Gasdynamics: Fundamentals and Applications
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Lecture 38
Varying area flow Numericals- II

So, we have discussed in detail about nozzle operation and diffuser operation. So, we did a numerical on nozzle operation. So, now we will focus on diffuser operation and further on we will do few more numericals covering combined concepts, several concepts that come together when discussing varying area flows. So, let us see one by one.

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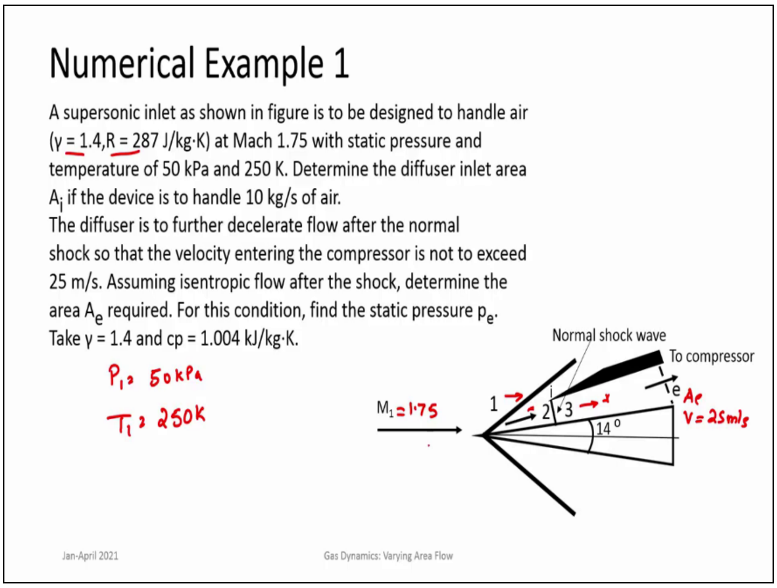

So, first here this is a case of a diffuser that is an intake diffuser. A supersonic inlet as shown in figure is to be designed to handle air. Properties of air are given $\gamma = 1.4,\ R = 287\ at\ Mach$ 1.75. So, this is entry Mach number is 1.75 with static pressure and temperature of 50 kPa and 250 K. So, P_1 is 50 kPa and T_1 is 250 K. Determine the diffuser inlet area. So, see notice the kind of inlet that is described over here.

There is a wedge that is protruding into the flow. So, this is more in the lines of a mixed compression kind of an intake, where there is an external compression taking place by means of an oblique shock. And further there is an internal compression also happening inside the duct. And there is a possibility of a normal shock also occurring here. So, there is a normal shock here. So, 2 shocks and then variable area duct.

Diffuser is further to so, the device is to handle 10 kg/s of air. The diffuser is to further decelerate flow after the normal shock. So, that the velocity entering the compressor is not to exceed 25 m/s. So, at the exit velocity is given 25 m/s. So, it is quite small velocity compared to the incoming velocity, assuming isentropic flow after the shock. So, here you have isentropic flow.

So, besides the shocks in other regions it is isentropic, determine that the area what area is required and find the static pressure at the exit P_e. So, this is the problem. So, in supersonic flows, since there is a problem of information, propagation from downstream to upstream, this way it will not go. So, always problem solving happens in a particular direction. You go from one region to the next. So, that is how we go through this.

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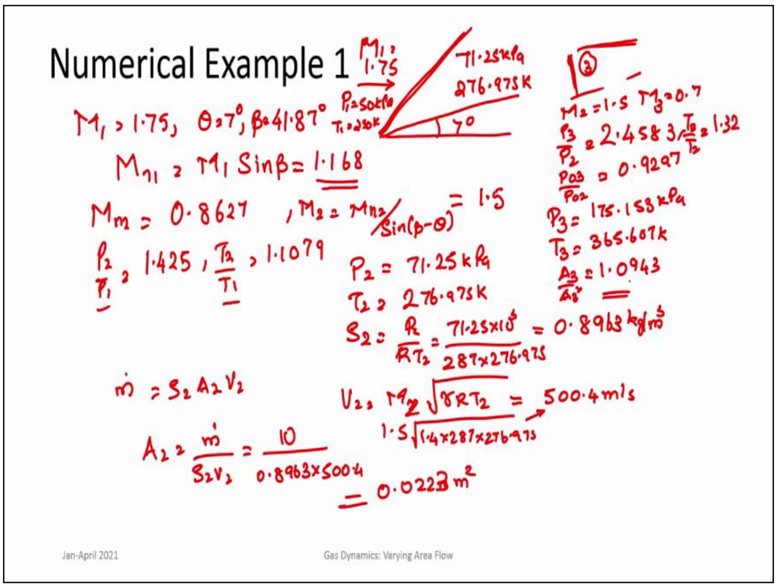

So, start in between region one and two which is bounded by an oblique shock ah. The angle semi angle(θ) of the wedge is 7°. So, corresponding to that there is an oblique shock. So, this is 1.75 Mach number M_1 it is 1.75. So, we know $M_1 = 1.75$ and θ = 7° degrees and β is 41.87 degrees. So, this is what is known. So, from this how do we go through with an oblique shock problem?

We have to find the normal component is $M_1 \sin\beta$ this is 1.168. So, once M_{n1} is known we can find M_{n2} by normal shock relations substitute the normal Mach number. So, it is M_{n2}= 0.8627, $\frac{P_2}{P_1}$ is known if you know this. So, it is $\frac{P_2}{P_1}$ =1.425 and $\frac{T_2}{T_1}$ is 1.1079 ok and $M_2 = \frac{M_{n2}}{\sin\beta-\theta}$. If you do this you get M_2 as 1.5. So, $\frac{P_2}{P_1}, \frac{T_2}{T_1}$ known, P_2 is 71.25 kPa.

So, 50 kPa which is given over here is the static pressure. $P_1\ is\ 50\ kPa,\ T_1\ is\ 250K$. So, these are static values. So, it is $P_2\ is\ 71.25kPa$ and T_2 is $276.975\ K$. So, $276.975\ K$ from here we can calculate what is density. Density is $\frac{P_2}{RT_2}$ which is $\frac{71.25*10^3}{287*276.975}$. It comes out to be $0.8963\ \frac{kg}{m^3}$.

Now we need to find what is mass flow rate $\dot{m} = \rho_2 A_2 V_2$. $\dot{m}$ is given it has to support 10 $\frac{kg}{s}$ of air. We need to find the area that is the area of the intake this A_i is, that is the area that we need to find. So, A_2 now we know properties in region 2 that is what is entering the intake. So, if we know $V_2,\ V_2 = M_2\left(\sqrt{\gamma R T_2}\right)$.

This is V_2 turns out to be $500.4\ \frac{m}{s}$, M_2 is 1.5, $V_2 = 1.5\left(\sqrt{1.4 * 287 * 276.975}\right)$. So, this gives $500.4\ \frac{m}{s}$. So, $A_2 = \frac{\dot{m}}{\rho_2 V_2}$. You can do this calculation $\frac{10}{0.8963*500.4}$. This is $0.0223\ m^2$. So, this is the area. So, at this point the Mach number is 1.5. So, at this point there is a normal shock.

So, you have a normal shock here. So, this normal shock stands here. So, we have to find out the properties across the normal shock in region 3. So, $M_2 = 1.5$. So, $M_3 = 0.7,\ \frac{P_3}{P_2} = 2.4583$. This is also useful $\frac{P_{03}}{P_{02}} = 0.9297$. So, from here we get P_3. $P_3 = 175.153\ kPa$ and similarly you can get $T_3 = 365.607\ K$. $\frac{T_3}{T_2} = 1.32$. So, you can get $P_3\ and\ T_3$ and also $\frac{A_3}{A_3^*}$.

You can get this value 1.0943. This is there in isentropic tables not in normal shock tables. So, once all this is known now we are faced with the point that you need to find out once the velocity goes to 25 m/s, what to do? So, let us just look at that.

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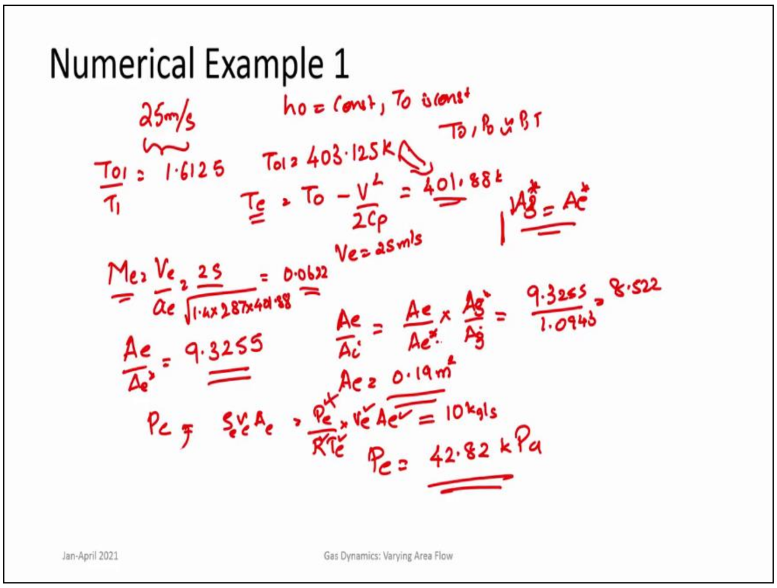

So, we will take a look at that point. So, $25\ m/s$. Looking at $25\ m/s$, it is quite small now, all through these processes they are all oblique shock waves and isentropic flows. So, they are adiabatic flows that means stagnation enthalpy is constant or T_0 is constant. This is the guiding principle. So, from here you we know what is the velocity at the exit.

If we find the T_0, T_0 is T_0 is for the main flow we can find $\frac{T_{01}}{T_1}$. This is for Mach 1.75 flow it is 1.6125. So, T_{01} is $403.125\ K$. And T_e can be found by $T_0 - \frac{V^2}{2c_P}$. Using the energy equation, this turns out to be $401.888\ K$. This is V_e is at the exit $25\ m/s$. So, T_e is known. So, what is the Mach number $M_e = \frac{V_e}{a_e}$ which is $\frac{25}{\sqrt{1.4*287*401.88}}$which is 0.0622. So, it is quite small.

At such small Mach numbers, you can also assume , that $T_0\ and\ P_0$ is approximately equal to $P\ and\ T$. It is not a bad assumption because you see the difference between them is hardly $2\ K$. So, that assumption can also be made or you can continue to pursue with actual numbers. So, if M_e is known then A_e can be found out $\frac{A_e}{A_e^*}$ star can be found. It is 9.3255.

Now we need $\frac{A_e}{A_i}$. That is what is the area ratio $\frac{A_e}{A_i}$. So, this is $\left(\frac{A_e}{A_e^*}\right)\left(\frac{A^*{}_e}{A_i}\right)$. Now for $\frac{A_e}{A_e^*}$, the value is known 9.3255. Now after the shock, the flow is isentropic. So, if we take the Mach number after normal shock. So, for that the $A^*, A_2^* and A_3^*$ will be equal to A_e^*.

And this we had found it is 1.0943. So, 9.3255 divided by 1.0943 is 8.522. So, you can get the exit area as 0.19 m^2. So, what is P_{exit}? This is what is needed. For this, we can use pressure ratios and calculate it. But now that we have calculated mass flow rates, static temperatures and velocity is 25 m/s, we can go ahead and convert this $\rho VA = \rho_e V_e A_e = \left(\frac{P_e}{RT_e}\right) V_e A_e$.

So, A_e is known, V_e is known, P_e is not known, T_e is known, R is known, $\dot{m}$ is 10 kg/s it is known. From here you can get what is P_e it is 42.82 kPa. So, you see. So, this particular concept had a diffuser problem. But the diffuser also had oblique shocks ahead of it and a normal shock at the diffuser. So, entry of the diffuser.

So, you see this has multiple concepts. So, from here on you see that problems involving all these applications, nozzles, diffusers they will not be having only one concept. They will involve multiple concepts. So, you have to take care of that. So, now let us go to the next problem.

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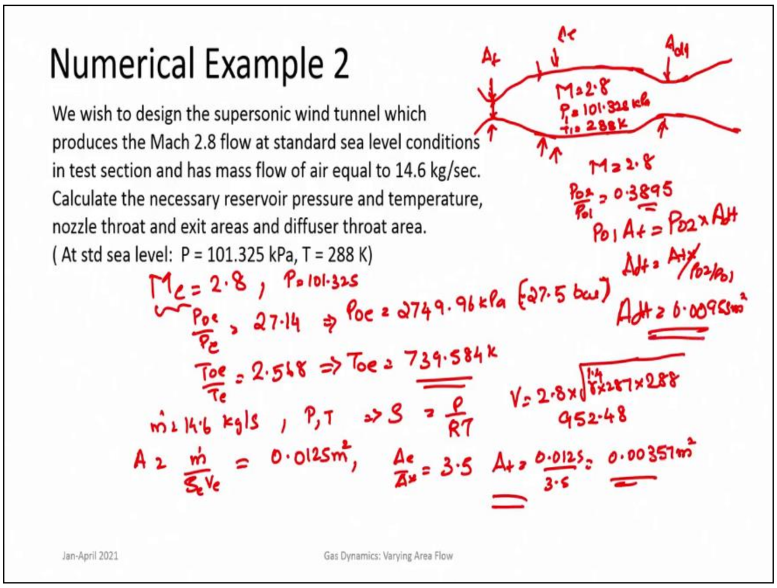

If we wish to design supersonic wind tunnel which produces Mach 2.8 flow at standard sea level conditions. That means at the test section, it is standard sea level conditions and Mach number is 2.8 and mass flow is given $\dot{m}$ = 14.6 kg/s. Calculate the necessary reservoir pressure and temperature nozzle throat and exit areas and diffuser throat area. So, basically it is a, you have this picture schematic should come there.

So, what is nozzle throat was is this nozzle exit area and given that this is $M = 2.8$ and the $P = P_{testsection}$. So, P_1 is standard sea level $101.325\ kPa$ and T_1 is taken as $288\ K$. So, these are the conditions and what should be the diffuser area. So, areas of the diffuser throat, diffuser throat so, what are these, what is required? So, we know the internal Mach number 2.8.

What should be reservoir pressure? We know pressure is 101.325 in ideal operating conditions, Mach number is 2.8. So, the pressure that should be given there at P_0 is going to be corresponding to 2.8. So, $\frac{P_{0e}}{P_e} = 27.14$ and that implies $P_0 = 2749.96\ kPa$. That is in bars, it will be $27.5\ bar$. It is quite high pressure you have to give 27.5 bars okay and what about T_{0e} ah?

If you have to achieve $288\ K$ at the test section then as flow expands through the nozzle temperature will reduce. So, that means much higher temperatures has to be given and that is this ratio is 2.568 implying T_{0e} is 739.584 Kelvin. So, much higher temperatures need to be provided. So, what is the area of the internal test section? That is the exit area of the nozzle.

We know mass flow rate $\dot{m}$ is $14.6\ kg/s$ and pressure and temperature are known. So, from this you can calculate density(ρ) $by\ \frac{P}{RT}$, Velocity should be found. So, velocity is Mach number $V = M(\sqrt{\gamma RT}) = 2.8(\sqrt{1.4 * 287 * 288})$. So, this is velocity. It turns out to be $952.48\ m/s.\ \gamma\ is$ 1.4. So, area is $A_e = \frac{\dot{m}}{\rho_2 V_e}$. This is $0.0125\ m$^2. Now for this Mach number, we know $\frac{A_e}{A_e^*}$ which is 3.5 . So, during correct operation the throat will work at Mach number 1. So, that is equal to A^*. So, throat area is A_{throat} is $\frac{0.0125}{3.5}$, this is $0.0035\ cm^2$. So, we know the throat area. Also now the next point here is we have to find what is the minimum area at the diffuser?

So, this should be such a way that the internal will start. It cannot be the same as the nozzle throat area. So, that is the highlight here. So, Mach number is 2.8. $M = 2.8$. So, the way it is designed is there is a normal shock standing at the test section. So, for Mach number equal to 2.8, if there is a normal shock $\frac{P_{02}}{P_{01}}$ across the normal shock is 0.3895 and we use the fact that $P_{01} A_{nozzlethroat} = P_{02} A_{diffuserthroat}$.

So, diffuser throat is nozzle throat divided by $\frac{P_{02}}{P_{01}}$. So, this is known. So, diffuser throat is $0.00955\ m^2$. So, diffuser throat is larger than the nozzle throat. So, the aspects of wind tunnel starting, is considered here in order to look at the nozzle and the diffuser throat areas. So, with this the aspect related to starting of diffusers should be covered and now we look at several problems related different concepts in varying area ducts.

So, we will go through a few simple problems and two problems that involve multiple concepts. So, that would be done in the coming classes. Thank you.