Gasdynamics: Fundamentals and Applications
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Lecture 33
Converging Nozzle and Chocking

So we are looking at varying area ducts and we have looked at their general behaviour how the flow to variable area ducts change. How do these ducts behave in different flow regime subsonic flow, supersonic flow they behave differently? And if you want to produce say a supersonic flow starting from a subsonic flow you need to give a convergent divergent duct. Similarly if you want to produce a subsonic flow starting with a supersonic flow again you have to give a converged divergent duct.

This minimum area point is important. So, various such concepts were introduced. Now let us go and start looking closer at nozzles and diffusers. And we will start with the converging nozzle and here we will come across a very important concept of mass flow rate choking. What do you mean by choking? And so other important point is all through these discussions we were looking only at effect of area ratio on various quantities particularly emphasizing on velocities and Mach number.

But we have to really see if you just put any nozzle or a diffuser it will not produce a certain flow in order to produce the flow you have to apply a certain pressure differential across it. Now this quite well known any student of fluid mechanics will know that pressure is what affects the flow changes. Now how are those pressure changes related to these changes in area.
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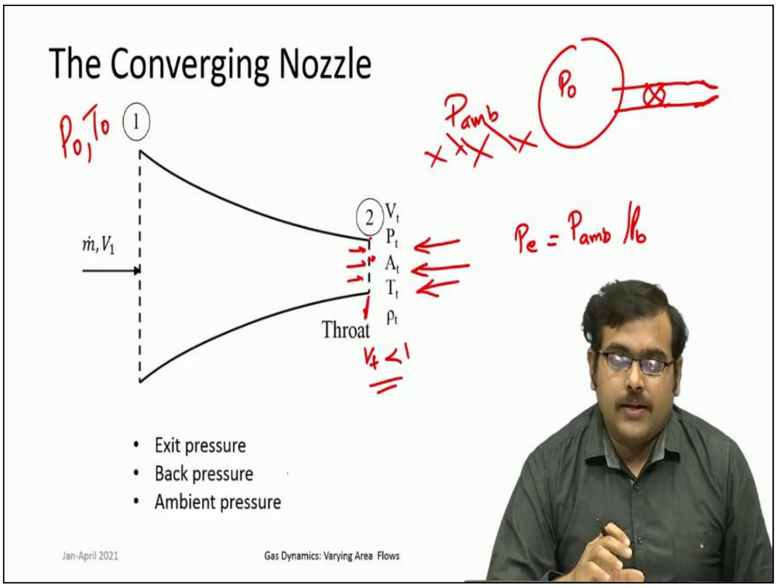

It is an isentropic flow but when the flow exhausts to certain ambient or back pressure then how does the flow behave? So these terminologies one has to understand so we are looking at convergent nozzles. So, converging nozzle or convergent nozzle, when we talk about a converging nozzle, you should understand by now that incoming flow is subsonic small velocities. And nozzle increases the flow so it increases to larger velocities.

A converging nozzle has a minimum area this is the minimum area here region 2. Now when we are looking along with pressure effects when we are seeing this together with pressure we should understand what we mean by various terminologies with respect to pressure? Now the flow coming out of such a nozzle can it can be attached to another device that is one kind of a system.

If that is so then at this point 2 it may be subjected to a certain pressure from the downstream device. So that kind of a pressure is known as back pressure or this particular it may be this particular duct may be opening out into an ambient or into a larger reservoir or ambient then pressure all around it that is getting imposed right all around it. The pressure is known as $P_{ambient}$ or $P_{ambient}$ pressure.

And pressure right at the exit of the nozzle in this case the minimum area is here. And very often the minimum area is also referred to as the throat. So that is also called another terminology. So at the throat in this case the throat is the exit. So, pressure of the flow at the exit at the throat that is exit pressures. Now there are various scenarios in the operation of nozzles when all these parameters need not always be the same.

So that is important and always there is a pressure that much higher pressure being provided at the inlet and usually you refer to these higher pressure in terms of the reservoir pressure $P_0 and T_0$, reservoir temperature. If the velocities at the inlet are very small then the static pressure and temperature are approximately equal to the stagnation pressure and temperature. But if they are not if they lie at higher subsonic velocities then you have to calculate either the stagnation pressure temperature or static pressure temperature to look at what is happening inside the duct.

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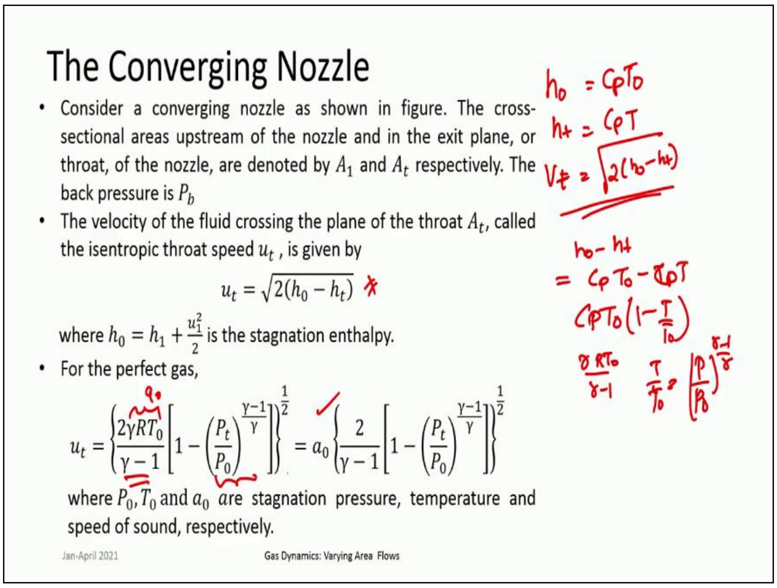

So, having understood these different terminologies about pressures let us go ahead and we want to express. We are mainly interested in what is happening to mass flow rate as it as you change pressure. So what we are looking at is initially if you consider that you have applied no pressure at all on to this converging nozzle. Then there will be no flow across the converging nozzle. So, velocity is 0 everywhere.

Now you start increasing pressure that is you have started increasing pressure at section 1 there is a pressure differential across the nozzle and flow starts begins and as a consequence you get increased velocity mass flow starts increasing you get higher and higher flows. It is very much similar to that you have a valve. So you have a large tank and then you have a duct you have a pipe and that pipe has a valve.

And then you have a nozzle over here initially you even though you have a larger pressure available to you the valve is closed and there is no flow. Slowly now the valve is opened this can is one of the scenario that you have then what happens. So this is what we are looking at. Now as long as the exit flow, the exit velocity V_t or which is less than 1 is subsonic. Then as long as there is a subsonic flow coming out or exit then the exit pressure P_e should be equal to the ambient pressure or back pressure.

So this is a very important principle in subsonic flows there cannot be any pressure differences between say back pressure or exit pressure because the information about pressure travels all through on all directions. But this situation is not true once you reach sonic conditions and go beyond sonic conditions. Because you already know from our earliest descriptions on flow regimes that in supersonic flows the information transfers only in a certain directions not in all directions.

So the exit knows about what is the ambient pressure or the back pressure as long as the flow is subsonic. But the moment the flow changes over and becomes sonic or supersonic then that information transfer gets cut off. So there are consequences of that how will that affect the flow here. So let us go ahead with that. So we are looking at now relating pressure ratios to what is happening in the nozzle.

So because of that we have to write all the equations that we had done earlier in terms of Mach number or velocities in terms of pressures. It can be done it is a isentropic flow the velocity at the exit. So given a stagnation enthalpy h_0 this is just nothing but $c_P T_0$ given this stagnation enthalpy and at the throat if the enthalpy is h_t the $c_P T$ then velocity can be found out at the throat V_t is nothing but $\sqrt{2(h_0 - h_t)}$.

This is directly from the energy equation you can get this. Now further on this can be expressed now $h_0 - h_t$ is you can express this as $c_P(T_0 - T)$, and taking $c_P T_0$ out this is $\left(1 - \frac{T}{T_0}\right)$ and c_P is $\frac{\gamma R}{\gamma - 1}$. So that is how this term $\frac{\gamma R T_0}{\gamma - 1}$ comes about. And it is an isentropic flow so $\frac{T}{T_0}$ can be expressed in terms of $\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$.

So that is how this term comes $\left(\frac{P_t}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$. You can also use the fact that $\sqrt{\gamma R T_0}\ is\ a_0$. So you get this particular equation. So here the velocity at the throat is now expressed in terms of pressure ratios. Earlier we were looking at area ratios.

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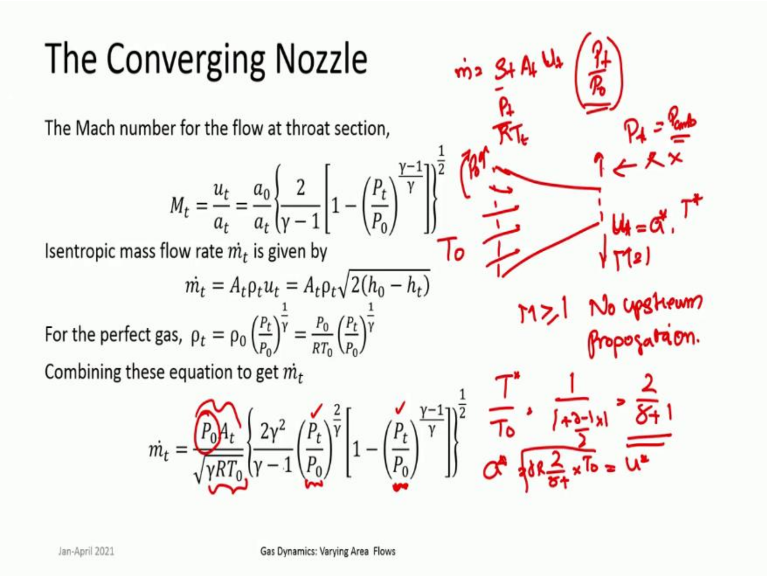

Now we will combine the two descriptions. So now you can write the mass flow rate at the throat $\dot{m} = \rho_{throat}A_{throat}V_{throat}$. And the density at the throat is $\frac{P_t}{RT_t}$ and this can be expressed in terms of $P_0\ \ and\ \ T_0$ this is something that we had done earlier also. So this $\frac{P_0 A_t}{\sqrt{\gamma R T_0}}$ this term has occurred before also.

And this term $\left(\frac{P_t}{P_0}\right)^{\frac{2}{\gamma}}$ comes from $\frac{\rho_t}{\rho_0}$. So how is $\frac{\rho_t}{\rho_0}$. expressed in terms of $\frac{P_t}{P_0}$. So here you see that the mass flow rate is expressed purely in terms of pressure ratios. So now let us see what happens. So you have a converging throat and now you are applying a certain; so in all these descriptions you find that pressure ratios are important.

So if you talk in terms of pressure ratio it is sufficient because pressure ratio is what figures in these equations. And pressure ratio is important absolute pressures of course you need to note them when you have to provide a particular pressure. But the performances of these devices are dependent on pressure ratios or temperature ratios or area ratios. So the ratio is very important. So if you can understand in terms of pressure ratios you can understand in general what happens to these flows.

So what you are doing is as you are increasing you can do two ways. So given that $\frac{P_t}{P_0}$. is what is important generally the P_t now this is exiting. So this is equal to ambient pressure initially $P_t = P_{amb}$ and you are increasing the stagnation pressure giving higher and higher pressures P_0 is rising. So $\frac{P_t}{P_0}$ decreases.

So your decreases it could you could also think of it that you have a high pressure at the upstream side and you are decreasing the ambient pressure. Both ways you can look at the problem as long as you understand it in terms of $\frac{P_t}{P_0}$ either way should help you understand the problem. So if you; now what we are looking at the problem is P_t that is P_{amb} is constant and we are increasing P_0.

So $\frac{P_t}{P_0}$ decreases. So as P_0 keeps increasing you can look at the mass flow rate equation here it has P_0 term over here. So P_0 as it increases mass flow through the system increases. So the mass flow through this system increases it can continue to increase and you also see there is $\frac{P_t}{P_0}$ term. As $\frac{P_t}{P_0}$ decreases you can look at what happens to velocity exit velocity.

T_0 is a constant $\frac{P_t}{P_0}$ decreasing implies $\left(1 - \frac{P_t}{P_0}\right)$ this value will increase that means u_t will increase. So your exit velocity will keep increasing. Now can this be done infinitely can you keep decreasing $\frac{P_t}{P_0}$ and forever will the velocity increase u_t and will mass flow rate forever increase to large values. Is this the question that we are trying to answer and the answer is no.

There is a certain limit, so if the $\frac{P_t}{P_0}$ approaches. So if this velocity at the exit u_t exactly becomes equal to a^* that is Mach number becomes equal to 1. When Mach number becomes equal to 1 then what happens is that the effect of downstream conditions whatever changes happens in the downstream will not travel upstream. This is a limitation imposed by flows which become where the Mach number becomes greater than equal to 1.

There is no upstream propagation of information. So a decrease in the pressure $\frac{P_t}{P_0}$ continuously increases u_t until the point that u_t becomes exactly equal to a^* or Mach number becomes equal to 1. Beyond that point any decrease in say if I keep P_0 constant and I decrease P_t beyond this particular point a^* there can be no change in velocity u_t at this particular point it will be equal to that of Mach 1.

So if you started off with an initial T_0 that is a temperature stagnation temperature you can find out what is the corresponding T^*. So $\frac{T^*}{T_0}$ this value is $\frac{1}{1+\frac{\gamma-1}{2}1^2}$ is $\frac{2}{\gamma+1}$. So this value is known and so correspondingly you can find what is a^* is $\frac{2}{\gamma+1}\gamma RT_0$. So square root of this velocity will be exactly equal to this and it will not change for; a given T_0.

So similarly the pressure at this exit P_e once it reaches Mach number equal to 1 the pressure will not change for a given P_0, $\frac{P^*}{P_0}$ is the same thing. So $\frac{P^*}{P_0}$ is $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$. So from here you can find out what is P^*. So, further changes will not change pressure temperature at the exit for constant P_0 and T_0.

That means if you look at and the Mach number is equal to 1. So if you go back and look at the mass flow rate relationship this all these becomes constant $\frac{P_t}{P_0}$ is $\frac{P^*}{P_0}$. And P_0 is constant T_0 is constant or in other terms $\dot{m}_t$ becomes constant.

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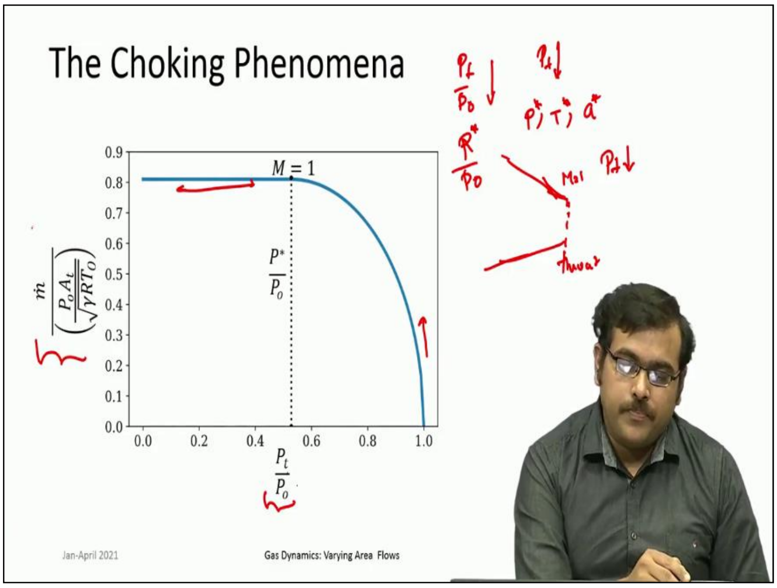

So if you do plot $\dot{m}_t$ versus $\frac{P_t}{P_0}$ this is what you observe that as you start decreasing $\frac{P_t}{P_0}$ from 1. So initial case you do not have you can take the entire thing to be at a constant pressure there is no flow happening. Once flow starts happening then $\frac{P_t}{P_0}$ will start reducing this can be either due to increase in P_0 or due to decrease in P_t it is valid in both case as long as you look at the ratio $\frac{P_t}{P_0}$.

So mass flow rate this is non dimensional mass flow rate normalized mass flow rate all other terms here are for a given constant P_0 and T_0. If you take constant P_0 and T_0 decrease of $\frac{P_t}{P_0}$ implies decrease implies P_t is decreasing or if the nozzle is exhausting into ambient or the back pressure or ambient pressure is decreasing.

So mass flow rate will increase more and more flow can now pass through the nozzle. It can continue to do that but only to a particular limit the moment that limit is reached which is $\frac{P^*}{P_0}$ the moment that limit is reached. Then from then on mass flow rate cannot increase because the exit velocity pressure and temperature becomes fixed at star values P^*, T^* and a^*.

Further decrease in pressure temperature downstream of the convergent nozzle if you continue to decrease P_t this decrease in pressure is not felt by the minimum area or the throat of the nozzle because the flow has become Mach number equal to 1 or it has become sonic. And upstream propagation of pressure is not possible. So any changes in downstream thus will not affect upstream.

It becomes the nozzle will operate in fixed conditions and mass flow rate becomes fixed. So this phenomena is known as the choking phenomena, mass flow rate becomes choked. This is something important in the context of variable area ducts. So one has to bear in mind what you mean by choking phenomena. And how is pressure ratio related to choking phenomena and when does this particular condition when is it achieved in the case of; here we are discussing the convergent nozzle.

But we will see this, this is actually true in other cases in other nozzles also. Now if you look at an increase in P_0. So we had looked at we were focusing on $\frac{P_t}{P_0}$ and this is normalized mass flow rate. Normalized mass flow rate of course will become constant.

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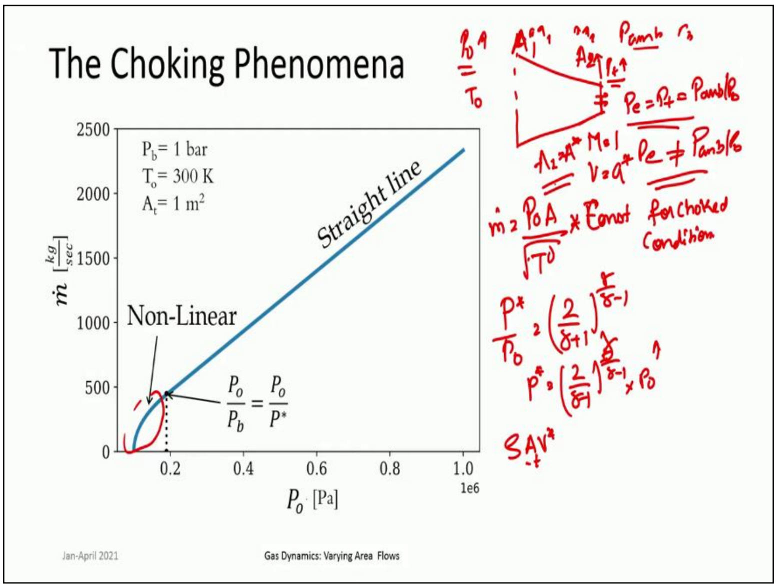

But what about absolute mass flow rate when you consider the case that the; convergent nozzle is there. Ambient pressure is held constant the ambient pressure is held constant and P_0 is kept on increasing. So if you consider this case as this particular case then as P_0 increases initially you have the mass flow rate increasing in a non-linear fashion. But once Mach number 1 is achieved that is Mach number becomes equal $\frac{P_0}{P_t}$ becomes equal to $\frac{P_0}{P^*}$.

So once that particular point is achieved then from then on Mach number at the exit or the throat cannot increase beyond 1. So Mach number will be fixed at 1. So if you have a constant T_0. So T_0 remains constant then velocity is also fixed V is fixed at a^*. But mass flow rate is not constant because mass flow rate if you look at the relationship it is $\frac{P_0 A}{\sqrt{T_0}}$ and some constant for conditions beyond some constant for choked conditions because Mach number is 1, so $\dot{m} \propto P_0$.

So it is a linear relationships so m dot will keep increasing as you increase P_0 given for a given fixed ambient condition or back pressure condition. So this you have to understand even though you understanding in terms of pressure ratio is useful to look at a general system. But specifically when you are looking at increase in P_0 and how does mass flow rate increase. Initially it will increase in a non-linear fashion but after achieving choking condition.

Choking condition is that Mach number equal to 1 at the exit of the convergent nozzle. If you continue to increase P_0 mass flow rate will continue to increase. The reason is that $\frac{P^*}{P_0}$ is a constant so $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$. Now this one is so P^* is now going to go as P star is $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} P_0$.

But P_0 is continuously increasing that means exit pressure keeps on increasing P_t keeps increasing. P_t keeps increasing density keeps increasing so ρAV^*, A is fixed A_t. So density is increasing mass flow rate will increase. So bear in mind about the relationships of the pressure ratios across these devices convergent devices or a variable area ducts. The relationship of the pressure ratio with mass flow rate and relationship of pressure ratio with velocities at the exit this has to be clearly borne in mind.

As long as you have a subsonic flow that is coming out at the exit P_e or equal to P_t is equal to P_{amb} or P_b back pressure. This boundary condition is correct because in subsonic flow the flow can always communicate upstream or downstream it will inform the nozzle the exit of the nozzle that see the exit pressure is as such and such value. And nozzle will adjust the mass flow rate in order to satisfy the pressure condition.

But once it has arrives it achieves Mach 1 the ambient pressure or the back pressure is never felt by the nozzle because it has achieved Mach 1. As a consequence of that the back pressure or the exit pressure of the nozzle is not necessarily equal to ambient pressure or back pressure. So this condition has to be understood then how do we calculate exit pressure? You know P_0 you know the area ratio for this $\frac{A_2}{A_1}$.

If you know $\frac{A_2}{A_1}$ you know P_0 you can find out the relationship between what is $M_2 \, and \, M_1$ if you know $\frac{M_2}{M_1}$, P_0 is a constant you can find what is the Mach number. If it is a convergent nozzle Mach number beyond chocking is always one here. So this A_2 becomes equal to A^* so that is the important point here.

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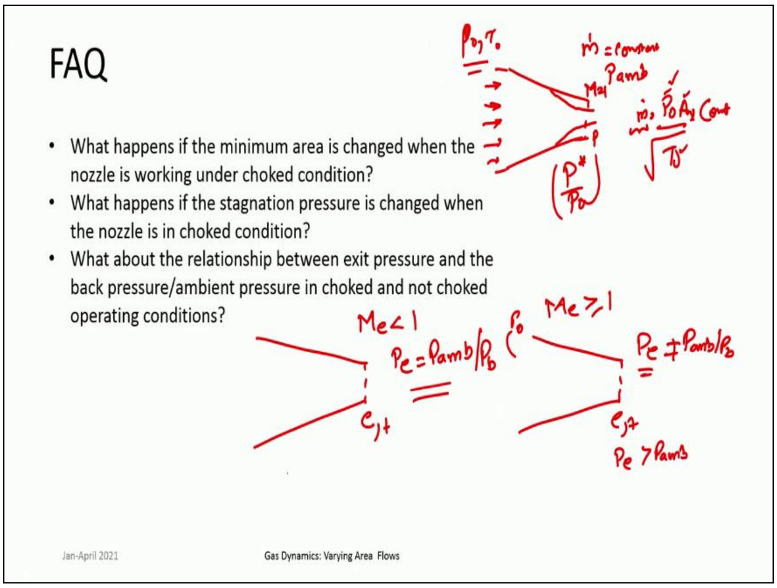

So often a lot of questions are asked because it is a new concept something that is confusing somewhat you have to understand this very closely. A lot of things that come into a picture here you have pressure ratios you have mass flow rate you have velocity all of them are changing together. So some questions that are usually asked when we do this particular section is suppose the nozzle is working in choked condition.

It is having a certain minimum area so this certain minimum area is there nozzle is working in choke condition. Nozzle working in choke condition with constant upstream pressures T_0, P_0 all these are constant that means then mass flow rate is constant is fixed. It does not change if you change the back pressure over here. Now what happens if suddenly the minimum area is changed?

This may happen in some engineering devices but it is certainly a good thought experiment to do. Of course, you have variable throat nozzles and devices of that kind. So what happens then is that provided that you are having the correct pressure differential in order to sustain choke conditions. So if you have sufficiently high pressure for given ambient conditions so that you have you are sure that $\frac{P^*}{P_0}$ is satisfied.

That means at all conditions this Mach number will be equal to 1. Then the moment the area is reduced then the Mach number should sorry the mass flow rate $\dot{m} = \frac{P_0 A}{\sqrt{T_0}}$ multiplied by some constant in choked condition $P_0\ is\ constant, T_0\ is\ constant$ but you decrease A that means mass flow rate has to decrease. And how does this decrease it will affect the upstream whatever velocity it was coming at initially that will decrease.

So automatically mass flow rate will decrease. So the other thing is what happens when stagnation pressure is changed when nozzle is in choked condition. So when you change stagnation pressure and nozzle is working then this convergent nozzle is working in choke condition then mass flow rate will increase. Again, you can go back to this particular form P_0 increases mass flow rate increases not only mass flow rate increases the pressure and density at exit will also increase.

And that need not be the same as P_{amb}. What about the relationship between exit pressure back pressure ambient pressure and choked and not choked operating conditions. So this is what we have been explaining till now that as long as your, nozzle or this convergent duct is operating such that $M_{exit} < 1$. This is the exit which, is also the throat.

As long as it is doing that $P_e = P_{amb} = P_{back}$ whatever is provided. This it satisfies this conditions the mass flow rate will adjust accordingly so that the this condition is satisfied. But the moment $M_exit \geq 1$, if you give the correct pressure ratio across the nozzle then $P_e \neq P_{amb} or P_{back}$. They need not be the same, what is P_e you have to calculate by isentropic relations if you are increasing P_0 then generally P_0, P_e will become greater than P_{amb}.

Then the nozzle can has enough pressure to expand further out of the nozzle. So it will expand out of the nozzle such kind of expansion is called under expansion and it will increase velocity significantly outside the nozzle. But we are talking about flows at the exit of nozzle and in the nozzle. So flow having done expansion outside we will have a little bit discussion in the next class about that.

So in next class we will see, so now we have been looking at converging nozzles we understood the concept of mass flow rate choking. And also, how pressures behave and because of pressure ratios how that nozzle behaves due to these changes for a convergent nozzle. Now we will go and look at convergent divergent nozzles how things change in convergent divergence? So, thank you.