Gasdynamics: Fundamentals and Applications
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Lecture 31
Varying Area Duct Flows- I

So in this module we look at flows in varying area ducts. A lot of engineering devices like nozzles diffusers wind tunnels all of them use these principles extensively. And so, whatever we had learnt in the previous classes for certain specific flow features like normal shocks, oblique shocks, expansion waves, their particular flow features. Now we carry all of them with us as we move into this module.

Also concepts like stagnation pressure, stagnation temperature and importantly the concept or the assumption of the quasi-1D flow. That is, flows that we assume that flow properties remain uniform across the cross section. So, with those assumptions we look at varying area flows. When we consider varying area duct flows, we are not considering any effects of friction or any other effects like heat addition or mass addition.

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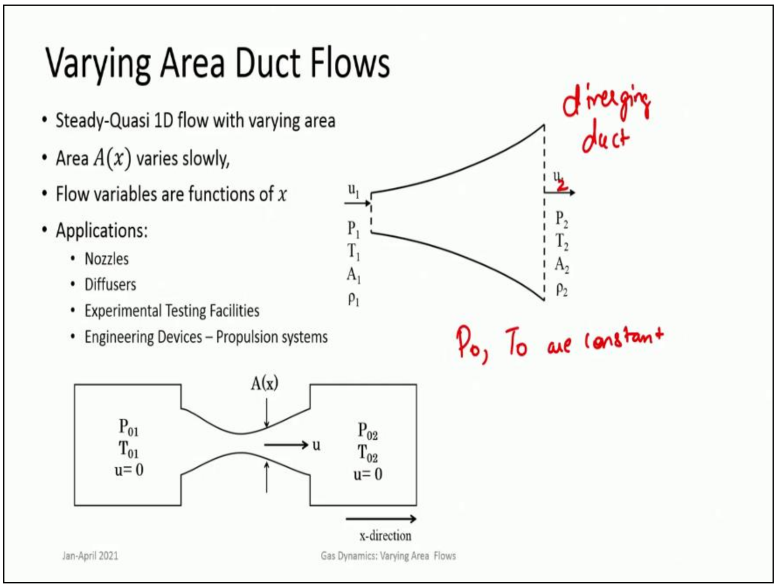

We are only considering that the area of the duct is changing. For quasi-1D assumption to hold good, the idea is that the area changes slowly it is not a sudden expansion or a sudden contraction. Typical change in area; so, a gradual change in area is represented in this figure for example where area slowly increases. So, terminology is often used this is a diverging duct area diverges, so diverging duct.

So, we will come to these terminologies as we go on. So we are interested in how what happens to flow as it goes starting, coming from incoming flow and as it passes through a change in area and what happens to the exit flow so this is $u2\ and\ p2$ so all properties are there. So $A1$ is changing to $A2$. Now since there are no frictional forces or any heat addition or any such entropy generating mechanisms the flow is ideal and so the flow is isentropic.

So, the entropy remains constant in these kind of flows the analysis done over here though we use quasi-1D assumption it is a very good representation of the flows that actually take place in nozzles. Certain 3-dimensional effects or certain losses are always present they are usually in engineering system they are accounted in terms of using a loss coefficient or an efficiency factor.

But we will take ideal flows. So, the main understanding that we have to get is how do compressible flow behave as it goes through varying areas. In this particular module you are actually clubbing several concepts you have been learning over the previous several modules. So, we will also come across certain counterintuitive flow behaviour that flows which is behaviour as it moves from subsonic flow to supersonic flow.

Other interesting flow phenomenon that occur is what is known as choking or mass flow rate choking. So, these principles are very much relevant for applications in nozzles, diffusers, propulsion systems and testing facilities. So, they are used extensively and so this is the typical kind of flow picture that you have to bear in mind that you are actually talking about a varying area duct.

And since the flow is going to be is isentropic the stagnation properties remain constant throughout the duct so $P_0\ and\ T_0$ are constant. Of course, this is a case when there are no shocks within the ducts. We will also consider cases when shocks can occur within such ducts.
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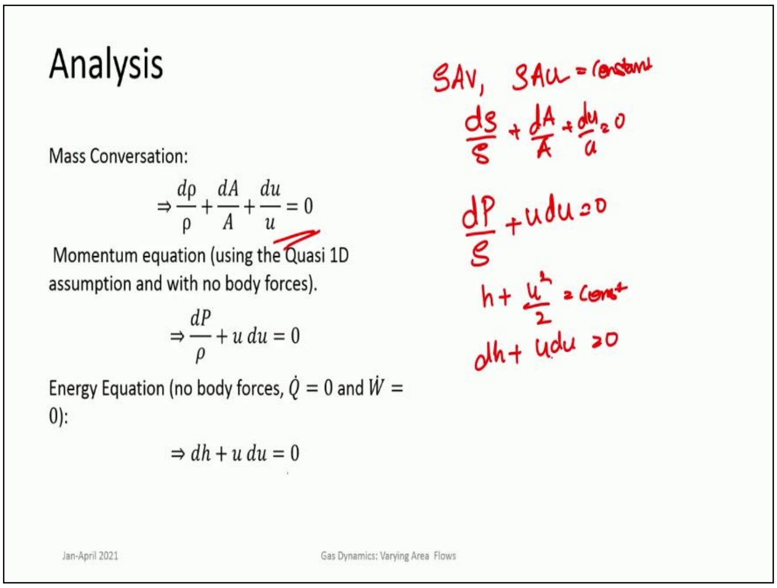

So let us go ahead so how do we analyse these flows. Again principles are the same - do the analysis of conservation equations. Mass, momentum and energy. The mass conservation or continuity equation $\rho A\ V$ or in this case $\rho A\ u$ is the terminology used here, is constant and you can differentiate this use a logarithm and differentiate $\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{du}{u} = 0$ that is the equation that comes here.

Momentum equation without any, body forces or frictional forces completely inviscid and using quasi-1D and the assumption it gives the Euler equation $\frac{dP}{\rho} + udu = 0$ we had done this in the initial classes. So, in case you need to refresh you can go back and check and there are no heat addition no work done. So total energy remains constant $h + \frac{u^2}{2} = constant$. So total enthalpy constant $dh\ +\ u\ du\ =\ 0$.

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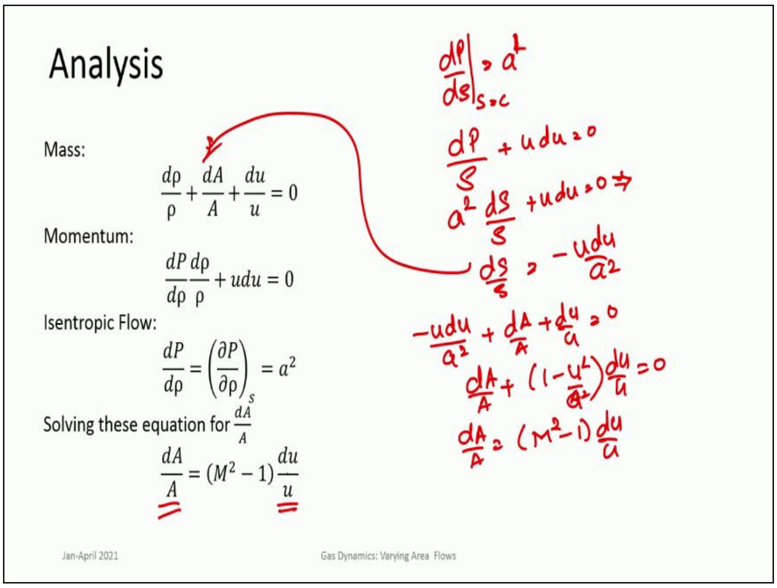

Now since the flow is isentropic, you also have the equation $\left(\frac{dP}{d\rho}\right)_{s=c} = a^2$, so this is the equation for speed of sound and an isentropic flow this is true $s = constant$. So, in the momentum equation $\frac{dP}{\rho} + udu = 0$, one can replace the dP with $a^2(d\rho)$, so this becomes $a^2\frac{d\rho}{\rho} + udu = 0$. Now with this equation from here you can get an equation for $\frac{d\rho}{\rho} = -\frac{udu}{a^2}$.

And that can be substituted here in the mass conservation equation. So, you get $\frac{-udu}{a^2} + \frac{dA}{A} + \frac{du}{u} = 0$. So, now taking $\frac{du}{u}$ as common here you get $\frac{dA}{A} + \left(1 - \frac{u^2}{a^2}\right)\frac{du}{u} = 0$ or $\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$. So, this relates the change in velocity of the flow to changes in area. So, this changes in area. So here the changes in area are important.

So it is like change in area is like a forcing function it changes the flow. And what we are interested in is how does area change affect the flow? So, this is what we are interested in.

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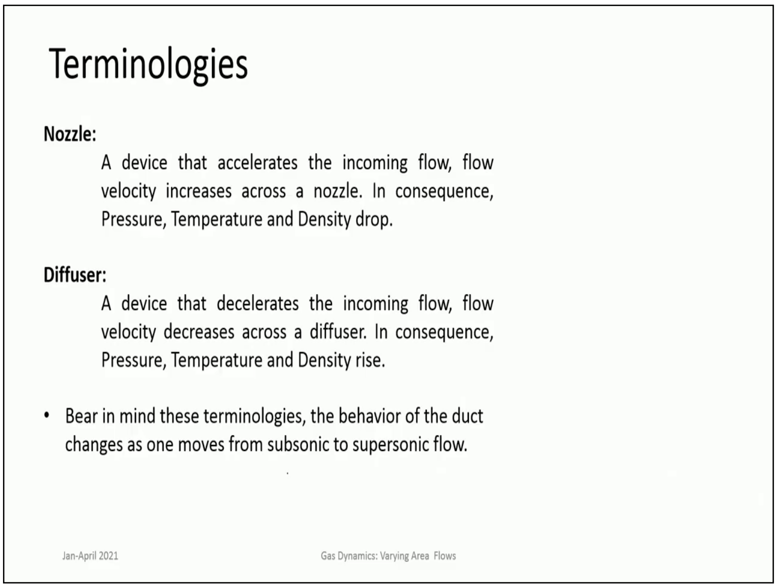

So, now let us come to certain terminologies their definitions and because this has to be understood really well. We will soon see how lot of counter intuitive behaviour will occur in compressible flow through varying area ducts. So, keep in mind these definitions.

Nozzle: a nozzle is a device which accelerates the incoming flow that means velocity has to increase.

Consequence is that pressure, temperature ad density they drop it is in a compressible flow. So, as velocity increases pressure, temperature and density they decrease. While in diffuser the opposite happens that is the incoming flow is decelerated velocity decreases across a diffuser pressure, temperature and density increase. So, this terminology distinction between nozzle and diffuser has to be borne in mind.

The way a duct behaves entirely depends on what happens to velocity as it flows through the duct. If it increases, then it is a nozzle. So, just by looking at the shape of the duct one cannot conclude whether it is a nozzle or diffuser this is a fact that you have to understand when we go through this varying area duct flows. So, things change as you move from subsonic to supersonic flows. So, let us look at this equation once again $\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$.

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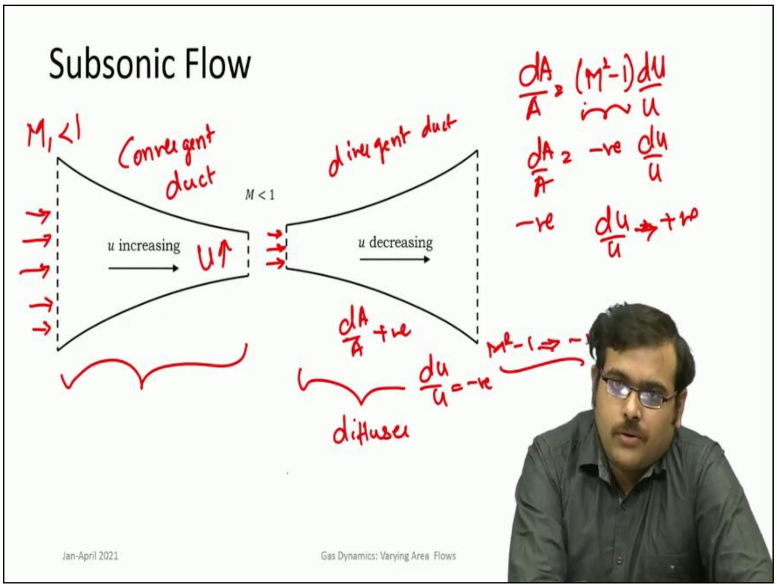

So now we have the two kinds of ducts over here and we are considering the incoming flow is subsonic. So subsonic flow incoming flow $M1\ < 1$ and the guiding equation for us here is $\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$. So, in incoming flow is subsonic. So we are having changes in area in a subsonic flow $M1\ < 1$ that means what happens to this sign; the sign of this is negative.

So, $\frac{dA}{A}$ is related to some negative sign $\frac{du}{u}$. Now let us check what happens? So, this particular duct that we are considering here its area decreases. So, area is decreasing. So, it is a convergent duct. So, area decreases so what about the sign of $\frac{dA}{A}$ area is decreasing so this is also negative. So, if you consider this in totality what about $\frac{du}{u}$? The sign of $\frac{du}{u}$ is positive. $\frac{du}{u}$ sign is positive that means velocity increases.

So, a convergent duct with an incoming flow subsonic incoming flow increases velocity. So, u increases, so a convergent duct in subsonic flow behaves like a nozzle. So, when we talk about convergent nozzles then the understanding that you should carry is the incoming flow is subsonic. So, a nozzle should always accelerate the flow and if the duct is convergent then the flow can be accelerated only if the incoming flow is subsonic.

So that is directly seen from these relations here. Now let us see the other case that you have a divergent duct. Here area is increasing so $\frac{dA}{A}$ is positive incoming flow is again subsonic. So, this is a subsonic flow and what happens to $\frac{du}{u}$. So, $\frac{dA}{A}$ is positive $(M^2 - 1)$ is negative. So, if this has to be satisfied $\frac{du}{u}$ should be negative. You can look at the equation and conclude from there this just looking at signs we are trying to understand in qualitative terms what happens to flow as it enters different kinds of ducts.

So, when area increases and incoming flow is subsonic, then velocity decreases or the flow behaves as a diffuser. So, this is a diffuser. So, you can have terminologies like a diverging diffuser, subsonic diffuser. A subsonic diffuser is a diverging diffuser so a diverging diffuser will always work in subsonic flows. So, these concepts you have to remember very carefully because once we move to the supersonic flow you can see something changes.

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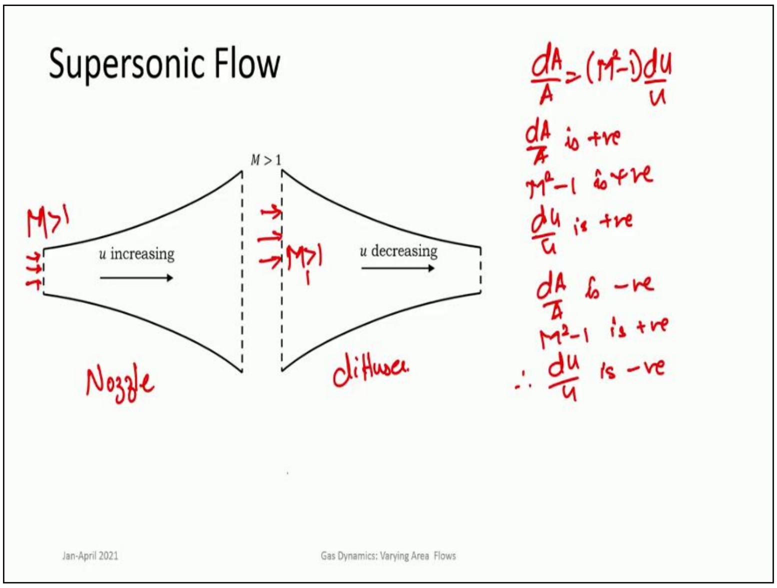

So let us go into supersonic flow. Again, our guiding equation is $\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$, so now again, we have the; same two kinds of ducts. So first we will consider the divergent duct here divergent duct means $\frac{dA}{A}$ is positive and $M > 1$ so $(M^2 - 1)$ is positive this can occur when $\frac{du}{u}$ is positive that means now you can see that here Mach number entering the duct is greater than one.

This is a divergent duct. So area is increasing and you find that velocity is increasing. So this is a nozzle. So now can you understand that whatever we have been talking about counter intuitive behaviour that is flow is switching the way a variable area forcing is working as it moves from subsonic to supersonic flow. We just discussed the subsonic flow in subsonic flow.

A divergent duct is a diffuser it decreases velocity but in a supersonic flow or when the incoming flow is supersonic then a diverging duct increases velocity it behaves like a nozzle.

Now we look at the convergent duct. So here $\frac{dA}{A}$ is negative $(M^2 - 1)$ is positive therefore $\frac{du}{u}$ is negative. So the convergent duct in incoming flow is greater than 1, $M > 1$ supersonic flow behaves as a diffuser. So this is something that you have to carry with you all the time in further discussions here on that what you call as nozzle or a diffuser the shape of the area or the shape of the device is completely dependent on the flow regime that it needs to work in.

So if you are looking at nozzles then nozzles are convergent ducts in subsonic flow but in supersonic flow they are divergent ducts that if the incoming flow is supersonic you have to further increase the velocity or Mach number then you have to use divergent ducts. Similarly about diffusers, diffusers decrease velocity and if the incoming flow is subsonic and you have to decrease velocity you should use a divergent duct.

But if the incoming flow is supersonic and you have to decrease velocity then the duct should be convergent. So this is something that you have to really understand and bear in mind from here on.

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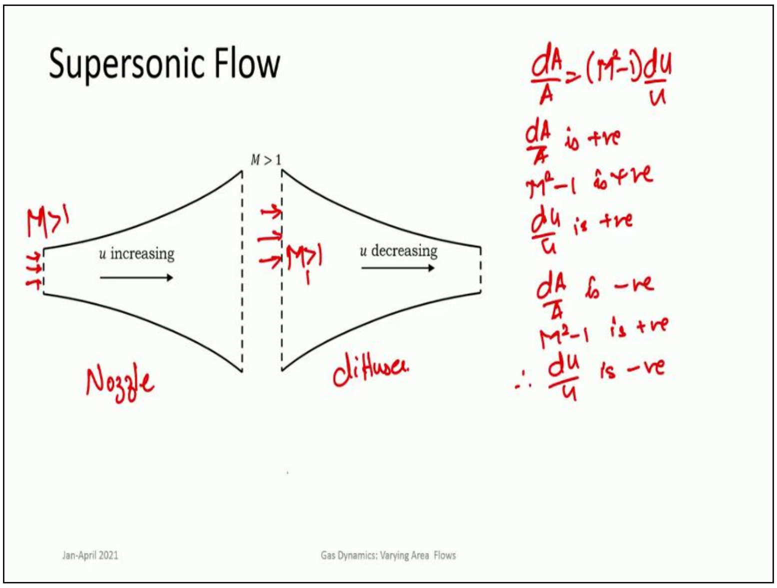

What happens when Mach number is exactly equal to 1 so $\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$ and when $M^2 = 1$ you get $\frac{dA}{A} = 0$. So this implies that the area reaches either minimum or maximum. So, when you reach Mach number equal to 1 or you reach the sonic point in a varying area duct then it reaches either minimum or maximum. In actual flows we will soon see we look at the equations and it will be clearly shown that the solution is always that area approaches a minimum when Mach number goes to one.

So, you have convergent divergent systems. So, a convergence system attaching to a diverging system Mach number will reach one at the minimum area. So, if you are looking to convert a subsonic flow to a supersonic flow or a supersonic flow to subsonic flow then you need combination of ducts. If you are accelerating then you have a convergent duct from a subsonic flow it accelerates all the way to minimum area where it becomes sonic and then it further if you want to accelerate the flow you have to provide a divergent duct.

So a C-D duct that is what is all sometimes word C-D, convergent divergent duct. Other terminology is used for nozzles especially its De-laval nozzle. De-laval is a convergent divergent nozzle. The same is for diffusers a convergent diffuser a convergent duct with an incoming supersonic flow first decreases the velocity it can decrease till a minimum area where it should approach Mach number equal to 1.

Further on if you want to decrease area decrease velocity you should from the; behaviour of ducts that we discussed just now the area should increase. So, a C-D duct a convergent divergent duct behaves both as a nozzle and a diffuser. So, it depends on what boundary conditions you give for the duct.

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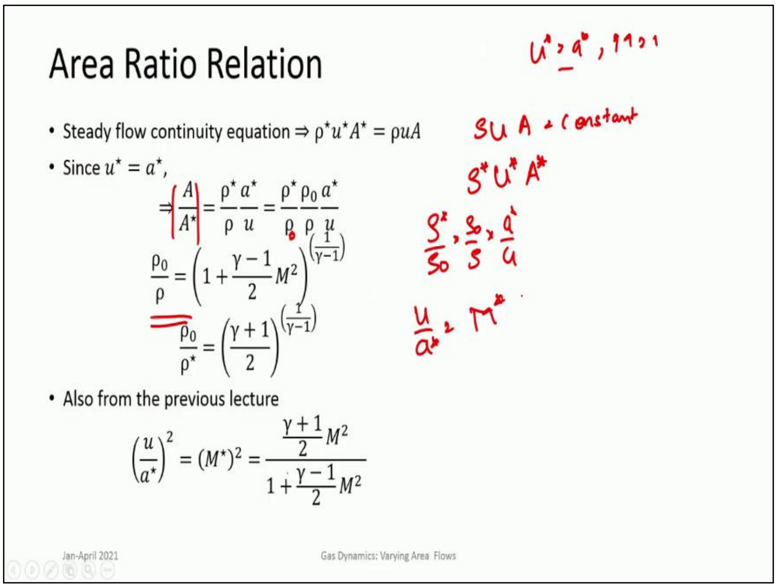

So that is qualitative that is in sense of an understanding of what do you mean by a varying area duct. How does flow behave? We saw that they behave differently in subsonic flow and supersonic flow. Now we need to look at - are there relations with which we can sort of do some analysis and we want to find out what should be the areas to provide a certain change in velocity or change in Mach number - can we do it?

So for that, approach is straight forward you have to take the continuity equation $\rho A\, u = constant$ and a good reference point; always is good to have a reference point and the good reference point is the point at which Mach number goes to 1 or u becomes equal to a^*. So $u^* = a^* or\ M = 1$. So that is given star values u^*, a^* and the area at which it happens is A^*. So, these two are equal mass flow does not change mass flow rate is constant.

So, now given that; you can write down $\frac{A}{A^*} = \left(\frac{\rho^*}{\rho}\right)\left(\frac{u^*}{u}\right)$. Now this is an isentropic flow. So, $\left(\frac{\rho^*}{\rho_0}\right)\left(\frac{\rho_0}{\rho}\right)\left(\frac{a^*}{u}\right)$. So, it is $\frac{\rho_0}{\rho}$ the equation is from isentropic relation $\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}$.

So $\frac{\rho_0}{\rho^*}$ is you have just put M equal to 1 here : $\left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}$ and $\frac{u}{a^*} = M^*$ which was also defined previously and this is the relation for M^*.

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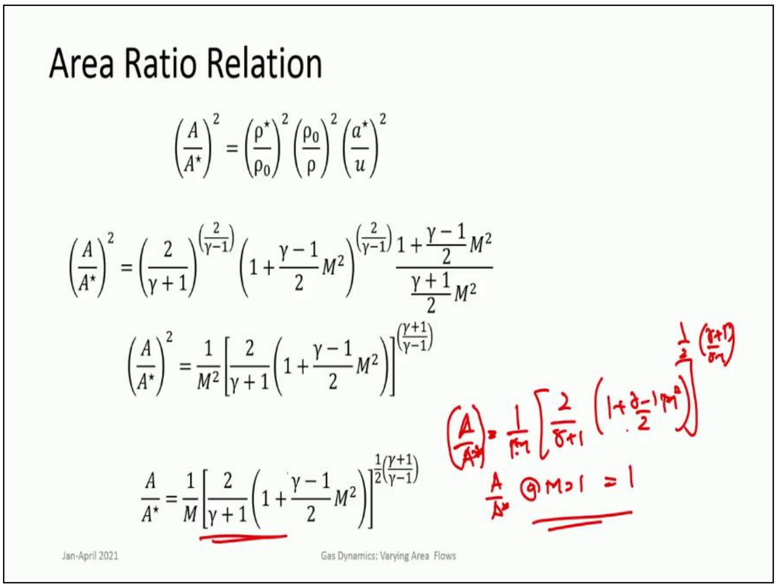

Now you can combine all of them together $\left(\frac{A}{A^*}\right)^2 = \left(\frac{\rho^*}{\rho_0}\right)^2\left(\frac{\rho_0}{\rho}\right)^2\left(\frac{a^*}{u}\right)^2$. So this is M^* put the numbers and do the algebra for this and ultimately this is the equation

$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$. So this is $\frac{A}{A^*}$. This is a very important term $\frac{A}{A^*}$, this is again plotted in isentropic charts $\frac{A}{A^*}$ is given.

For every Mach number you can find an $\frac{A}{A^*}$ when Mach number is equal to 1 this is one. So, if you put one here this is $\frac{\gamma+1}{2}$ gamma + 1 by 2, $\frac{2}{\gamma+1}$ multiplied by $\frac{\gamma+1}{2}$, $\frac{A}{A^*}$ at $M = 1$ is equal to 1. So that is with the reference area as A^*. So, this is correct. So, you will get $\frac{A}{A^*}$ as 1 at $M = 1$. So now in the next class we will see further how this relation varies what are the characteristics of this relation?