Gasdynamics: Fundamentals and Applications

Prof. Srisha Rao M V
Aerospace Engineering
Indian Institute of Science – Bangalore

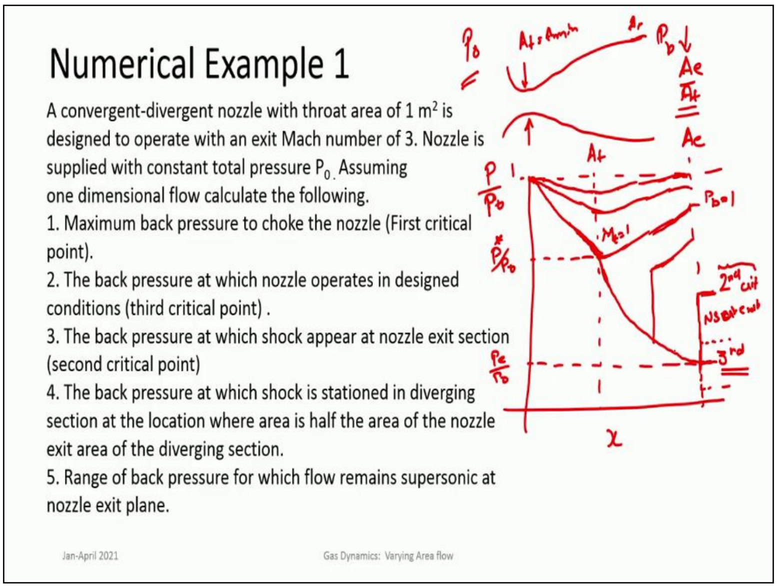
Lecture 35
Varying area flow- Numericals- I

In the previous class we had looked at the operating characteristics of nozzles. We did both converging nozzles and convergent diversion nozzles. Convergent diversion nozzles convert a low speed or subsonic flow all the way to supersonic velocities. But we also became aware that it is not just enough to look at area ratios and their relationship with Mach number it is important to understand how pressure ratios affect the operation of the nozzle.

And an important phenomenon that occurs in these flows is the flow choking that is the mass flow rate becomes a constant once a pressure certain pressure ratio is exceeded or. So, once at the minimum area Mach 1 is achieved then further on any changes downstream of the throat will not affect the mass flow rate that is mass flow rate choking. So, with these ideas in mind let us look at a problem where we look at all these various regimes for a convergent divergent nozzle.

So, we will go through these steps once again but now with a numerical example. So, that you understand the concepts involved in operation of convergent divergent nozzles or convergent nozzles the principles are similar.

(Refer Slide Time: 02:14)



So, the numerical example is a convergent divergent nozzle with throat area of 1 m^2 is designed to operate with an exit number of Mach 3 nozzle is supplied with constant total pressure. So, supply pressure P_0 is total pressure is constant assuming one dimensional flow that is a quasi-1D approximation calculate the following maximum back pressure to choke the nozzle that is the point at which the mass flow rate of the nozzle becomes a constant in this case it does become a constant because P_0 is taken as constant a constant supply pressure.

So, that is first critical point. So, if we do go and draw those curves again, so, we are looking at a convergent divergent nozzle the minimum area is at this point which is throat. At throat A_t, area is a minimum and this sort of picture qualitative picture of how the nozzle behaves one should have. So, this point let us say corresponds to the location of throat. So, A_t is located here and A_{exit} is located here.

And this is $\frac{P}{P_0} = 1$ when $\frac{P}{P_0} = 1$, that is there will be no flow and the moment the back pressure. So, here supply pressure is constant back pressure that is imposed on the nozzle is reduced. So, once it starts reducing a flow begins in the nozzle in the initial stages the mass flow rate is quite small flow velocities are small. So, as a consequence the flow is completely subsonic in this case mass flow rate continues to increase slowly.

So, there is a particular point when you reduce P_0 to such an extent that at the throat. So, at the throat you achieve $\frac{P^*}{P_0}$. So, once the that is achieved that is you are able to reach the critical pressure then Mach number at the throat becomes equal to 1. So, $M_t = 1$ when this is done then mass flow rate becomes choked further decrease in back pressure will not change mass flow rate it will not affect the upstream also.

So, the upstream will behave the same. So, this particular point this particular back pressure is first critical point the designed pressure ratio according to the area ratio you will have a designed pressure ratio $\frac{P_e}{P_0}$. So, if that particular pressure ratio is given or it is reduced to such a pressure ratio then nozzle will designed will work in the design condition and there will be no shocks within the nozzle you will have a complete isentropic flow.

And this particular pressure ratio can be calculated by isentropic relations and that is known as the third critical point. So, this is the third critical point while in between the first critical point and third critical point the back pressures are high still high and they do not support complete supersonic flow all the time. So, one case is that the flow expands completely supersonic until the exit of the nozzle but right at the exit you will have a normal shock.

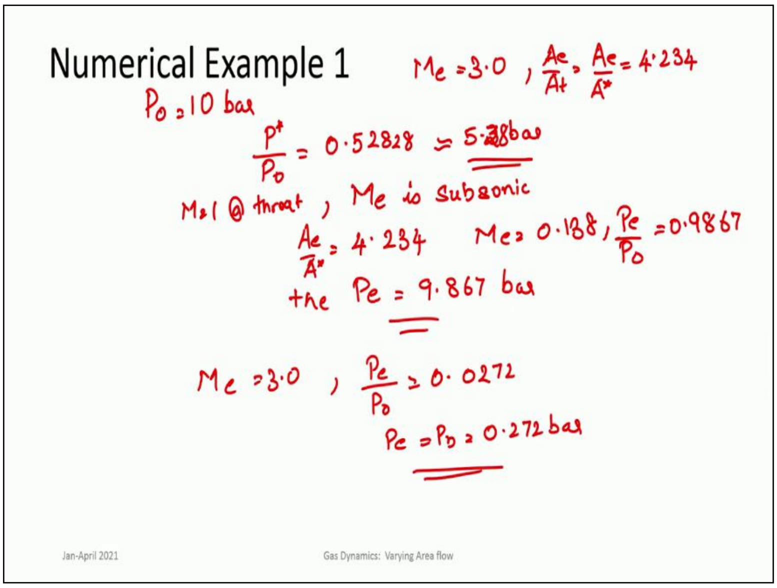
So, normal shock at exit that particular location is second critical point in between the second critical point and the first critical point shock can be located anywhere inside the nozzle and after the shock. So, shock is an entropy generating process. So, you will across the shock there is entropy generation but after that you can consider an isentropic process again in between.

So, at if the pressure is slightly above the third critical point then that flow is called an over expanded flow. And oblique shocks are generated because the pressure at the exit of the nozzle is now no longer equal to that of the back pressure, back pressure is slightly higher exit pressure is slightly lower than back pressure and a oblique shocks develop as a consequence. While the under expanded flow occurs if the back pressure is reduced below the third critical point then the nozzle has not completely expanded to the ambient or the back pressure it has capacity to expand and it will expand outside the nozzle.

So, these are the highlights of the various operating regimes. So, what we can do is for this Mach 3 nozzle can we look at various such points in a numerical sense and also the 4th question is the back pressure at which shock is stationed in the diverging section at the location where the area is half the area of the nozzle exit area. So, the area ratio for the nozzle at which the shock stands is half.

So, when we talk about the area ratio of the nozzle it refers to $\frac{A_e}{A_t}$ when nozzle is operating in correct conditions then the exit Mach number will correspond to the Mach number associated with $\frac{A_e}{A_t}$ because that wrote the Mach number will be 1. So, let us go ahead with this.

(Refer Slide Time: 09:18)



So, that is one important point that we should always look for M_{exit} is known to be 3.0. So, immediately we should look at what is the area ratio $\frac{A_e}{A_t}$ which is if it is under correct operation will be equal to $\frac{A_e}{A^*}$ and for Mach 3 it is 4.234 just to get numbers in place. So, that you have an idea what are these numbers we are talking about let us assume that P_0 is about 10 bar we could discuss it in non-dimensional numbers also $\frac{P}{P_0}$ is enough.

But this gives a feel for numbers what are we talking about. So, when the first critical point occurs when the Mach number at throat is 1 and at the exit the flow is still subsonic. So, the flows there are no shocks inside. So, flow is isentropic and what is the pressure at which flow chokes at the nozzle that is equal to $\frac{P^*}{P_0}$ and for air this is constant. So, it is for air it is with $\gamma = 1.4$, $\frac{P^*}{P_0} = 0.52828$.

So, you can see that it is about 5.3 bar(5.28 bar). So, when this particular pressure is achieved at the throat the throat Mach number becomes equal to 1 the flow chokes. From that point onwards if the exit back pressure is continuously reduced it will not affect upstream sections from the throat further reduction from this point will start supersonic flow in the diverging section.

But the flow cannot be supersonic all through the nozzle because the back pressure does not support it at the exit you still need to have the back pressure quite low. So, as a consequence shocks develop in the diverging portion. So, these series of development can be seen when one starts a nozzle in certain applications like wind tunnels and such applications if there is a supersonic nozzle and before operating the wind tunnel everything will be closed high pressure will be there in the reservoirs and then the nozzle is the valve is opened.

So, slowly the pressure starts being felt by the nozzle and all these various steps that we discuss here can be encountered in such an operation. So, this is 5.28 bar but this is at the throat. So, what will it be at the exit? So, at the exit it will be subsonic but Mach number equal to 1 at throat but M_e is subsonic. So, very if you look at this you know $\frac{A_e}{A^*}$ because this is already fixed by the nozzle the design of the nozzle that is 4.2, but we also know that for a given $\frac{A_e}{A^*}$.

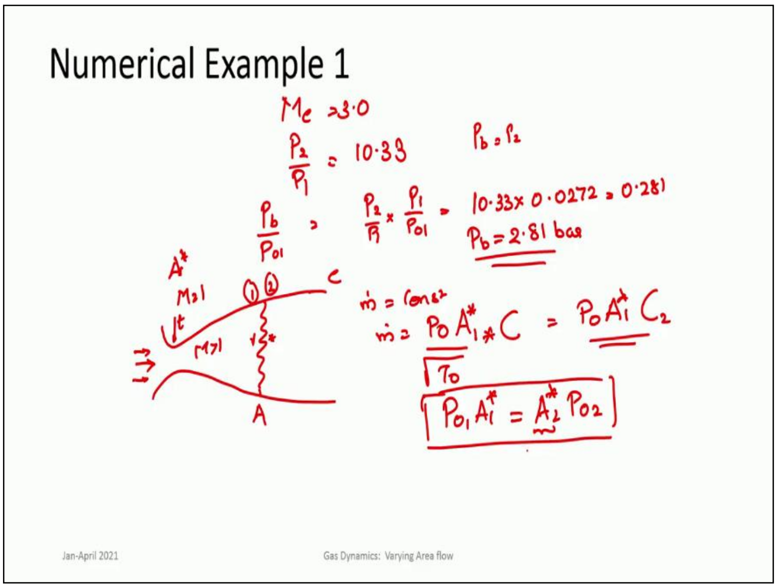
There are 2 possible solutions one is in the subsonic domain one is in the supersonic domain while designing this nozzle the designers would have designed it for the supersonic operation but currently it is operating subsonic condition with Mach number one at the throat. So, we look for the subsonic solution for this given $\frac{A_e}{A^*}$ and that corresponding subsonic value is 0.138 now the flow is completely subsonic therefore you can calculate what is $\frac{P_e}{P_0}$ this turns out to be 0.9867.

So, you get the exit pressure if we take P_0 is around 10 bar exit pressure is 9.86767 bar. So, it will be around this value that is for the case when the throat is at Mach 1 and exit is subsonic now the second case is that pressure ratio is exactly equal to complete expansion there are no shocks and it expands completely. So, that is the third critical point actually. So, for that we know the Mach number M_e is 3.0 corresponding to these isentropic relations what is $\frac{P_e}{P_0}$ this is 0.0272.

So, you see that for 10 bar upstream pressure you really need to give quite low pressures in order to achieve Mach 3 complete expansion without shocks in the nozzle and you can get this number it will be the exit back pressure should be equal to exit pressure and it will be 0.272 bar. So, that is the point at which there will be complete supersonic flow through the nozzle and at the exit as well now the let us go back to the picture that we had done before.

So, that is a third critical point now what we need to find out is what is this pressure at which the; flow is completely supersonic throughout the nozzle divergent portion of the nozzle but at the exit it encounters a normal shock. So, still at the exit if it does that at the exit the flow is subsonic when flow is subsonic at the exit then the pressure at the exit will match the back pressure. So, that pressure we have to find out.

(Refer Slide Time: 16:46)



So, here the nozzle operates. So, that within the nozzle divergent portion of the nozzle you have complete supersonic flow. So, $M = 3.0$ but right at the exit there is a normal shock normal shock is a discontinuity. So, you have to calculate normal shock at 3.0 and pressure ratio across the normal shock $\frac{P_2}{P_1}$ is 10.33. So, if one were to calculate what would be the exit back pressure.

So, $\frac{P_b}{P_{01}}$ at this particular condition it would be $P_b = P_2$. So, it is $\left(\frac{P_2}{P_1}\right)\left(\frac{P_1}{P_{01}}\right)$ where $\frac{P_1}{P_{01}}$ corresponds to a complete isentropic expansion and $\frac{P_2}{P_1}$ is across the shock. So, 10.33 multiplied by 0.0272 is 0.281. So, the back pressure when this would be achieved is 2.81 bar. So, you see its considerably high compared to a completely expanded flow.

Now let us consider the case where you may have a normal shock within the divergent portion there is a subsonic flow there is a supersonic flow that starts from the throat but if the back pressure does not support. So, until you reach back pressures of at least 2.81 bar it will not support full supersonic flow that means exit will be subsonic. So, this can be achieved by a normal shock. So, this is a model of the flow that happens within nozzles it is.

So, if you look at this section then the flow has become it has achieved Mach 1 at throat it continues to expand. So, m is greater than one but somewhere in the nozzle there is a shock. So, here there is a shock sitting at certain area ratio ok. So, you can take this as throat this point is one 2 and this is exit ok. So, shock is a discontinuity. So, it has the same it is at that particular area ratio the mass flow rate that is supplied is choke.

So, mass flow rate does not change here. So, $\dot{m} = constant$. So, before the nozzle the before the shock as it goes through the throat the $\dot{m}$ can be written as $P_0 A_t$. Now that is in if it is 1, it is A^*. So, for it is for location if you consider one for point one it is ${A_1}^*$. ${A_1}^*$ is the same in an isentropic flow the star area does not change $\frac{P_0 A_1^*}{\sqrt{T_0}}$ and it gets multiplied by constant this constant I will say is C.

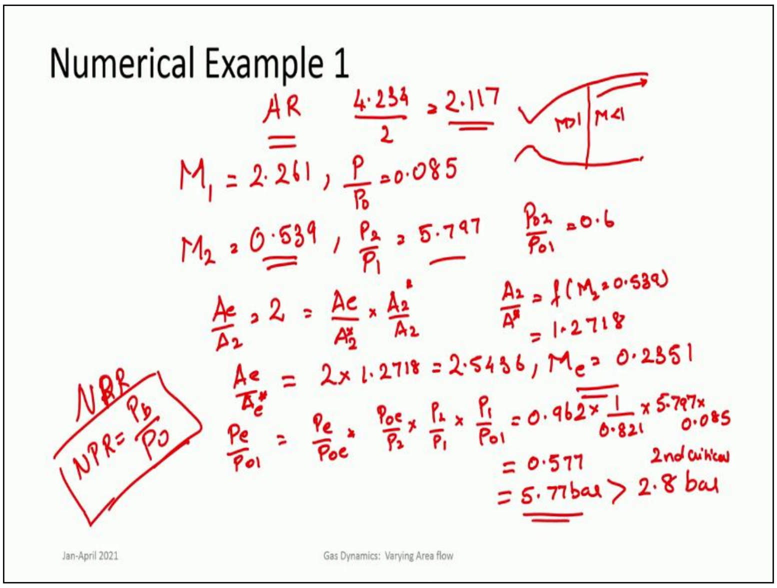
So, stagnation temperature is a constant it is an adiabatic flow. So, essentially it is dependent on $P_0 A_1^*$ and everything else is a constant. But now you have a shock in between across the shock mass is conserved. So, your mass flow rate is conserved. So, across the shock if I take 2 then at 2 also mass flow rate should be the same but what about P_0. So, P_0 across the shock is not constant because there is entropy generation.

So, what has happened now? So, P_0 is not constant rest everything they are constants. So, something else must have changed. So, yes it would have changed that is the area ratio for that particular Mach number after the shock. So, before the shock there is you have a supersonic Mach number after the shock there is a subsonic Mach number if you calculate the star area for that subsonic Mach number it will not be equal to A_1^*.

So since mass flow rates are equal you have $P_{01} A_1^*$ equal, to this is a general formula it is not restricted to any case early requirement is it should be a star area. So, what is A_2^* it should be multiplied by P_{02}. So, $P_{01}A_1^* = P_{02}A_2^*$ is equal. So, this is the guiding principle which will allow us to solve such problems. There is an entropy generation at the shock.

As a consequence the star area changes this principle if you remember many cases in area variable area ducts which we will discuss more often in diffusers where shocks can be easily seen in variable area ducts. And the way to solve such problems is to use this guiding principle. So, how do we go with this problem?

(Refer Slide Time: 23:07)



So, with this particular problem the area ratio at which the shock stance is given its half that of the exit area ratio. So, it is 4.234 divided by 2 which is 2.117. So, we know the area ratio at this point and the flow is supersonic until this particular area ratio. So, that means at the throat it was 1 and till 2.117 the area ratio of 117 it is supersonic. So, we can calculate what was the Mach number at that particular area ratio?

So, M_1 is known. So, you can find that out it is 2.261 and corresponding to this area ratio the $\frac{P}{P_0}$ is 0.085 now shock is at this particular area ratio. So, M_1 is 2.261. So, you can find what is M_2 after the shock M_2 after the shock is 0.539 and $\frac{P_2}{P_1}$ is 5.77 and you can also get $\frac{P_{02}}{P_{01}}$ which is 0.6 all these are available in normal shock tables. So, now you look at this problem you know that $\frac{A_e}{A_2}$ is 2 this can be written as $\left(\frac{A_e}{A_2^*}\right)\left(\frac{A_2^*}{A_2}\right)$.

Now $\frac{A_2^*}{A_2}$ this should now correspond to M_2 because the flow has passed through a normal shock is for M_2 that is 0.539. So, now if we do calculate for M_2, the $\frac{A_2}{A_2^*}$ t is 1.2718. So, $\frac{A_e}{A_2^*}$ this is also equal to A 2 star is now (2)*(1.2718) which is 2.5436 and for this the subsonic Mach number M_{exit} will be 0.2351. So this is how we can calculate the exit Mach number when there is a shock sitting in the divergent portion of the convergent area convergent divergent duct.

What is we are interested in $\frac{P_e}{P_{01}}$ this is $\left(\frac{P_e}{P_{0e}}\right)\left(\frac{P_{0e}}{P_2}\right)\left(\frac{P_2}{P_1}\right)\left(\frac{P_1}{P_{01}}\right)$. Now this all these numbers we know for 0.2351 for this Mach number $\frac{P_e}{P_{0e}}$ is 0.962 and P_{0e} is the same as P_{02} that is for the value of $\frac{P_2}{P_{02}}$ that is 0.821 and $\frac{P_2}{P_1}$ is that is known $\frac{P_2}{P_1}$ is 5.797 and $\frac{P_1}{P_{01}}$ is known 0.085.

So, finally from this we get 0.577 that means it is 5.77 bar. So, you can see that this value is greater than the second critical point the value for the second critical point where it was just after that is the second critical point. So, the shock is at the exit while shock here is at half that of the divergent portion after the shock you still have a divergent area. So, shock is somewhere here.

So the flow is supersonic all through until the shock then it becomes subsonic then it undergoes isentropic diffusion that is it undergoes deceleration velocity decreases pressure increases. So, through this elaborate example I hope that the concepts related to operation of convergent divergent nozzles are clear. So, there are various; as the pressure ratio across the nozzle changes various operating conditions take place.

The first important point is the point at which mass flow rate chokes in the nozzle but flow may be still subsonic once pressure ratio is decreased continuously decrease after that supersonic flows start in the divergent portion. But it may not last until they exit the limit of that is supersonic flow happens all through the divergent portion. But there is a shock at the exit the exact operating condition is when there are no shocks at all and the nozzle expands.

So, that the exit pressure matches the back pressure at supersonic flows that is known as the correct designed operating conditions. The pressure ratio across the nozzle is often referred to as nozzle pressure ratio that is NPR. So, many places you will find NPR. NPR is always back pressure by stagnation pressure. So, or vice versa $\frac{P_0}{P_b}$ that is not the exit pressure. So, exit pressure can be different when these nozzle operates in the under expanded or over expanded regimes.

So, with this discussion on nozzles we will move ahead with diffusers. In diffusers we face some other problems in nozzles we had shocks in the divergent section. Let us look at diffusers in the coming class and see what problems we face there, thank you.