the exit, T_e is 172 K and the area is $1.687 * 1000 = 1687\ cm^2$. So, what is on the mass flow rate? Mass flow rate it can be calculated at throat = $\rho_t V_t A_t$ all things are calculated here. Density at the throat ρ_t can be calculated by $\frac{P_t}{RT_t}$ it is 0.49 kg/m^3. And V_t is known 323 m/s and A_t is known 1000 cm^2. So, that is 0.1 m^2.

So, this turns out to be $\dot{m} = 15.827\ kg/s$. So, here in this problem directly we are using a variable area duct principles, mass flow rate principles and isentropic flow through variable area duct, so, a straight numerical example.

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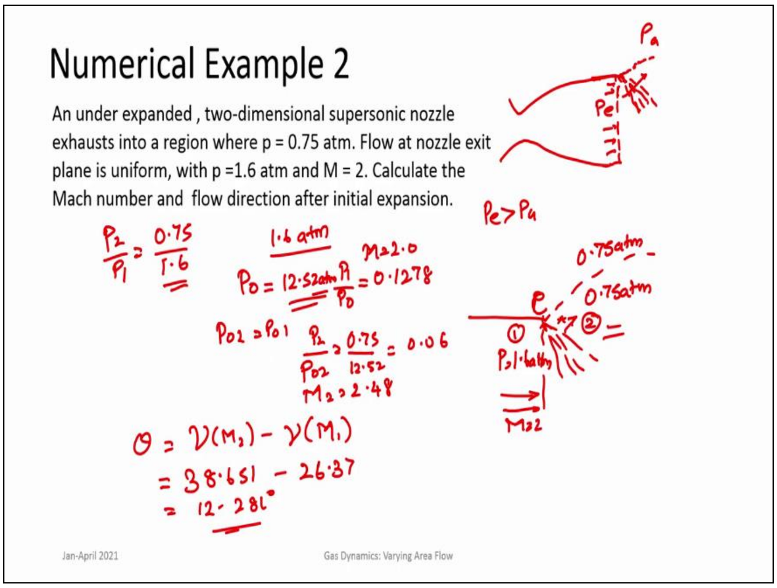

Second one is an under expanded 2 dimensional supersonic nozzle. So, here we have an under expanded jet. So, an under expanded flow; in under expanded nozzle flow the nozzle pressure at the exit. So, P_{exit} at nozzle will be greater than the ambient pressure $P_{amb,A}$ if it is ambient pressure then P_{exit} will be greater than ambient pressure. So, there is still pressure left for expansion.

So, as a result the nozzle under the flow undergoes an expansion through a centered expansion fan if we take the, so, if we take that quasi-1D flow through nozzle at the exit you will have a uniform flow. And thereafter you can take a centered expansion fan and the nozzle the flow will expand. So, flow will turn across the centered expansion fan. So, if you zoom out, zoom into this, so, it will be like this.

So, here we use principles not only of quasi 1D nozzle flows, we also use expansion wave principles here. So, what is given? Exhaust is a region where pressure is $0.75\ atm$. So, here pressure is $0.75\ atm$. That means at the edge here also it should be $0.75\ atm$ but at the nozzle exit plane at P_e at exit at the section exit pressure is $1.6\ atm$. So, clearly it is a case of an expansion under expansion.

And Mach number is 2. So, within the nozzle a flow is quasi-1D it is uniform but outside the nozzle it will now expand. So, calculate Mach number and flow direction after the initial. So, this expansion happens. So, what is the Mach number here after the expansion and the flow direction? So it has now expanded it flows outward. So, what is that direction?

We know pressure ratio, pressure ratio is known.

So, if I take this as region 2 and region 1 , $\frac{P_2}{P_1}$ is given $\frac{P_2}{P_1} = \frac{0.75}{1.6}$ this is given to us. So, how do we go about this problem? Flow through the nozzle is, flow through the nozzle as well as the expansion centered expansion is isentropic. So, it uses isentropic relations. If exit pressure is 1.6 atm then, the corresponding P_0 for M_2 operation, we can find this out.

Because $\frac{P_1}{P_0} = 0.1278$ for $Mach\, number\ =\ 2$. So, $P_0 = 12.52\ atm$ and this P_0 remains the same across the centered expansion also. So, $P_{02} = P_{01}$. So, $\frac{P_2}{P_{02}} = \frac{0.75}{12.52} = 0.06$. So, here you get Mach number is 2.48. So, now we have found out the Mach number at region 2. So, now we need to know what is change in the angle theta?

Theta is from Prandtl Meyer expansion relations $\nu(M_2) - \nu(M_1)$, where ν is the Prandtl Meyer angle and Prandtl Meyer angle you can read out from tables or you can use a calculator $38.651° – 26.37°$ which is $12.281°$. So, you have used 2 concepts here one is a quasi-1D flow in the nozzle and then an under expanded jet and in the initial expansion you used Prandtl Meyer expansion relations, so, connected concepts being used here.

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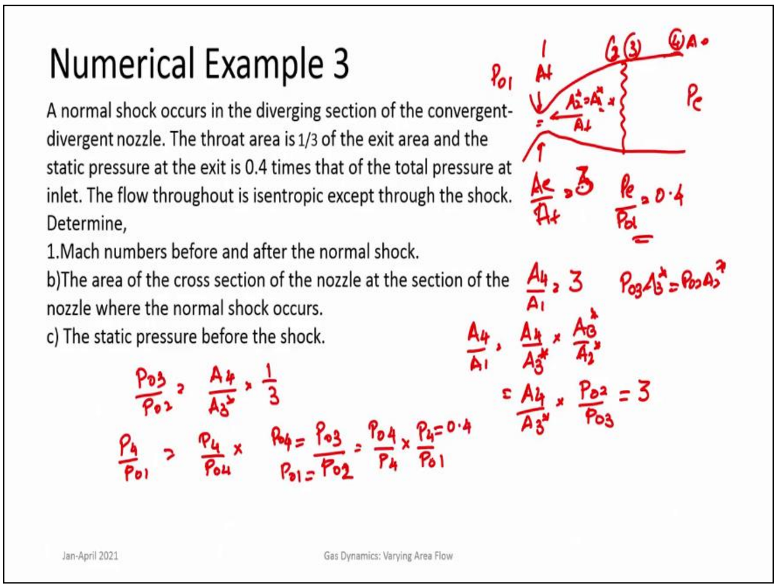

So, in the next problem, a normal shock occurs in the divergent section of the convergent divergent nozzle. Throat area is one third of the exit area. And static pressure at the exit is 4 times the total pressure at inlet. Flow throughout is isentropic except through the shock.

Determine be Mach numbers before and after the shock, area of cross section of nozzle at the section, where the normal shock occurs, static pressure before the shock.

So, what is given, let us see. This is a nozzle and area ratio of this nozzle is given, exit area and throat area . $\frac{A_e}{A_t}$, A_t is $\frac{1}{3}$ of A_e , $\frac{A_e}{A_t} = 3$. So, that is $\frac{A_e}{A_t} = 3$ and somewhere here now, one shock occurs and the $\frac{P_{exit}}{P_{01}}$ is 0.4 So, this is given. How do we go about this problem? So, this uses concepts of shock in divergent passage. So, what we know is so, if I name these as one station number 1, 2, 3, 4.

So, this is what I would say. So, what we are given here is $\frac{A_4}{A_1}$ is 3. It is given $\frac{A_4}{A_1}$ is 3 and what is $\frac{A_4}{A^*}$ in this particular case? It will be $\frac{A_4}{A_1} = \left(\frac{A_4}{A_3^*}\right)\left(\frac{A_3^*}{A_2^*}\right)$, because there is a normal shock here. So, A_2^* will be equal to A_1 because this is a supersonic flow here. Mach number is 1 and here all through this region $A_2^* = A_1^* = A_1$. But $A_3^* \neq A_2^*$, because there is a stagnation pressure loss across this shock. So, this is the principle we use. So, now what is the guiding principle here? This $\frac{A_3^*}{A_2^*}$ is related to $\frac{P_{02}}{P_{03}}$. $P_{03}A_3^* = P_{02}A_2^*$. So, $\frac{A_3^*}{A_2^*} = \frac{P_{02}}{P_{03}}$

So, this is $\left(\frac{A_4}{A_3^*}\right)\left(\frac{P_{02}}{P_{03}}\right)$. So, if we can find out what is $\frac{P_{02}}{P_{03}}$ then, we can find what is the Mach number but this is $\frac{A_4}{A_1}$. So, this is equal to 3, now what is given about $\frac{P_{02}}{P_{01}}$. So, now $\frac{P_{03}}{P_{02}} = \left(\frac{A_4}{A_3^*}\right)\frac{1}{3}$. That, this is known now we know $\frac{P_4}{P_{01}}$ is given. So, $\frac{P_4}{P_{01}}$ is given can be written as $\left(\frac{P_4}{P_{04}}\right)\left(\frac{P_{04}}{P_{01}}\right)$.

So, $\frac{P_4}{P_{04}}$ is. So, this is known P 0 1 is known. So, $\frac{P_{03}}{P_{02}}$ can we write it in terms of $\frac{P_4}{P_{04}}$ this is what needs to be understood. $P_{03} = P_{04}.\frac{P_{03}}{P_{02}}$, I can write it as $\left(\frac{P_{04}}{P_4}\right)\left(\frac{P_4}{P_{01}}\right)$. So, we can give this value because $P_{02} = P_{01}$ and $\frac{P_{03}}{P_{02}} = \left(\frac{P_{04}}{P_4}\right)0.4$, So, we can because $\frac{P_4}{P_{01}} = 0.4$ is given.

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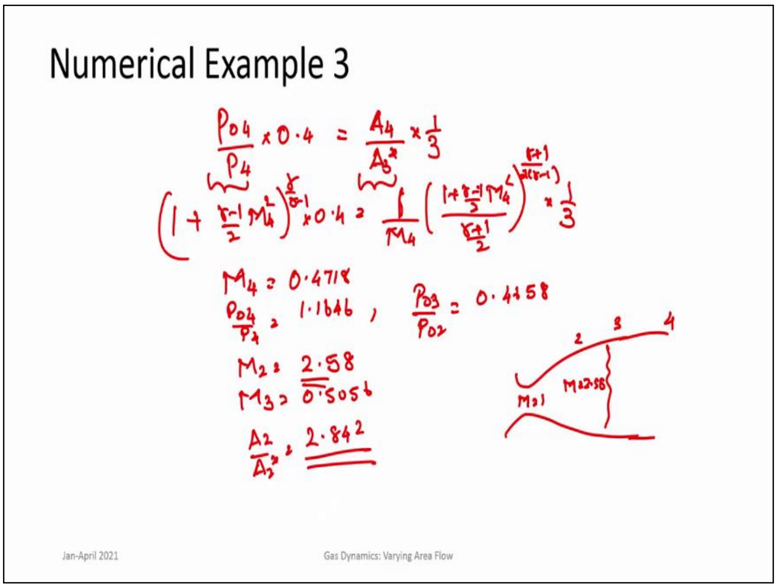

So, now we get an equation $\left(\frac{P_{04}}{P_4}\right)0.4 = \left(\frac{A_4}{A_3^*}\right)\frac{1}{3}$. Now this can be expressed only in terms of Mach number M_4. This can be also expressed only in terms of the Mach number ok. So, this can be expressed. So, we can write this as $\left(1+\frac{\gamma-1}{2}M_4^2\right)^{\frac{\gamma}{\gamma-1}}0.4 = \frac{1}{M_4}\left(\frac{1+\frac{\gamma-1}{2}M_4^2}{\frac{\gamma=1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\frac{1}{3}$.

So, this you can solve for M_4 and M_4 turns out to be 0.4718. So, now if we know this value then $\frac{P_{04}}{P_4}$ is known is 1.1646 therefore$\frac{P_{03}}{P_{02}}$ can be found out 0.4658. Now once $\frac{P_{03}}{P_{02}}$ is known then Mach number is known. So, the Mach number of the shock is 2.58. So, M_2 is 2.58 and M_3 is 0.5056. Now we know M_2 is 2.58. So, this is the case Mach number equal to 1, at section 1, here at section 2 Mach number is 2.58.

And there is a shock there and then you have section 3 and 4. So, if you know 2 .58 you know the area ratio. So, area ratio is $\frac{A_2}{A_2^*}$ is 2.842. So, this is the area ratio at which the shock occurs. So, this example is a nice example where connected concepts are given about shock waves in divergent ducts and pressure ratio across the nozzle which creates that shock.

And you have to find the area ratio and Mach number at which the shock is located. And so, with this these simple problems involved direct concepts of area ratios. Now let us see when this kind of concepts are applied into multiple examples or cases where you need multiple concepts or you want to apply it in some situations, then, how will we apply these principles in the next class.